CME306 / CS205B Homework 6

Essentially Non-Oscillatory Schemes

Given the following data for ϕ^n, write down the interpolating polynomial that third order HJ ENO would construct in order to compute ϕ_i^{n+1} in approximating the equation $\phi_t + \phi_x = 0$.

$$\phi_{i-3}^n = 5, \phi_{i-2}^n = 5, \phi_{i-1}^n = 4, \phi_i^n = 5, \phi_{i+1}^n = 1, \phi_{i+2}^n = -2, \phi_{i+3}^n = 0$$

Weighted ENO

If we consider an upwind discretization of ϕ_x, we have three possible third-order interpolating polynomials, given by

$$\phi_x^1 = \frac{v_1}{3} - \frac{7v_2}{6} + \frac{11v_3}{6}$$
$$\phi_x^2 = -\frac{v_2}{6} + \frac{5v_3}{6} + \frac{v_4}{3}$$
$$\phi_x^3 = \frac{v_3}{3} + \frac{5v_4}{6} - \frac{v_5}{6}$$

Where $v_j = D^*\phi_{i+j-3}$, and $D^*\phi$ is the first-order upwind discretization of ϕ_x.

However, the philosophy of picking exactly one of the three candidate stencils is overkill in smooth regions of ϕ where ϕ is well-behaved. Instead, we can take a convex sum of the three stencils,

$$\phi_x = \omega_1\phi_x^1 + \omega_2\phi_x^2 + \omega_3\phi_x^3 \quad (1)$$

Where $0 \le \omega_i \le 1$, $\omega_1 + \omega_2 + \omega_3 = 1$. It has been shown that we can pick $\omega_1 = .1, \omega_2 = .6, \omega_3 = .3$ and achieve a 5^{th} order accurate approximation of ϕ_x.

1. Show that if we perturb ω by $\mathcal{O}(\Delta x^2)$ we still get a 5^{th} order approximation to ϕ_x.

2. Why is this a bad idea in non-smooth areas of the flow? In order to demonstrate this, consider $\phi_t + \phi_x = 0$ for a heaviside step function, with initial data given by:

$$\phi_{i-3}^n = 0, \phi_{i-2}^n = 0, \phi_{i-1}^n = 0, \phi_i^n = 1, \phi_{i+1}^n = 1, \phi_{i+2}^n = 1, \phi_{i+3}^n = 1$$