CME306 / CS205B Homework 2

Rotation Matrices

1. The ODE that describes rigid body evolution is given by $R' = w^\star R$.

   (a) Write down the forward Euler update for this equation.

   (b) Show that the updated "rotation" matrix computed from this update is not orthogonal (you may assume that R^n *is* orthogonal).

   (c) How might one "fix" this matrix that is almost a rotation to make it rotation?

2. Another update equation is $R^{n+1} = e^{\Delta t(\omega^n)^\star} R^n$. Here, $\omega^\star$ is the cross product matrix, where $\omega^\star v = \omega \times v$. The exponential map e^A can be defined for square matrices using the taylor series expansion

$$e^{\Delta t A} = I + \Delta t A + \frac{\Delta t^2}{2} A^2 + \frac{\Delta t^3}{6} A^3 + ...$$

   (a) Show that this update equation is a first order approximation of the ODE $R' = w^\star R$ that it is meant to solve.

   (b) Show that the result of this update is orthogonal.

   (c) Find a closed form expression for this update rule. *(Hint: the cross-product matrix is closed under addition; i.e.* $(\omega^\star)^3 = -|\omega|^2 \omega^\star$*... and don't forget your trigonometric identities!)*

   (d) Give an intuitive description of what this update rule is doing.

Modified Equations

Consider the advection equation

$$u_t + au_x = 0.$$

The numerical methods below satisfy *modified equations* to higher order than the advection equation itself. See Leveque §8.6 and the discussion notes for more on modified equations. You should assume $\lambda = \frac{\Delta t}{\Delta x}$ is a constant.

A. Find a modified equation for which explicit central differencing gives an $\mathcal{O}(\Delta t^2)$ approximation (we never want more than first-order derivatives in time, so if your solution contains u_{tt}, convert them to spatial derivatives). What modification to the explicit central differencing scheme does this suggest to make it a stable numerical scheme for the advection equation?

B. Find a modified equation for which your proposed method from A gives an $\mathcal{O}(\Delta t^3)$ approximation.