Lecture 7 - SISO Loop Design

- Design approaches, given specs
- Loopshaping: in-band and out-of-band specs
- Design example
- Fundamental design limitations for the loop
  - Frequency domain limitations
  - Structural design limitations
  - Engineering design limitations

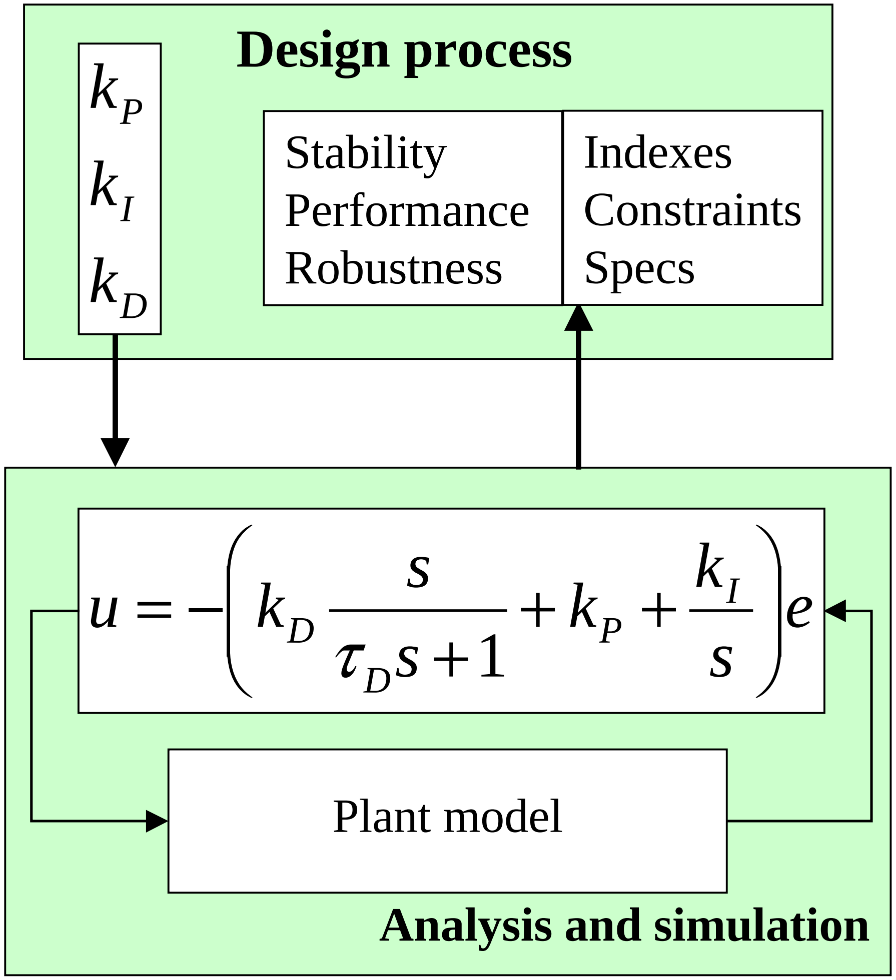
Modern control design

- Observable and controllable system
  - Can put poles anywhere
  - Can drive state anywhere
  - Can design ‘optimal control’

- Issues
  - Large control
  - Error peaking in the transient
  - Noise amplification
  - Poor robustness, margins
  - Engineering trade off vs. a single optimality index

Feedback controller design

- Conflicting requirements
- Engineers look for a reasonable trade-off
  - Educated guess, trial and error controller parameter choice
  - Optimization, if the performance is really important
    - optimality parameters are used as tuning handles



Design process

k_P
k_I
k_D

Stability
Performance
Robustness

Indexes
Constraints
Specs

$$u = -\left(k_D \frac{s}{\tau_D s + 1} + k_P + \frac{k_I}{s} \right) e$$

Plant model

Analysis and simulation

Loopshape requirements

$$L(i\omega) = P(i\omega)C(i\omega)$$

Performance

$$S(i\omega) = [1 + L(i\omega)]^{-1}$$

- Disturbance rejection and reference tracking
  - $|S(i\omega)| \ll 1$ for the disturbance d; $|P(i\omega)S(i\omega)| \ll 1$ for the load v
  - **satisfied for $|L(i\omega)| \gg 1$**
- Noise rejection
  - $|T(i\omega)| = |S(i\omega)L(i\omega)| < 1$ is Ok unless $|1 + L(i\omega)|$ is small
- Limited control effort
  - $|C(i\omega)\, S(i\omega)| < 1$
  - works out with large $|C(i\omega)|$ for low frequency, where $|P(i\omega)| > 1$

Loopshape requirements

Robustness

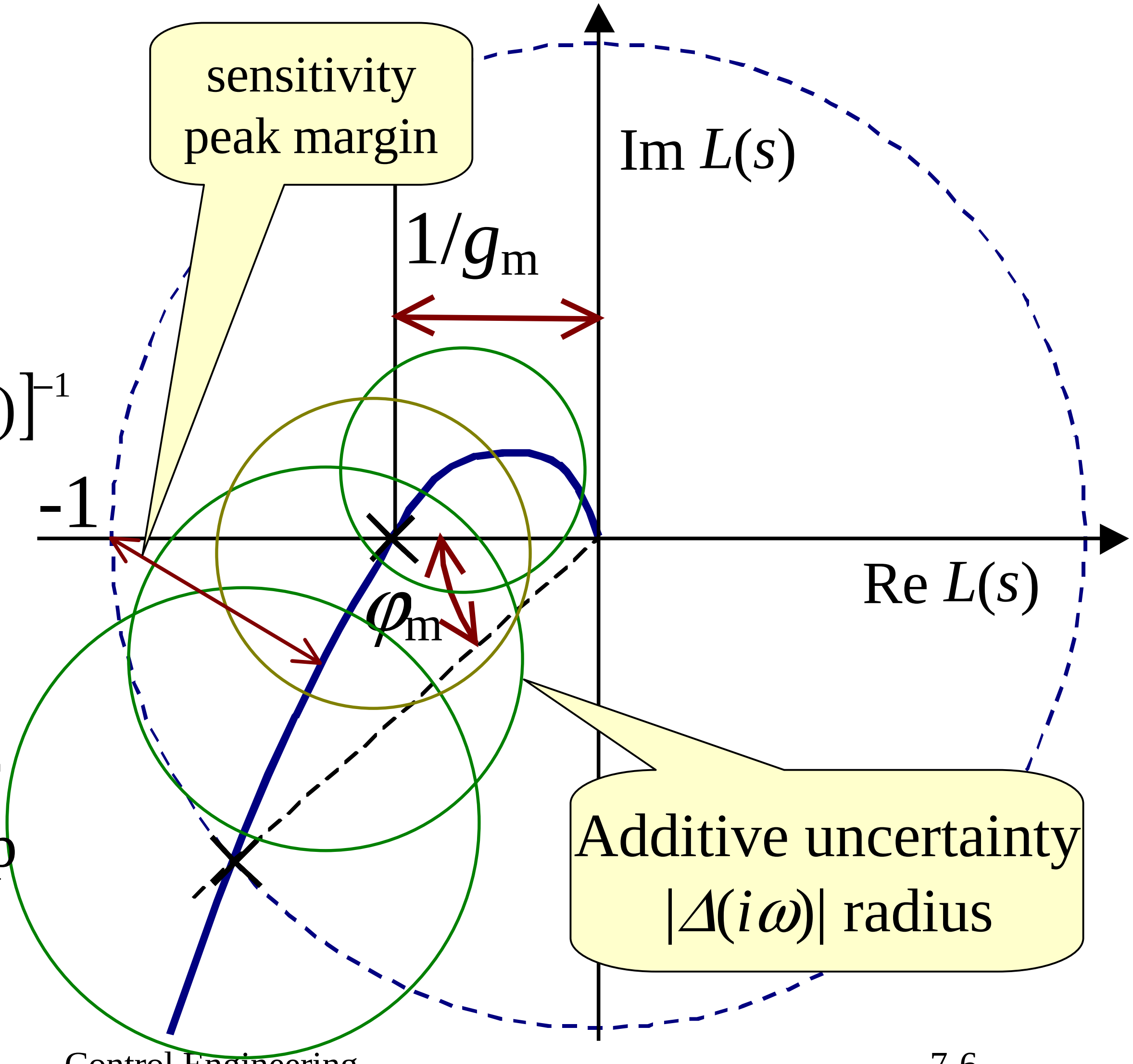
- Multiplicative uncertainty
  - $|T(i\omega)| < 1/\delta(\omega)$, where $\delta(\omega)$ is the uncertainty magnitude
  - at high frequencies, relative uncertainty can be large, hence, $|T(i\omega)|$ must be kept small
  - **must have $|L(i\omega)| \ll 1$ for high frequency, where $\delta(\omega)$ is large**
- Additive uncertainty
  - $|C(i\omega)\, S(i\omega)| < 1/\delta(\omega)$, where $\delta(\omega)$ is the uncertainty magnitude
- Gain margin of 10-12db and phase margin of 45-50 deg
  - this corresponds to the relative uncertainty of the plant transfer function in the 60-80% range around the crossover

Gain and phase margins

- Are less informative than the noise sensitivity

$$S_u(s) = C(s)\left[1 + P(s)C(s)\right]^{-1}$$

- Can use uncertainty characterization and the sensitivity instead
- Margins are useful for deciding upon the loop shape modifications

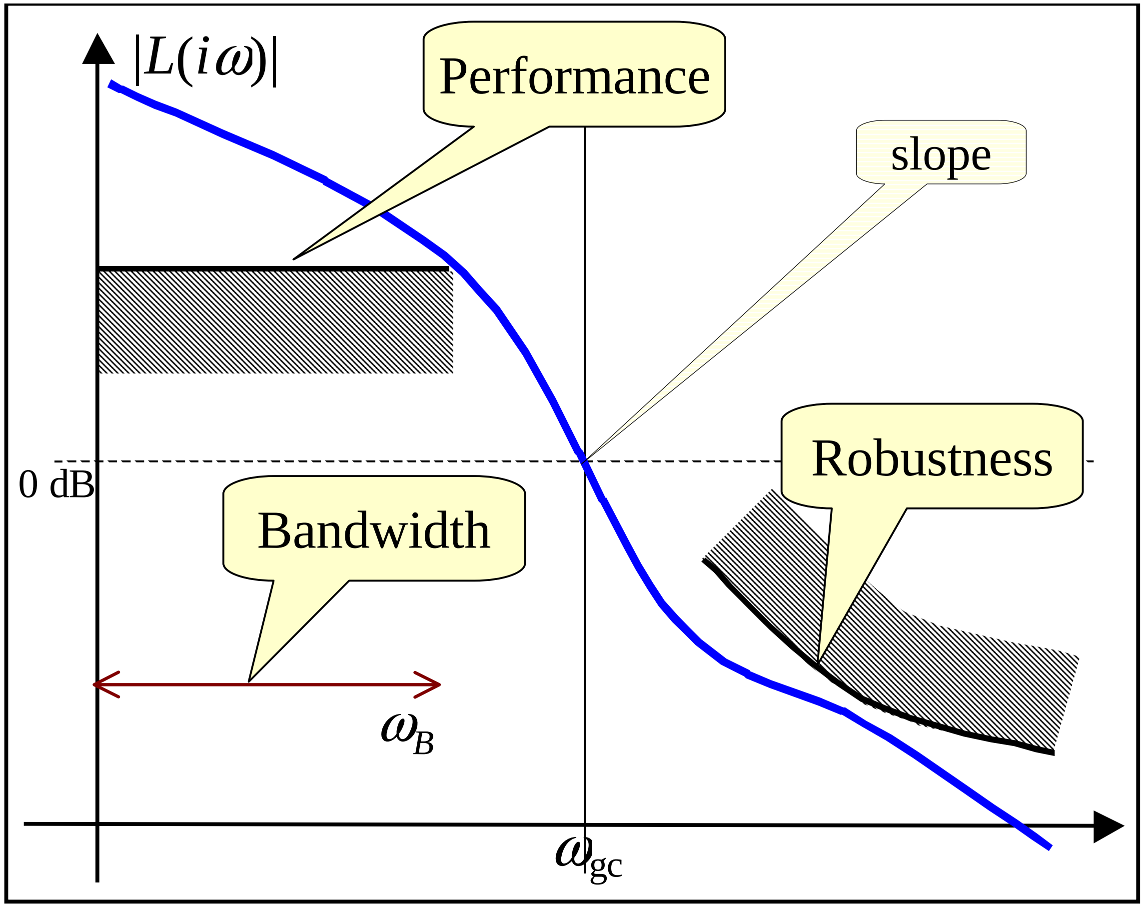


sensitivity peak margin

Additive uncertainty $|\Delta(i\omega)|$ radius

Loop Shape Requirements

- Low frequency:
  - high gain L
    = small S
- High frequency:
  - small gain L
    = small $T \cdot$ large δ
- Bandwidth
  - performance can be only achieved in a limited frequency band: $\omega \leq \omega_B$
  - ω_B is the bandwidth



Fundamental tradeoff: performance vs. robustness

Loopshaping design

- Loop design
  - Use P,I, and D feedback to shape the loop gain
- Loop modification and bandwidth
  - Low-pass filter - get rid of high-frequency stuff - robustness
  - Notch filter - get rid of oscillatory stuff - robustness
  - Lead-lag to improve phase around the crossover - bandwidth
    - P+D in the PID together have a lead-lag effect
- Need to maintain stability while shaping the magnitude of the loop gain
- Formal design tools H_2, H_∞, LMI, H_∞ loopshaping
  - cannot go past the fundamental limitations

Example - disk drive servo

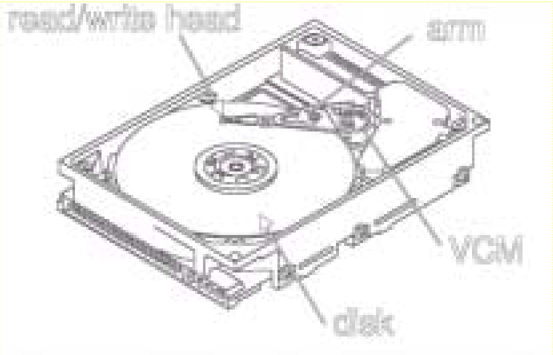
- The problem from HW Assignment 2
  - data in `diskPID.m, diskdata.mat`
- Design model: $\Delta P(s)$ is an uncertainty

$$P(s) = \frac{g_0}{s^2} + \Delta P(s)$$

- Analysis model: description for $\Delta P(s)$
- Design approach: PID control based on the simplified model

$$C(s) = k_P + \frac{k_I}{s} + k_D \frac{s}{\tau_D s + 1}$$

Disk servo control



$$J\ddot{\varphi} = T_{VCM} + T_{DISTURBANCE}$$

Voice
Coil
Motor

Disk drive servo controller

- Start from designing a PD controller
  - poles, characteristic equation

$$1 + C(s)P(s) = 0 \Rightarrow \left(k_P + sk_D\right) \cdot \frac{g_0}{s^2} + 1 = 0$$

$$s^2 + sg_0k_D + g_0k_P = 0$$

- Critically damped system

$$k_D = 2w_0 / g_0; \qquad k_P = w_0^{\;2} / g_0$$

where frequency w_0 is the closed-loop bandwidth

- In the derivative term make dynamics faster than w_0. Select $\tau_D = 0.25 / w_0$

$$k_D \frac{s}{\tau_D s + 1}$$

Disk drive servo

- Step up from PD to PID control

$$1+\left(k_P+sk_D+\frac{1}{s}k_I\right)\cdot\frac{g_0}{s^2}=0$$

$$s^3+s^2g_0k_D+sg_0k_P+g_0k_I=0$$

- Keep the system close to the critically damped, add integrator term to correct the steady state error, keep the scaling

$$k_P=w_0^2/g_0;\qquad k_D=aw_0/g_0;\qquad k_I=bw_0^3/g_0\qquad \tau_D=c/w_0$$

where *a*, *b*, and *c* are the tuning parameters

- Initial guess: w_0 =2000; a=2; b=0.1; c =0.25
- Tune *a*, *b*, *c* and w_0 by watching performance and robustness

Disk drive - controller tuning

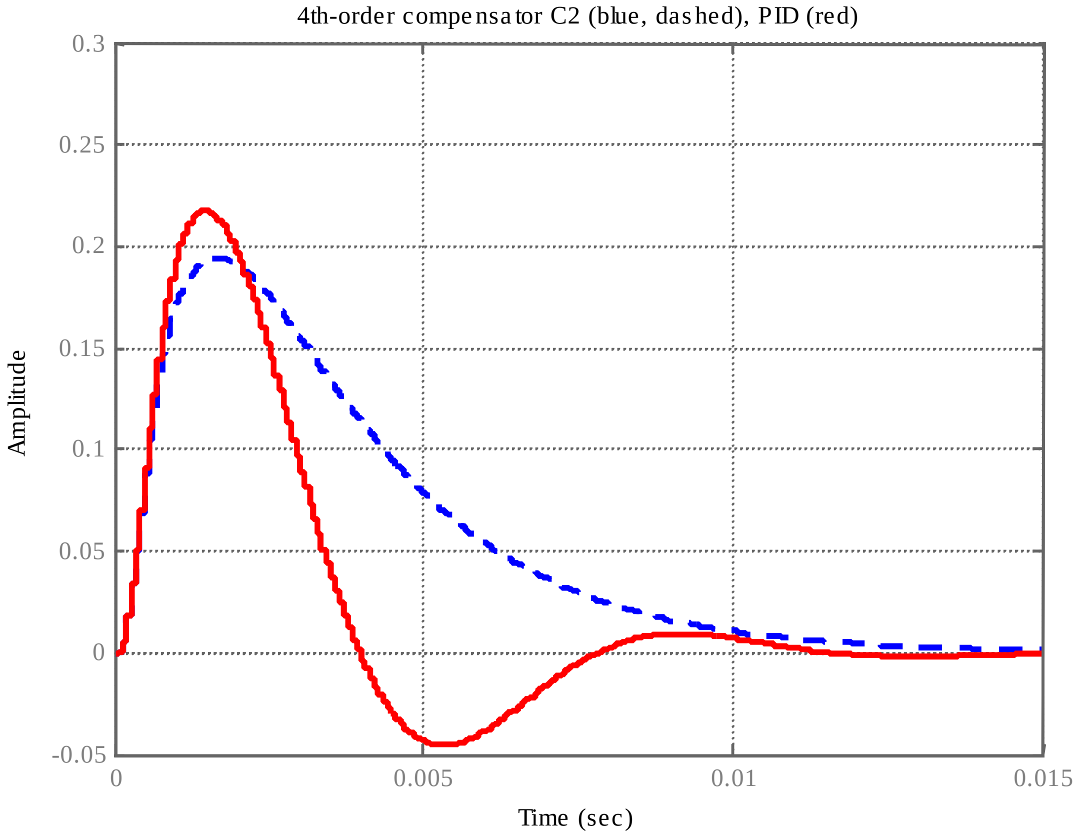
- Tune a, b, w_0 , and τ_D by trial and error
- Find a trade off taking into the account
  - Closed loop step response
  - Loop gain - performance
  - Robustness - sensitivity
  - Gain and phase margins
- Try to match the characteristics of C2 controller (demo)

- The final tuned values:

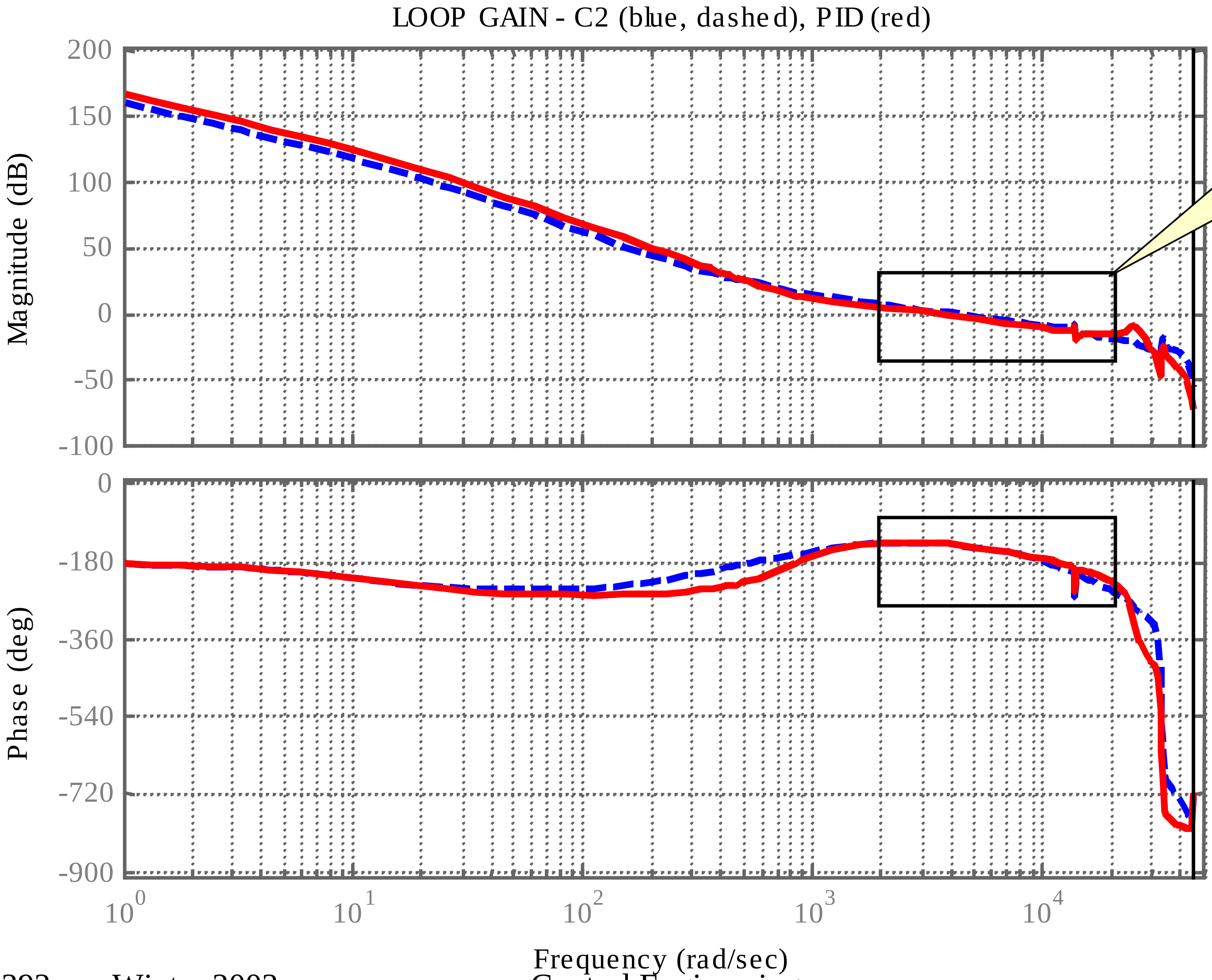
$$w_0 = 1700; \; a = 1.5; \; b = 0.5; \; c = 0.2$$

Disk servo - controller comparison

- PID is compared against a reference design
- Reference design: 4-th order controller: lead-lag + notch filter
  - Matlab `diskdemo`
  - Data in `diskPID.m`, `diskdata.mat`



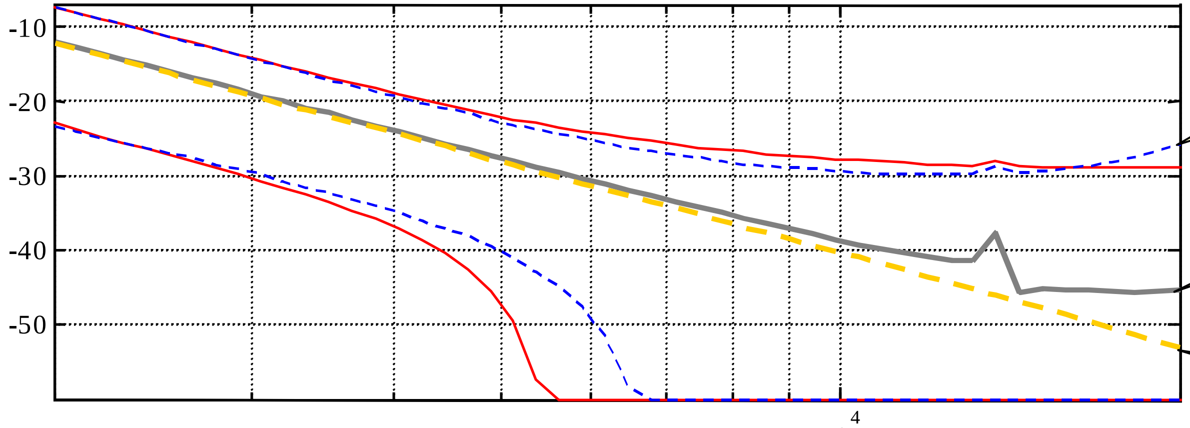
Loop shape, margins



Crossover area of interest

Disk drive servo - robustness

TRANSFER FUNCTION AND ACCEPTABLE UNCERTAINTY - C2 (blue, dashed), PID (red, dotted)

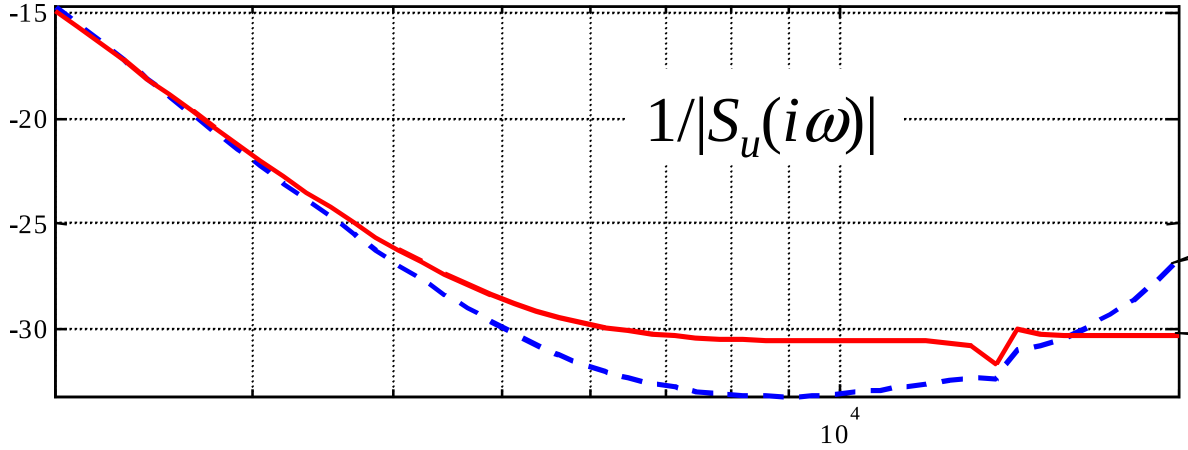


Robust stability bounds

Full model

Simple model

ROBUSTNESS TO PLANT UNCERTAINTY (dB) - C2 (blue, dashed), PID (red)



C2 - Matlab demo

PID

```
[m2,ph2]=bode(feedback(C2,Gd),w))
[mP,phP]=bode(feedback(PIDd,Gd),w))     plot(w,1./mP,w,1/m2)
```

Fundamental design limitations

- If we do not have a reference design - how do we know if we are doing well. May be there is a much better controller?
- Cannot get around the fundamental design limitations
  - frequency domain limitations on the loop shape
  - system structure limitations
  - engineering design limitations

Frequency domain limitation

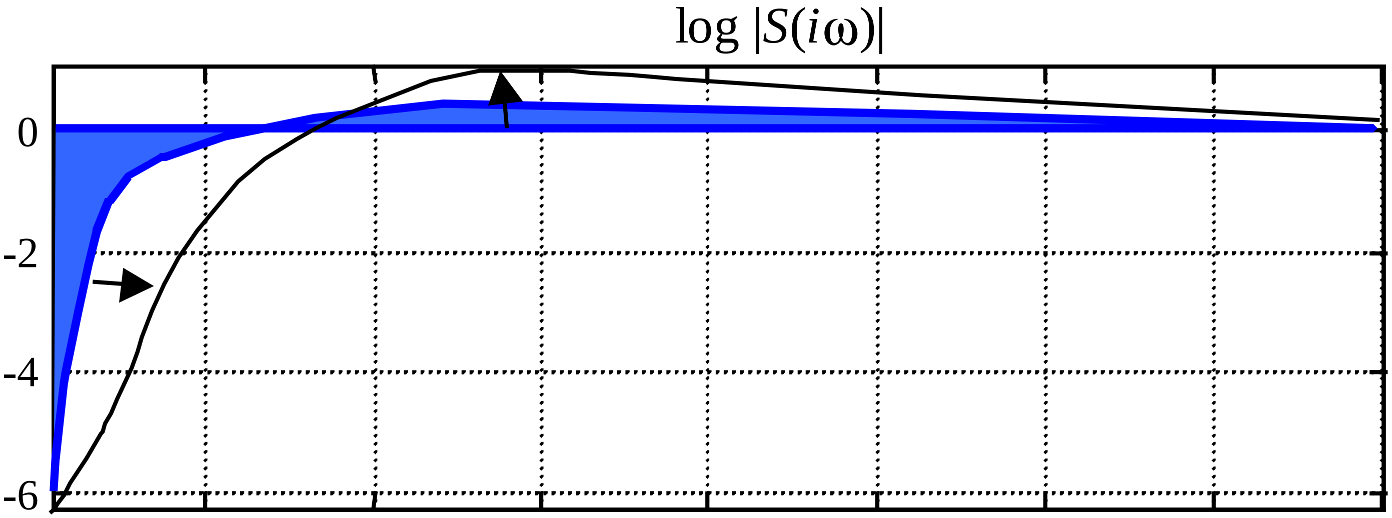
$S(i\omega) + T(i\omega) = 1$

Performance: $|S(i\omega)| \ll 1$

Robustness: $|T(i\omega)| \ll 1$

- Bode's integral constraint - waterbed effect

$$\int_0^\infty \log|S(i\omega)|\,d\omega = 0$$ (for most real-life stable system, or worse for the rest)



Structural design limitations

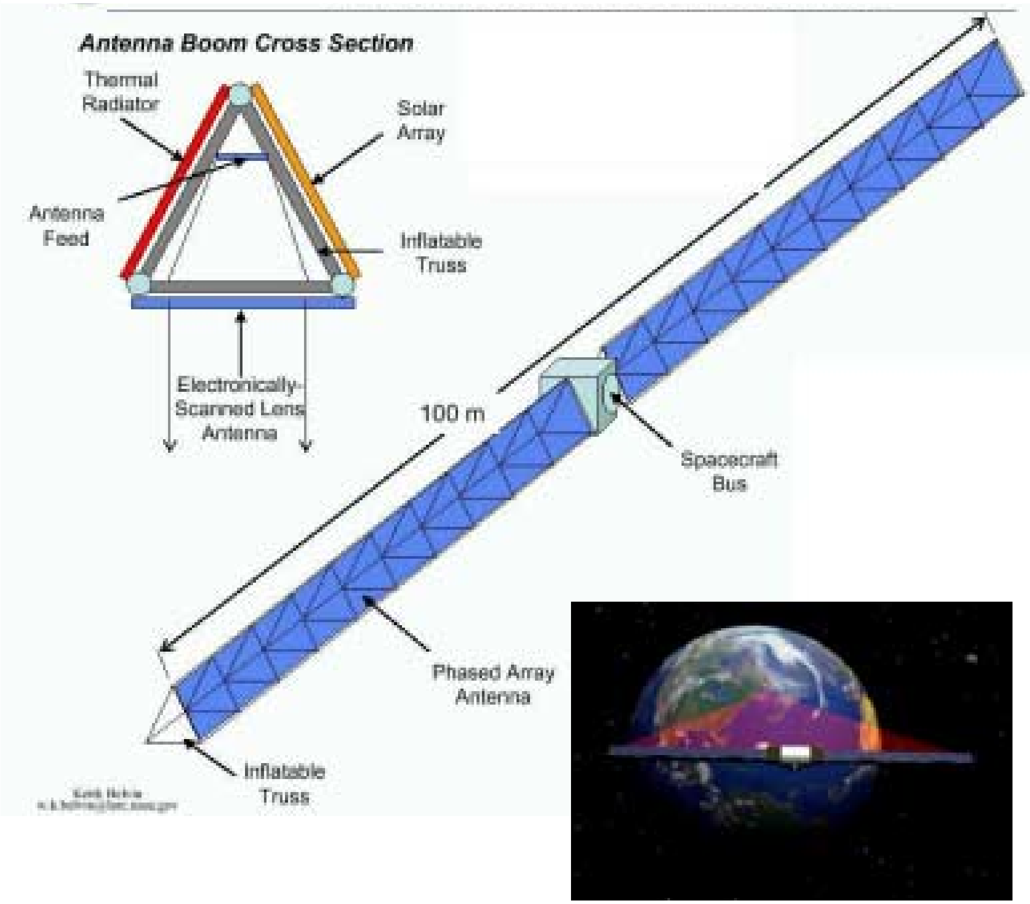
- Delays and non-minimum phase (r.h.s. zeros)
  - cannot make the response faster than delay, set bandwidth smaller
- Unstable dynamics
  - makes Bode’s integral constraint worse
  - re-design system to make it stable or use advanced control design
- Flexible dynamics
  - cannot go faster than the oscillation frequency
  - practical approach:
    - filter out and use low-bandwidth control (wait till it settles)
    - use input shaping feedforward

Unstable dynamics

- Very advanced applications
  - need advanced feedback control design

EC91 491-6 Photographed 1991
X-29 Smoke Test

Flexible dynamics



- Very advanced applications
  - really need control of 1-3 flexible modes

NASA Dryden Flight Research Center Photo Collection
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Pathfinder-Plus flight in Hawaii

Engineering design limitations

- Sensors
  - noise - have to reduce $|T(i\omega)|$ - reduced performance
  - quantization - same effect as noise
  - bandwidth (estimators) - cannot make the loop faster
- Actuators
  - range/saturation - limit the load sensitivity $|C(i\omega)\ S(i\omega)|$
  - actuator bandwidth - cannot make the loop faster
  - actuation increment - sticktion, quantization - effect of a load variation
  - other control handles
- Modeling errors
  - have to increase robustness, decrease performance
- Computing, sampling time
  - Nyquist sampling frequency limits the bandwidth