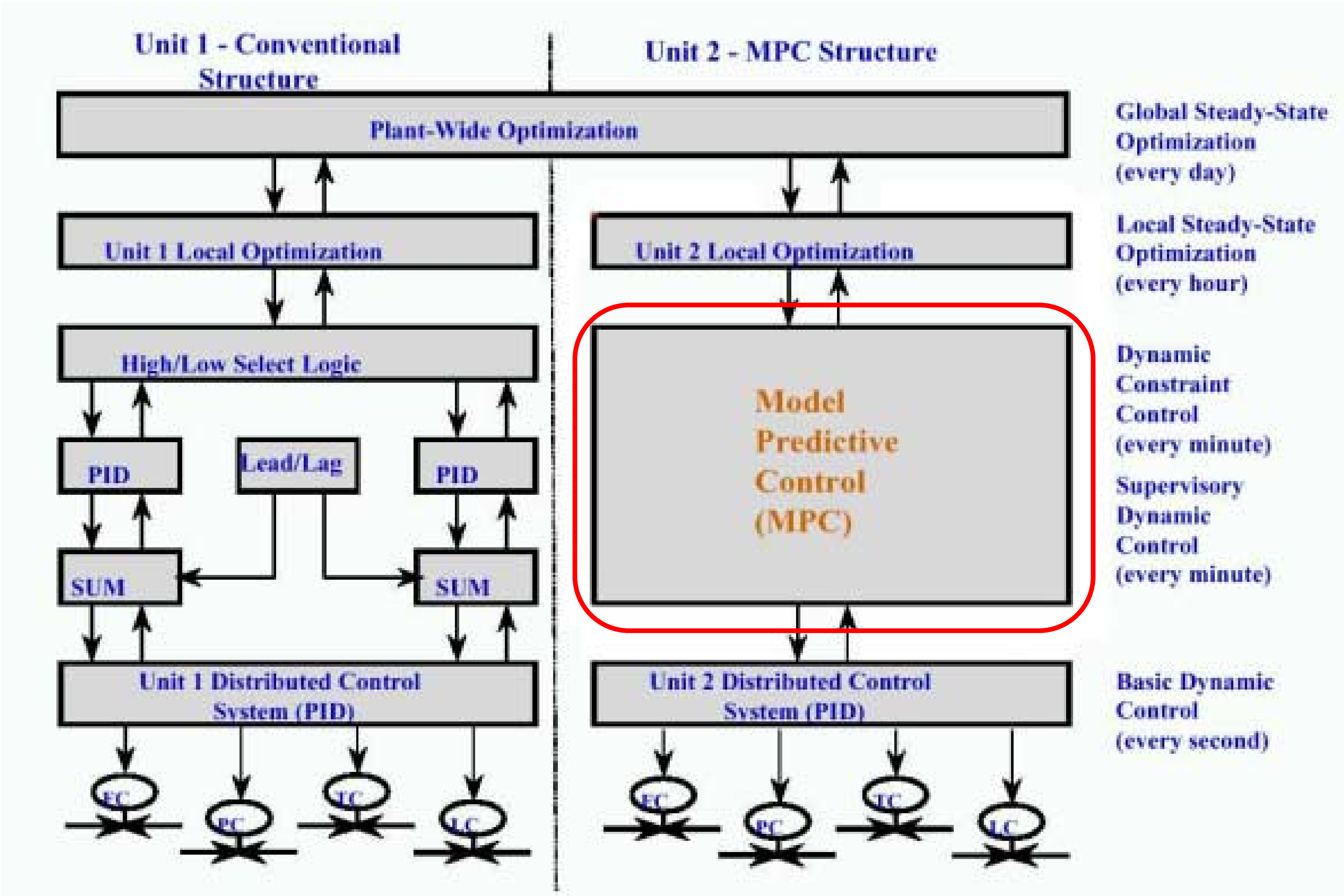
Lecture 12 - Model Predictive Control

- Prediction model
- Control optimization
- Receding horizon update
- Disturbance estimator - feedback
- IMC representation of MPC
- Resource:
  - Joe Qin, survey of industrial MPC algorithms
  - http://www.che.utexas.edu/~qin/cpcv/cpcv14.html

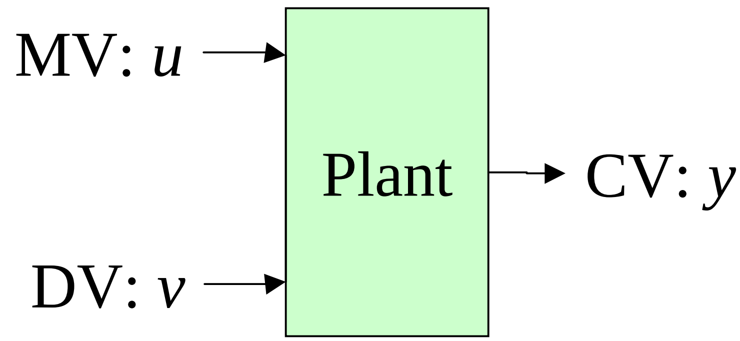
Control Hierarchy



Models for MPC

Plant structure:

- CV - controlled variables - *y*
- MV - manipulated variables - *u*
- DV - disturbance variables - *v*



- FSR - Finite Step Response model

$$y(t) = \sum_{k=1}^{N} S^U(k)\Delta u(t-k) + \sum_{k=1}^{N} S^D(k)\Delta v(t-k) + d$$

  – compact notation

$$y(t) = (s^U * \Delta u)(t) + (s^D * \Delta v)(t) + d$$

$$h = \Delta s;$$

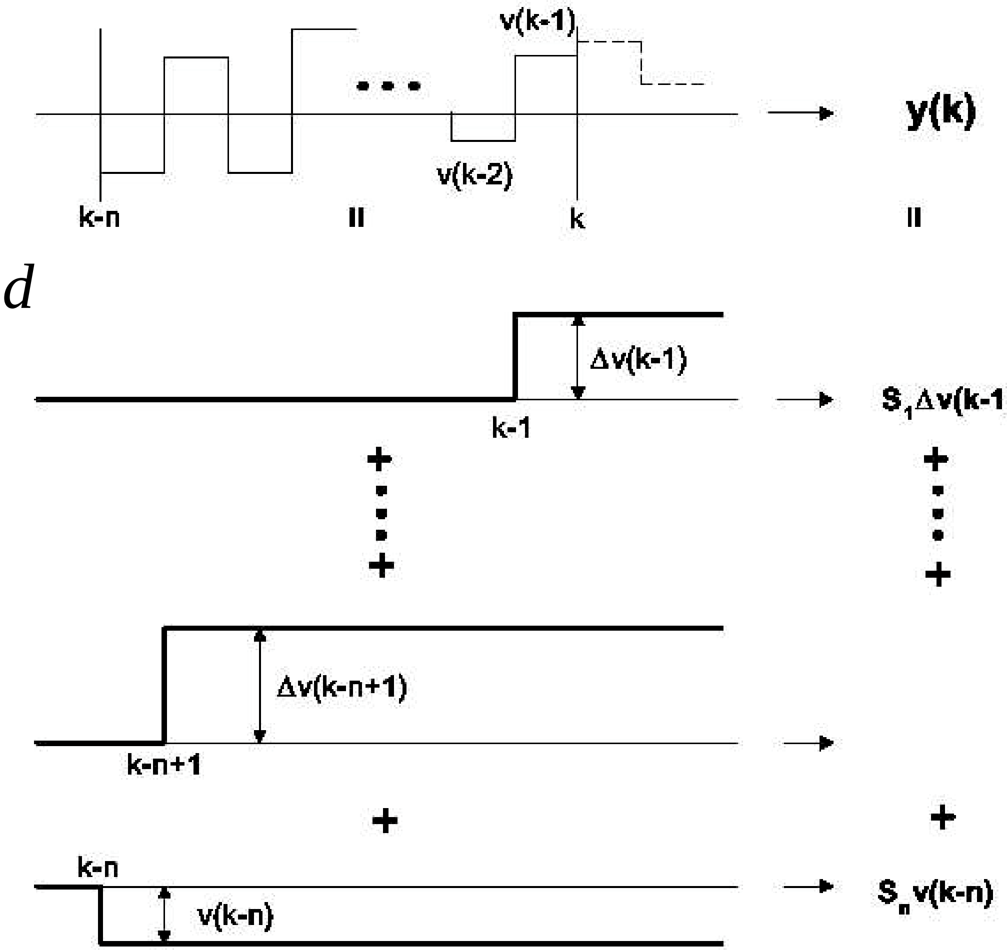
$$\Delta = 1 - z^{-1}$$

FSR Model

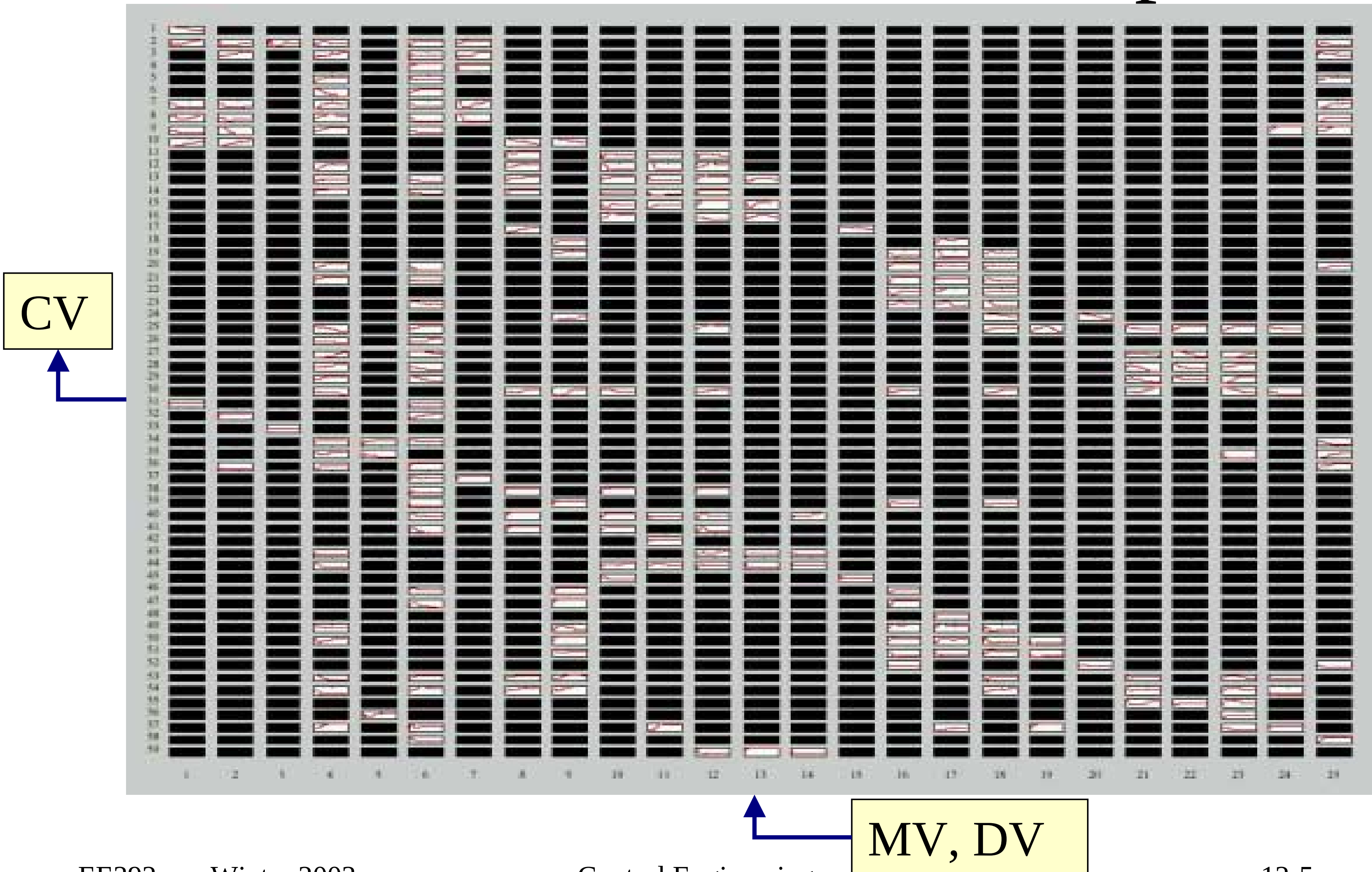
FSR model

$$y(t) = \sum_{k=1}^{n} S(k)\Delta v(t-k) + d$$

- Ignores anything that happened more than *n* steps in the past
- This is attributed to a constant disturbance *d*

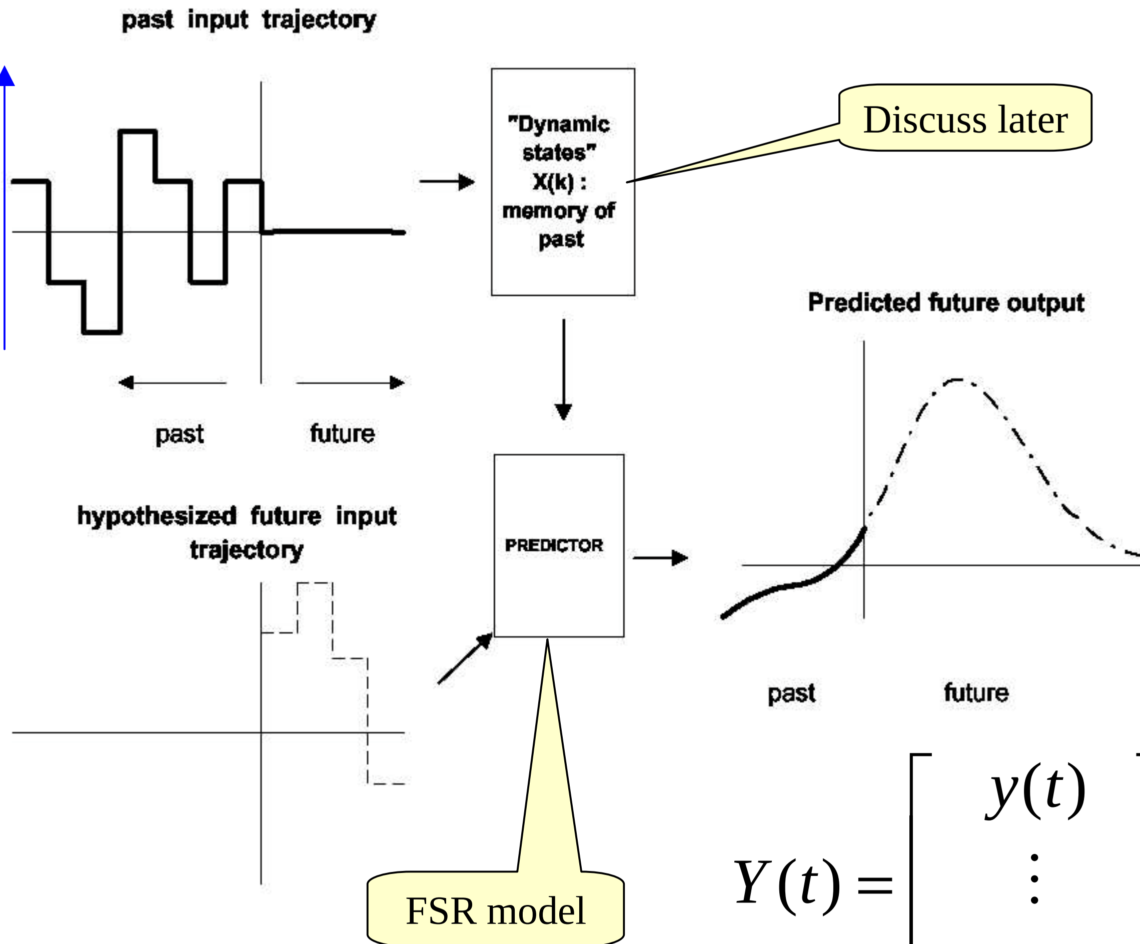


MPC Process Model Example



Prediction Model

$$\underline{U}(t) = \begin{bmatrix} \Delta u(t-1) \\ \vdots \\ \Delta u(t-N+1) \end{bmatrix}$$



$$U(t) = \begin{bmatrix} \Delta u(t) \\ \vdots \\ \Delta u(t+n) \end{bmatrix}$$

$$Y(t) = \begin{bmatrix} y(t) \\ \vdots \\ y(t+n) \end{bmatrix}$$

Prediction Model

$$y(t) = (s^U * \Delta u)(t) + (s^D * \Delta v)(t) + d$$

CV Prediction; Toeplitz matrix; Future MV; Hankel matrix; Past MV; Hankel matrix; Past DV; Unmeasured disturbance

$$Y(t) = \Psi U(t) + \Phi \cdot \underline{U}(t) + \Phi^D \underline{V}(t) + Dd$$

$$\Psi = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ S^U(1) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ S^U(n) & S^U(n-1) & \cdots & 0 \end{bmatrix}$$

Toeplitz matrix

$$\Phi^D = \begin{bmatrix} S^D(1) & S^D(2) & \cdots & S^D(N) \\ S^D(2) & S^D(3) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ S^D(N) & 0 & \cdots & 0 \end{bmatrix}$$

Hankel matrix

$$D = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

Future impact of the disturbance

Optimization of future inputs

$$Y(t) = \Psi U(t) + \underbrace{\Phi \cdot \underline{U}(t) + \Phi^D \underline{V}(t) + Dd}_{Y^*(t)}$$

- Optimization problem

$$J = \left(Y(t) - Y_d(t)\right)^T Q \left(Y(t) - Y_d(t)\right) + U^T(t) R U(t) \rightarrow \min$$

$$Y_d(t) = \begin{bmatrix} y_d(t) \\ \vdots \\ y_d(t+N) \end{bmatrix}$$

$$Q = \begin{bmatrix} Q^y & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Q^y \end{bmatrix}, R = \begin{bmatrix} R^u & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R^u \end{bmatrix}$$

Optimization constraints

- MV constraints

$$-\Delta u_{\max} \le \Delta u(t) \le \Delta u_{\max}$$

$$u_{\min} \le u(t) \le u_{\max}$$

$$\Longrightarrow$$

$$-\Delta u_{\max} \le U(t) \le \Delta u_{\max}$$

$$u_{\min} \le \Sigma U(t) - C \le u_{\max}$$

$$u(t+k) = u(t-1) + \sum_{j=1}^{k} \Delta u(t)$$

- CV constraints

$$y_{\min}(t) \le y(t) \le y_{\max}(t) \quad \Longrightarrow \quad Y_{\min} \le Y(t) \le Y_{\max}$$

- Terminal constraint:

$$y(t+k) = y_d; \Delta u(t+k) = 0 \ \text{ for } \ k \ge p$$

QP solution

- QP Problem:

$$Ax \le b$$

$$A_{eq}x = b_{eq}$$

$$J = \frac{1}{2}x^T Qx + f^T x \to \min$$

$$x = \begin{bmatrix} U(t) \\ Y(t) \end{bmatrix} \quad \text{Predicted MVs, CVs}$$

- Standard QP codes are available

Receding horizon control

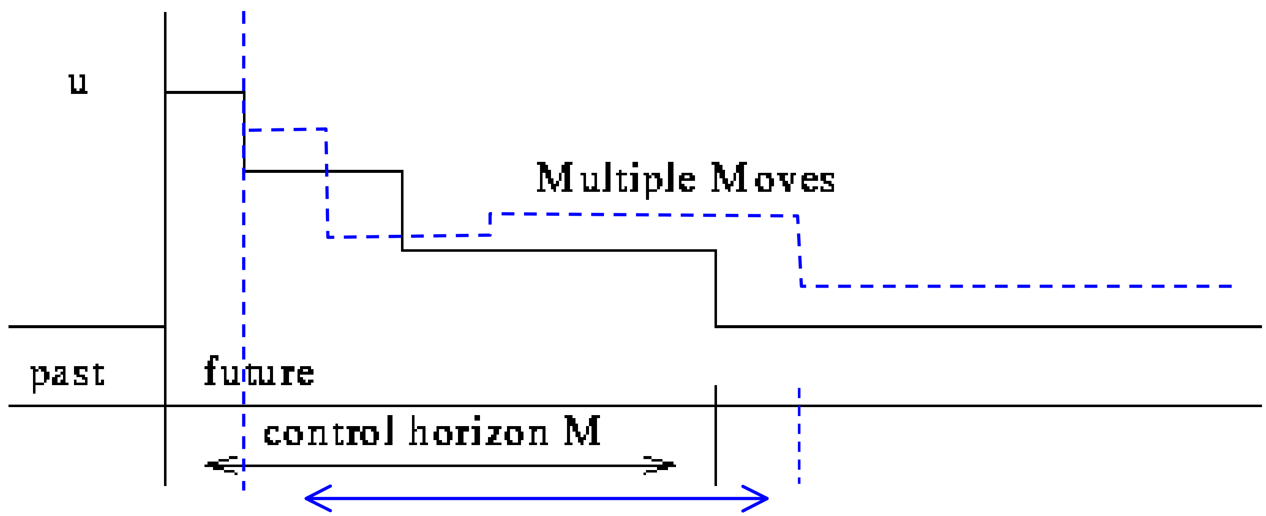
- Optimization problem solution at step t :

$$J(U) \to \min \quad \Rightarrow \quad U = U_{OPT}(t)$$

- Use the first computed control value only

$$u(t) = [1 \quad 0 \quad \ldots \quad 0] \cdot U_{OPT}(t)$$

- Repeat at each t



Control dynamics

- System dynamics as an equality constraint in optimization

$$Y(t) = \Psi U(t) + Y^*(t)$$

$$Y^*(t) = \begin{bmatrix} \Phi & \Phi^D \end{bmatrix} \cdot X(t) + d(t)$$

- Update of the system state

$$X(t+1) = AX(t) + B\Delta u(t) + B^D \Delta v(t)$$

- Optimization problem solution at step t :

$$J(U; Y(U)) \to \min \qquad \Rightarrow \qquad U = U_{OPT}(t)$$

- Use the first of the computed control values

$$u(t) = [1 \quad 0 \quad \ldots \quad 0] \cdot U_{OPT}(t)$$

State update and estimation

- State update - shift register

$$X(t) = \begin{bmatrix} \underline{U}(t) \\ \underline{V}(t) \end{bmatrix} \qquad \underline{U}(t) = \begin{bmatrix} \Delta u(t-1) \\ \vdots \\ \Delta u(t-N+1) \end{bmatrix} \qquad \underline{V}(t) = \begin{bmatrix} \Delta v(t-1) \\ \vdots \\ \Delta v(t-N+1) \end{bmatrix}$$

$\Delta u(t)$

$\Delta v(t)$

- Disturbance estimator (feedback)

$$d(t+1) = d(t) + \left(y_m(t) - y(t) \right)$$

Unmeasured disturbance

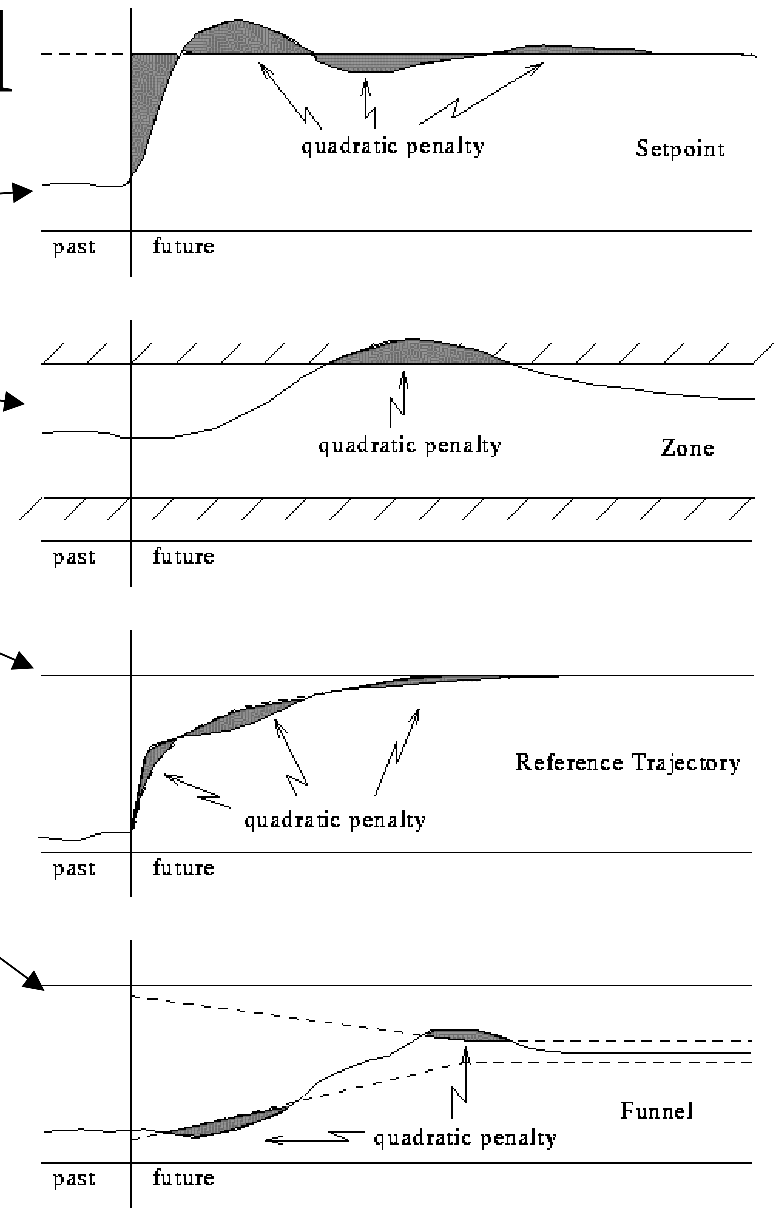
Actually measured CV

CV Prediction based on the current state *X*(t)

- Integrator feedback

Optimization detail

- Setpoint
- Zone
- Trajectory
- Funnels



- Soft constraints (quadratic penalties) and hard constraints for MV, CV
- Regularization
  – penalty
  – singular value thresholding

Advantages and Conveniences

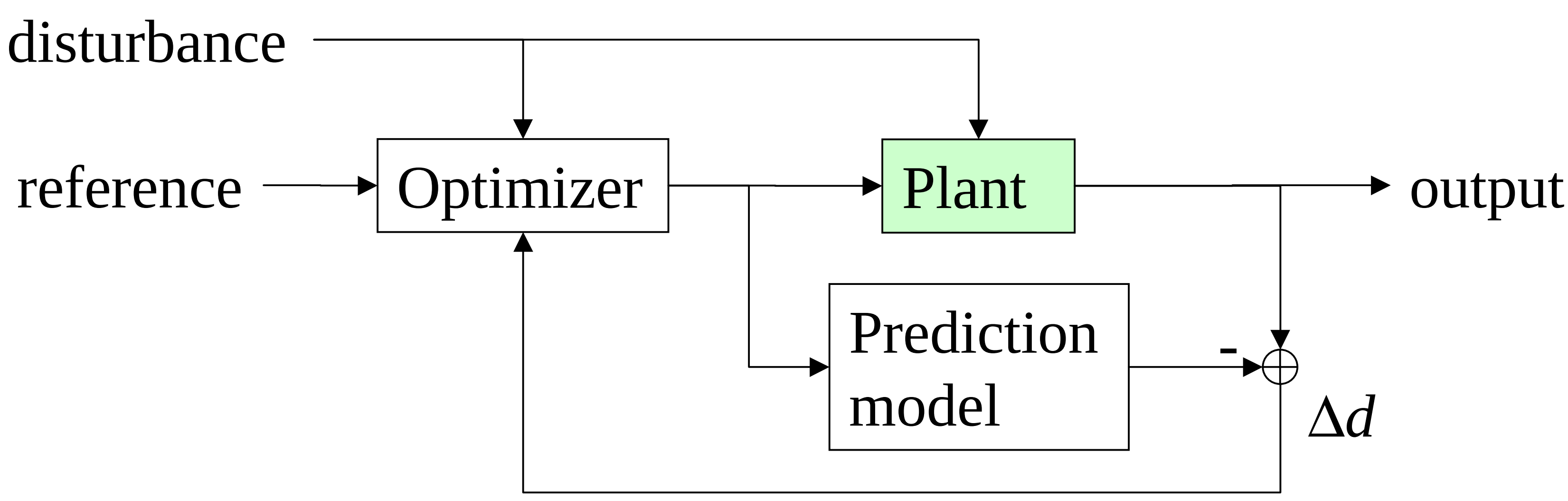
- Industrial strength products that can be used for a broad range of applications
- Flexibility to plant size, automated setup
- Based on step response/impulse response model
- On the fly reconfiguration if plant is changing
  - MV, CV, DV channels taken off control / returned into MPC
  - measurement problems, actuator failures
- Systematic handling of multi-rate measurements and missed measurement points
  - do not update d if no data

Technical detail

- Tuning of MPC feedback control performance is an issue.
  - Works in practice, without formal analysis
  - Theory requires
    - Large (infinite) prediction horizon
    - Terminal constraint
- Additional tricks for
  - a separate static optimization step
  - integrating and unstable dynamics
  - active constraints
  - regularization
  - shape functions for control
  - different control horizon and prediction horizon
  - ...

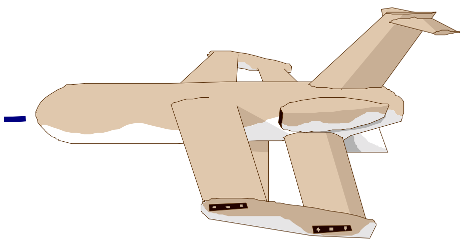
MPC as IMC

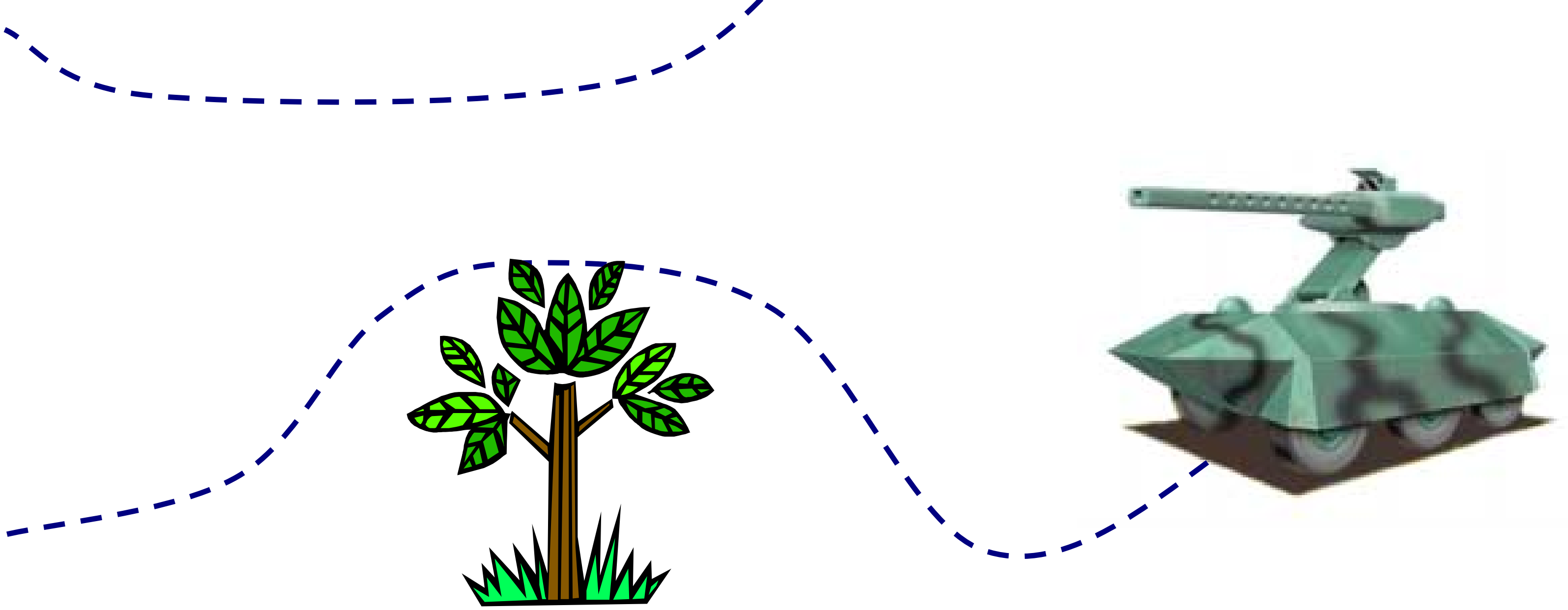
- MPC is a special case of IMC
- Closed-loop dynamics (filter dynamics)
  - integrator - in disturbance estimator
  - *N* poles z=0 - in the FSR model update



Emerging MPC applications

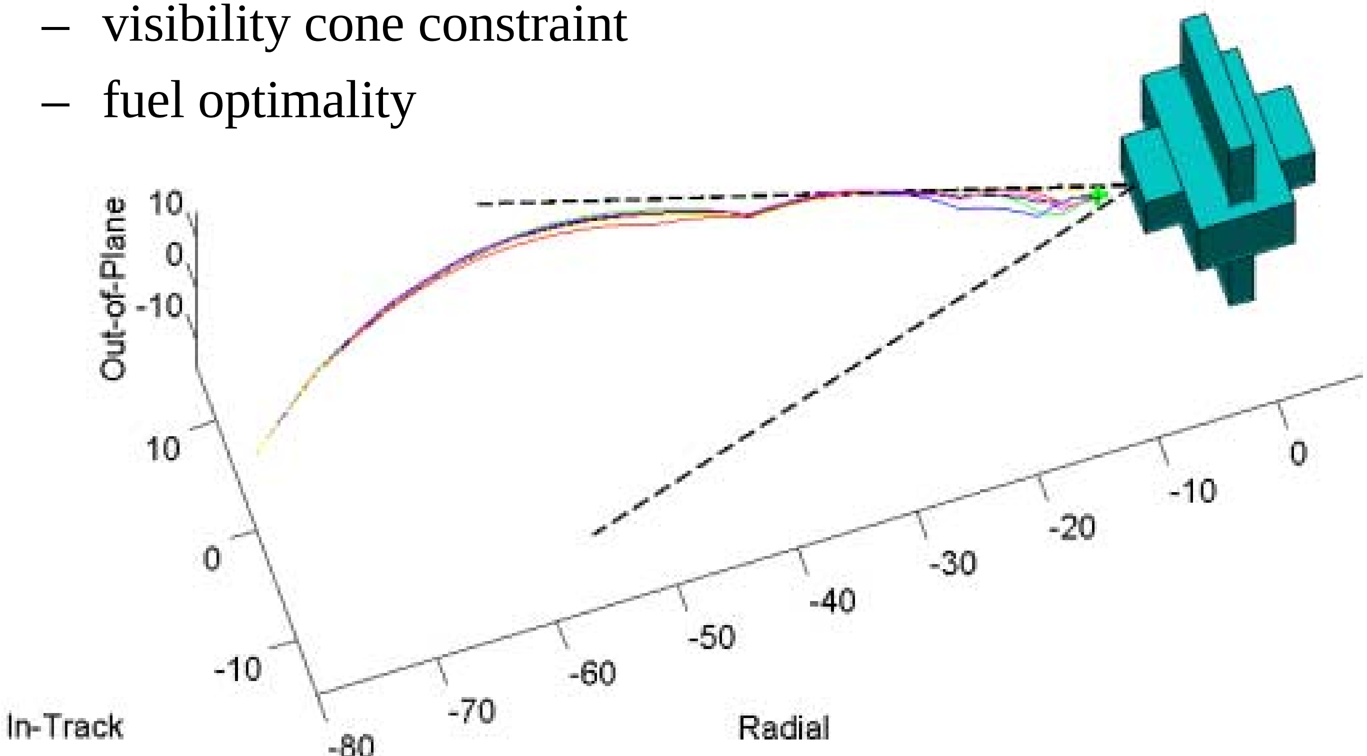
- Vehicle path planning and control
  - nonlinear vehicle models
  - world models
  - receding horizon preview





Emerging MPC applications

- Spacecraft rendezvous with space station
  - visibility cone constraint
  - fuel optimality



From Richards & How, MIT

Emerging MPC applications

- Nonlinear plants
  - just need a computable model (simulation)
- Hybrid plants
  - combination of dynamics and discrete mode change
- Engine control
- Large scale operation control problems
  - operations management
  - campaign control