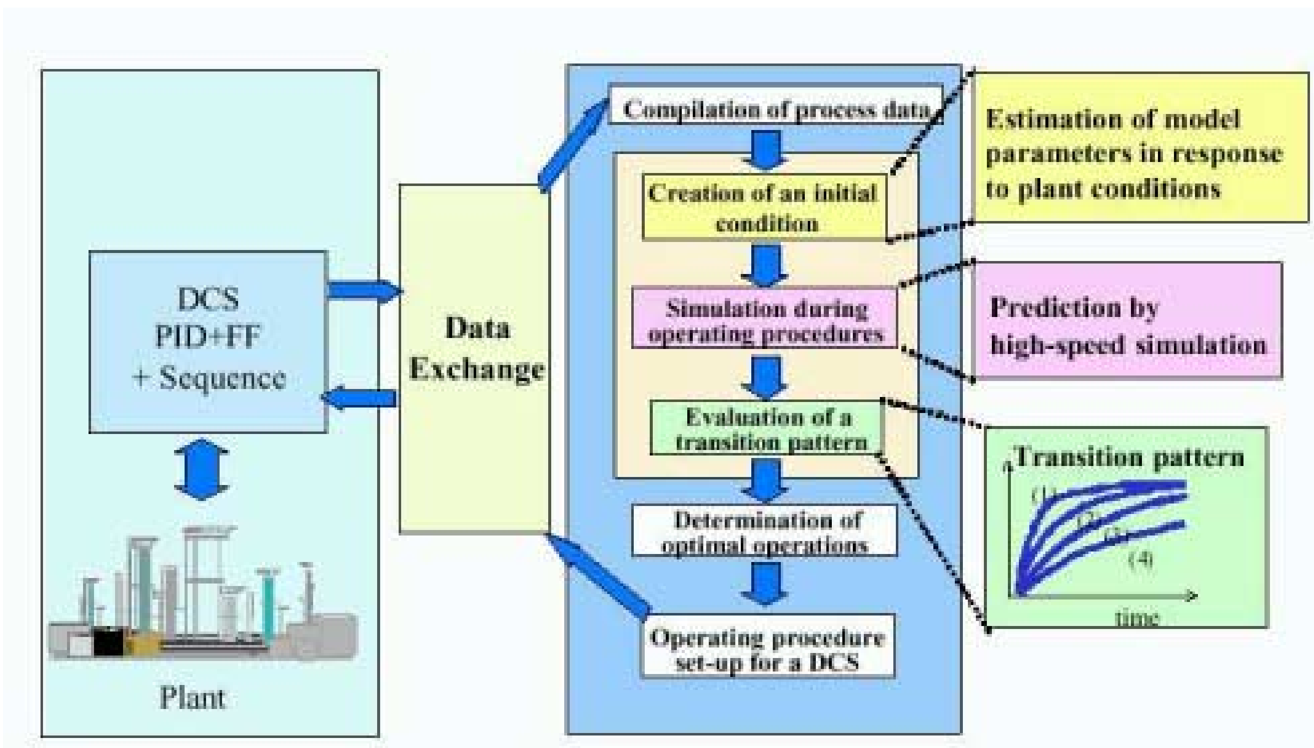
Lecture 11 - Optimal Program

- Grade change in process control
  - example
- QP optimization
- Flexible dynamics: input shaping, input trajectory
  - example
- Rocket, ascent
- Robotics

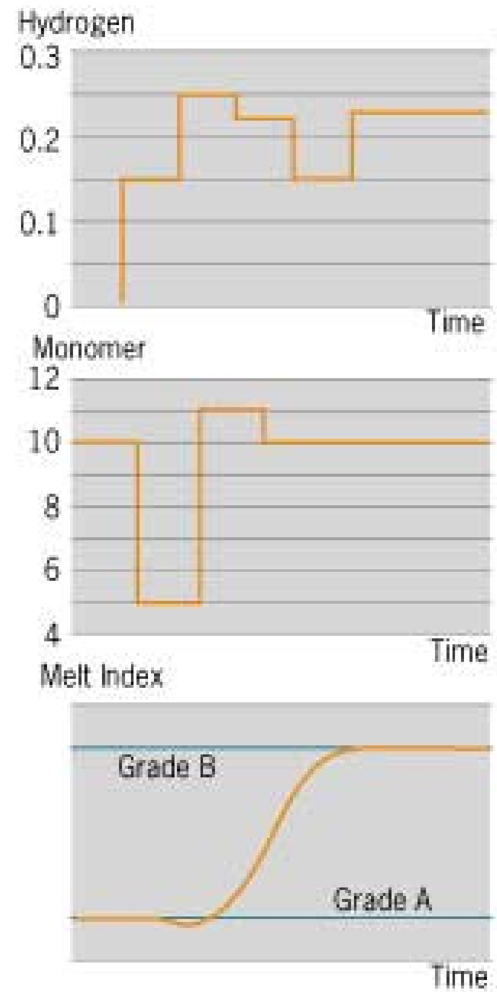
Optimization of process transitions

- Process plants manufacture different product varieties (grades)
- Need to optimize transitions from grade to grade



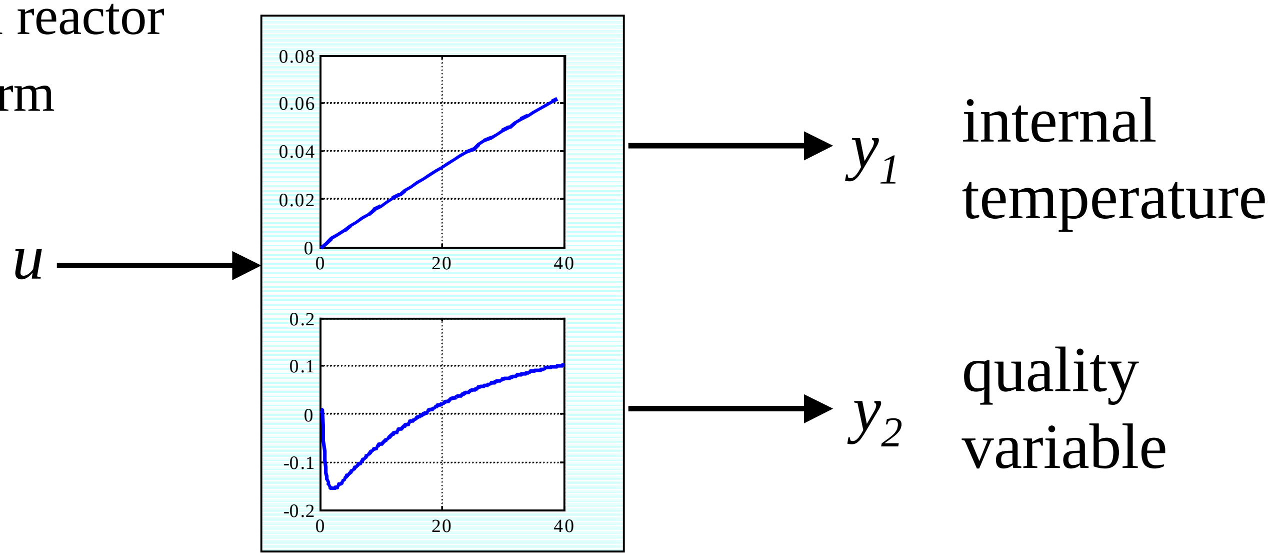
Product Grade Change

- The requirement: to change manufacture from grade A to grade B with the minimum off-spec production.
- The implementation: using detailed models of process and operating procedures.
- The results: optimum setpoint trajectories for key process controllers during the changeover, resulting in minimum lost revenue.



Grade change control example

- Simple process model:
  - chemical reactor
  - server farm



- The process is the initial steady state: $u = 0; y_1 = y_2 = 0$
- Need to transition, as quickly as possible, to other steady state: u = const; y_1 = const; $y_2 = y_d$

Grade change control example

- Linear system model in the convolution form

$$y = h * u$$

- Quadratic-optimal control

$$\int \left(\left| y_2(t) - y_d \right|^2 + r \left| \dot{u}(t) \right|^2 \right) dt \to \min$$

- Equality constraint (process transitioning to the new grade)

$$\dot{y}_1(t) \equiv 0, \; y_2(t) \equiv y_d, \; \text{for } \; T \le t \le T + T_f$$

- Inequality constraints
  - Control $\quad \left| u(t) \right| \le u_*$
  - Temperature $\quad \left| y_2(t) \right| \le d_*$

Grade change control example

- Sampled time: $t = k\tau$, (k=1,…,N);
  - Y is a $2N$ vector
  - H is a block-Toeplitz matrix

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} H_1 U \\ H_2 U \end{bmatrix} = HU$$

$$U = \begin{bmatrix} u(\tau) \\ \vdots \\ u(N\tau) \end{bmatrix}, Y_1 = \begin{bmatrix} y_1(\tau) \\ \vdots \\ y_1(N\tau) \end{bmatrix}, Y_2 = \ldots$$

$$H_{1,2}U = h_{1,2} * U$$

- Dynamics as an equality constraint:

$$HU - Y = 0$$

Grade change control example

- Quadratic-optimal control

optional

$$(Y_2 - Y_d)^T (Y_2 - Y_d) + rU^T D^T DU + w(Y_1 - Y_{d1})^T (Y_1 - Y_{d1}) \to \min$$

$$U^T D^T DU + Y_2^T Y_2 - 2Y_d^T Y_2 + \ldots \to \min$$

$$Y_d = y_d \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & -1 \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$

- Inequality constraints
  - Control $-u_* \le U \le u_*$
  - Temperature $0 \le Y_1 \le T_*$

Terminal constraint

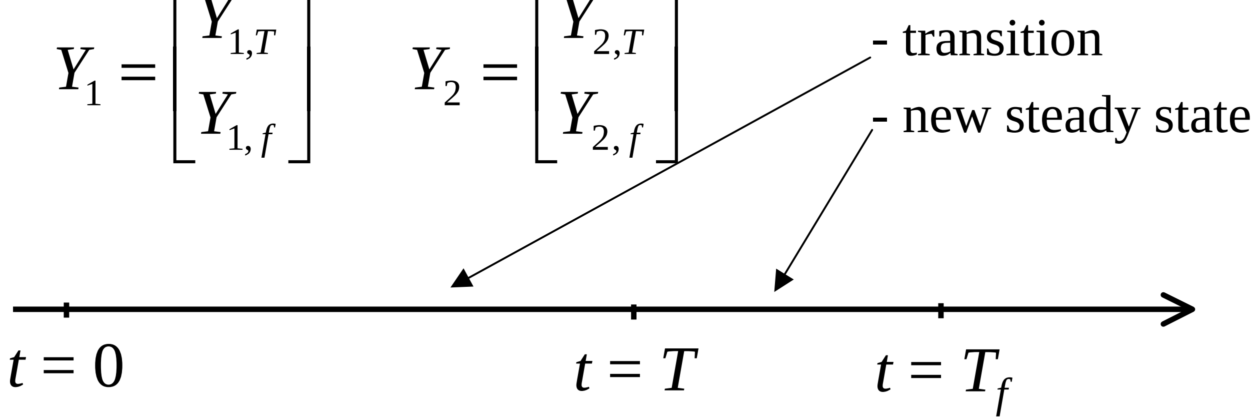
- Equality constraints (new grade steady state)

$$DY_{1,f} = 0$$ - steady in the end

$$Y_{2,f} = Y_{d,f}$$ - at target in the end

$$D = \begin{bmatrix} 1 & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & -1 \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} Y_{1,T} \\ Y_{1,f} \end{bmatrix} \qquad Y_2 = \begin{bmatrix} Y_{2,T} \\ Y_{2,f} \end{bmatrix}$$



Quadratic Programming

- QP Problem:

$$Ax \le b$$

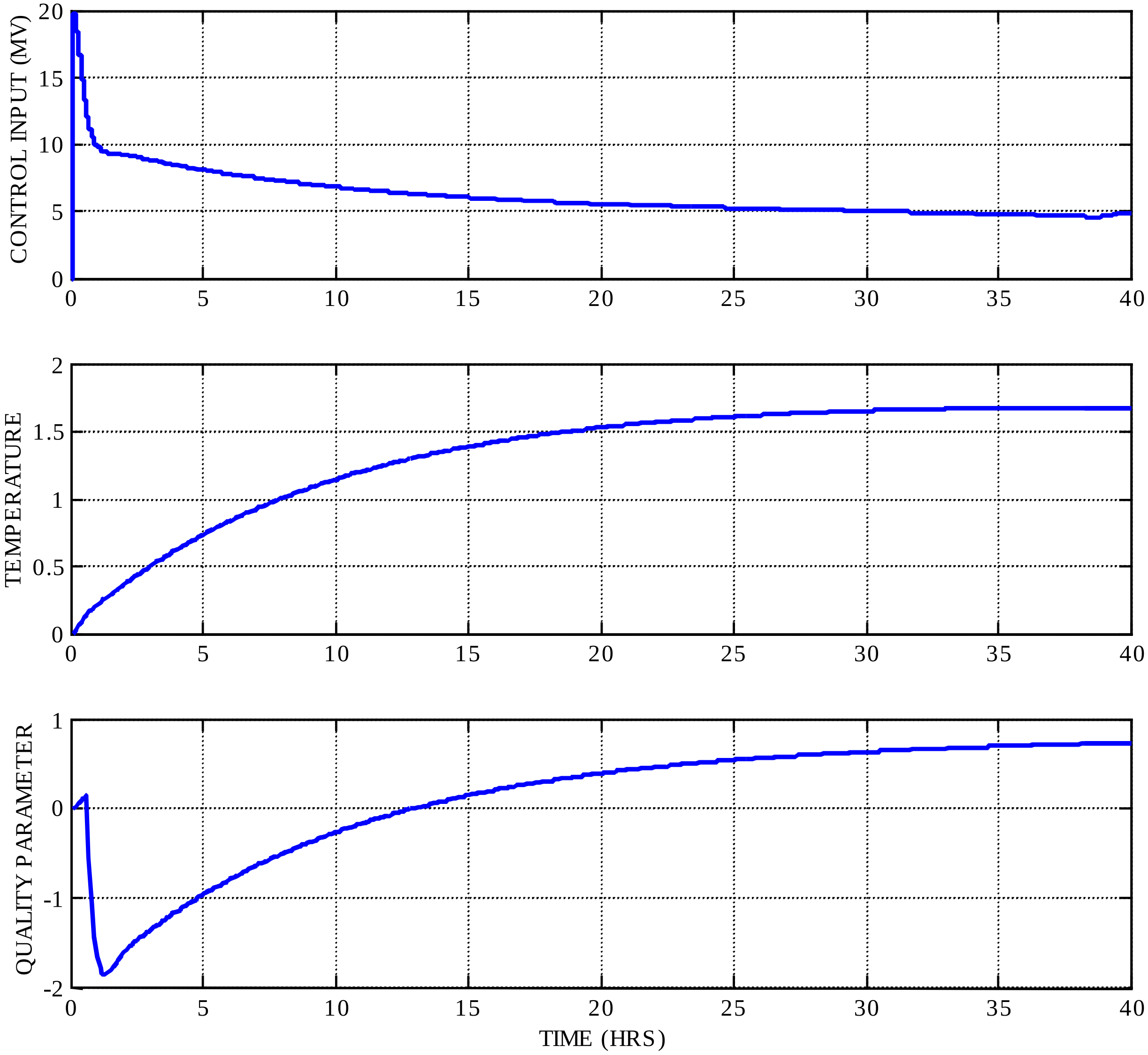
$$Gx = h$$

$$J = \frac{1}{2} x^T H x + f^T x \to \min$$

- Matlab Optimization Toolbox: **`QUADPROG`**

Sim

QP Program for the grade change, no terminal constraint



$y_d = 0.75$

$\tau = 0.1$

$r = 0.05$

$T_* = 2$

$u_* = 20$

Sim

QP Program for the grade change with a terminal constraint at $T = 8$

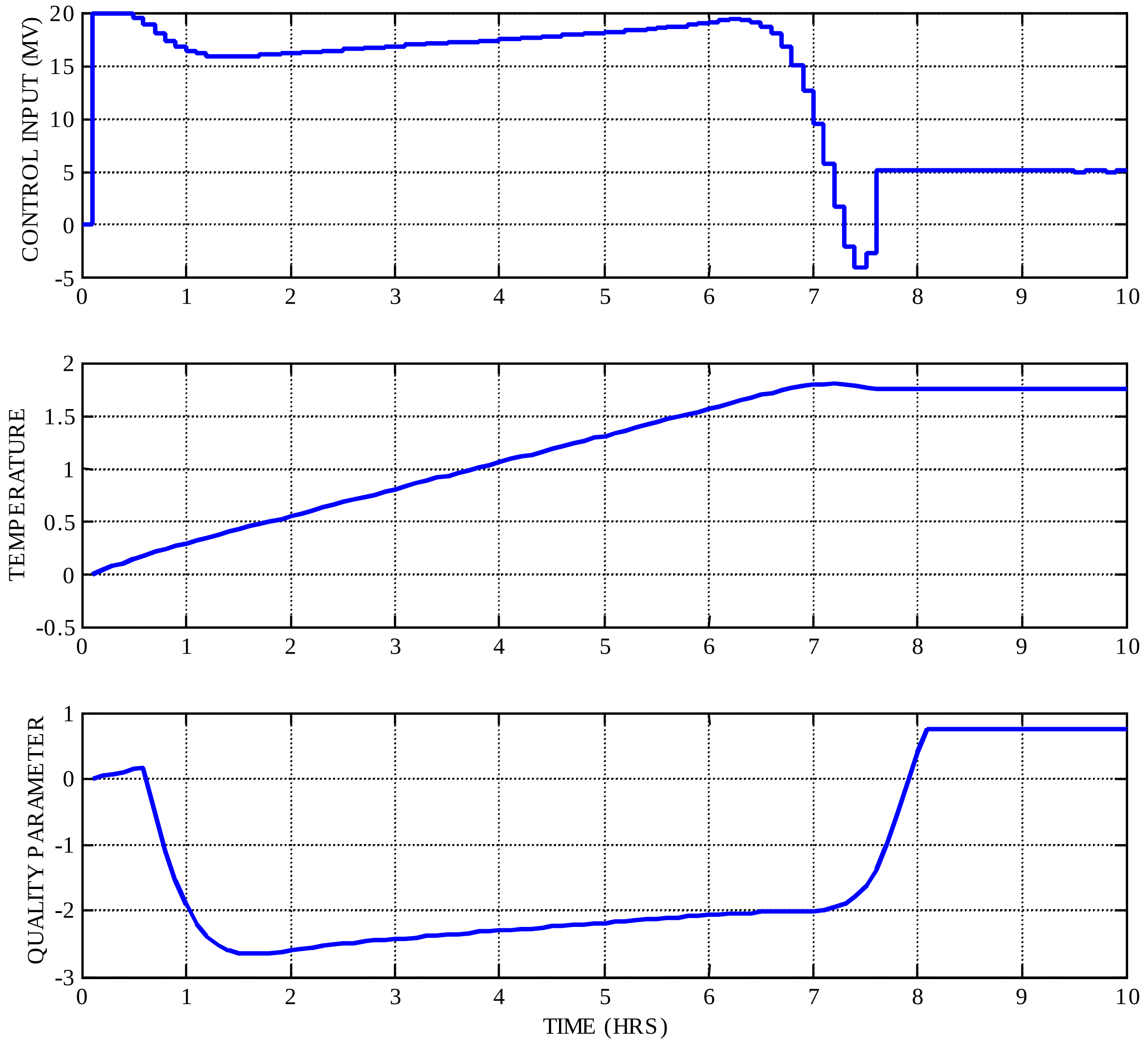
$y_d = 0.75$

$\tau = 0.1$

$r = 0.05$

$T_* = 2$

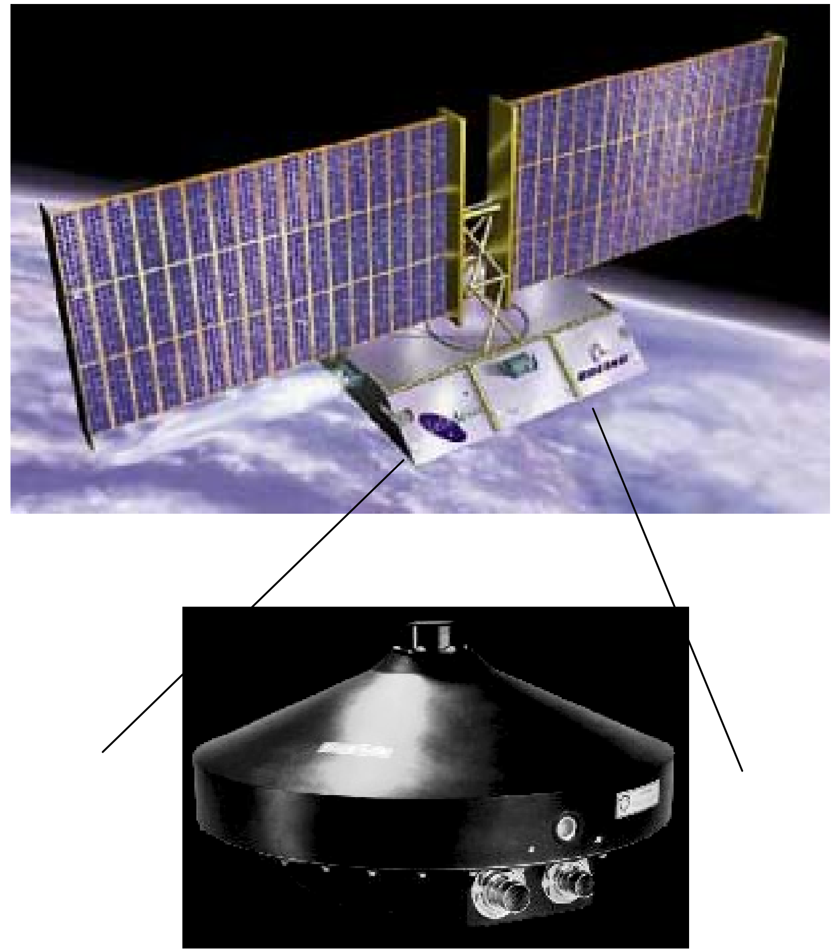
$u_* = 20$

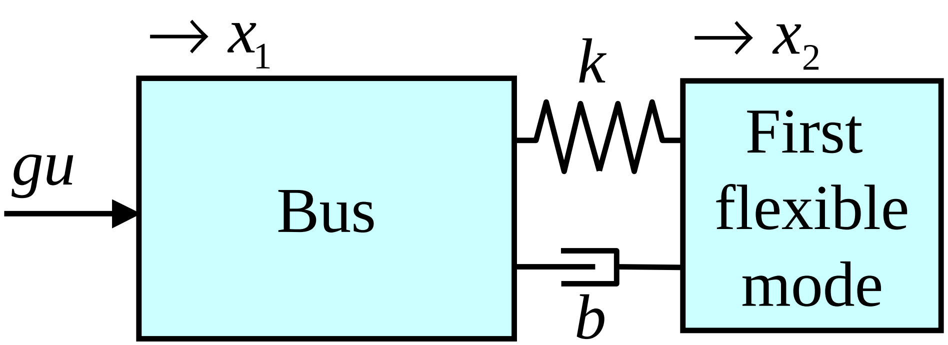


Flexible Satellite Slew Control

- Single flexible mode model
- Franklin, Section 9.2

$$J_1\ddot{x}_1 = -k(x_1 - x_2) - b(\dot{x}_1 - \dot{x}_2) + gu$$
$$J_2\ddot{x}_2 = k(x_1 - x_2) + b(\dot{x}_1 - \dot{x}_2)$$





Flexible Satellite Slew Control

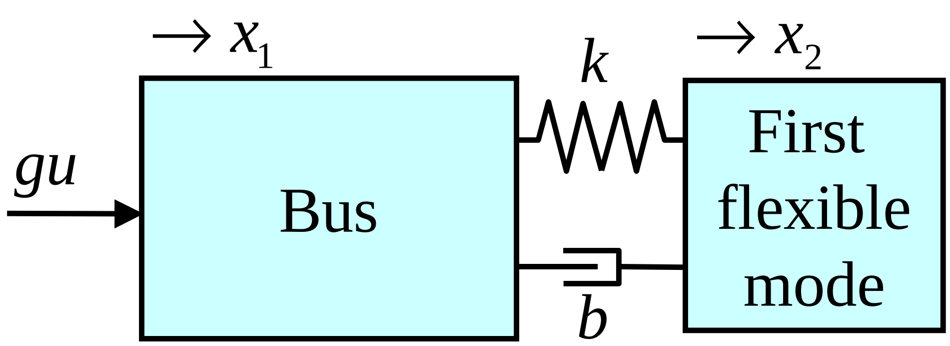
- Linear system model

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$x = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/J_1 & -b/J_1 & k/J_1 & b/J_1 \\ 0 & 0 & 0 & 1 \\ k/J_2 & b/J_2 & -k/J_2 & -b/J_2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ g/J_1 \\ 0 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} x_1 \\ x_2 - x_1 \\ \dot{x}_1 \end{bmatrix}$$

- slew angle
- deformation
- slew rate



Flexible Satellite Slew Control

- Linear system model in the convolution form

$$y = h * u$$

- Quadratic-optimal control

$$\int |u(t)|^2 dt \to \min$$

- Equality constraint (system coming to at target slew angle)

$$y(t) \equiv y_d, \text{ for } T \le t \le T + T_f$$

- Inequality constraints
  - Control $|u(t)| \le 1$
  - Deformation $|y_2(t)| \le d_*$
  - Slew rate $|y_3(t)| \le v_*$

Flexible Satellite Slew Control

- Sampled time: $t = k\tau$, (k=1,…,N); Y is a 3N vector; H is a block-Toeplitz matrix

$$Y = HU$$

- Quadratic-optimal control

$$U^T U \to \min$$

- Equality constraint (system coming to at target slew angle)

$$SY = Y_d$$

- Inequality constraints
  - Control $-1 \le U \le 1$
  - Deformation $d_* \le S_2 Y \le d_*$
  - Slew rate $v_* \le S_3 Y \le v_*$

This is a QP problem

Sim

QP Program for the flexible satellite slew

$g = 0.02$

$J_1 = 1$

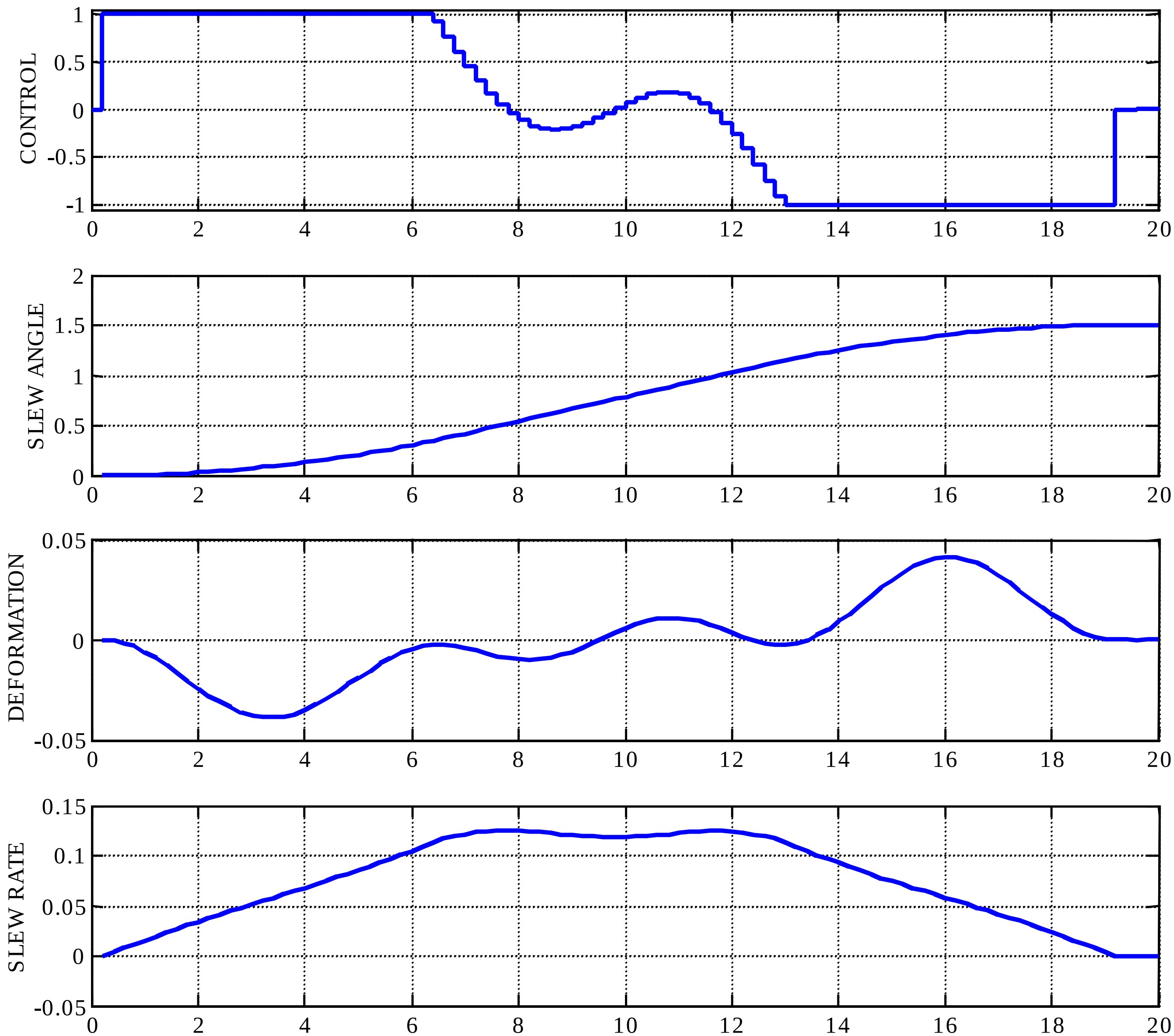
$J_2 = 0.1$

$k = 0.091$

$b = 0.0036$

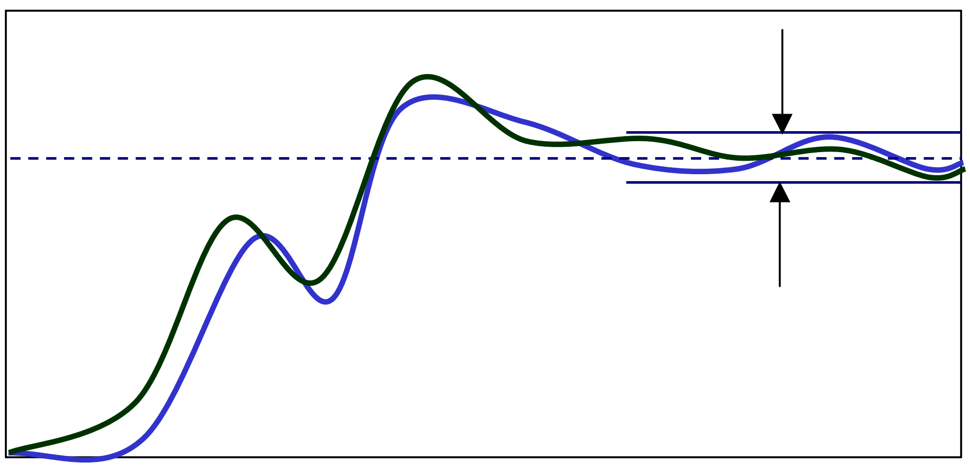
$d_* = 0.02$

$v_* = 0.2$



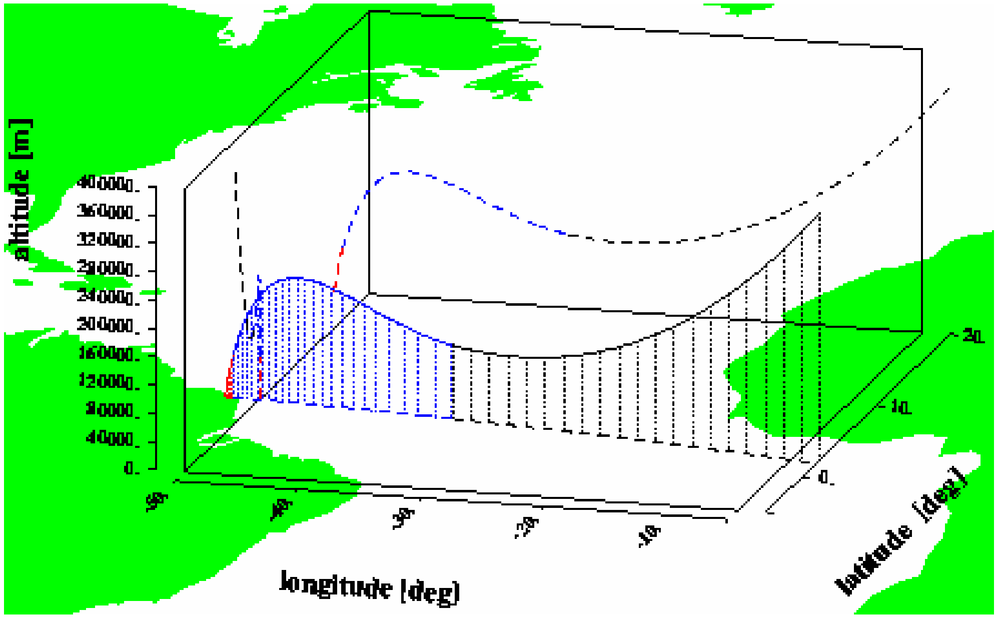
Robust design approach

- Replace exact terminal constraint by a given residual error
- Consider the system for several different values of parameters and group the results together
- As an optimality index, consider the average performance index or the worst residual error

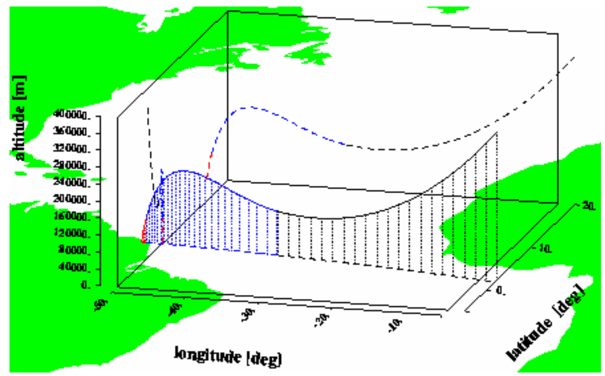


Ascend trajectory optimization

- Rocket launch vehicles
  - fuel (payload) optimality
  - orbital insertion constraint
  - flight envelope constraints
  - booster drop constraint



Ascend trajectory optimization



- Nonlinear constraint optimization problem
  - not QP, not LP
  - iterative optimization methods: Gradient, Newton, Levenberg-Marquardt, SQP, SSQP
  - can get results if supervised by a human
  - QP, LP are guaranteed always produce a solution if the problem is feasible - suitable for one-line use inside control loop

Mobile Robot Path Planning

$$F(\xi(\cdot), \eta(\cdot), t_f) \rightarrow \text{min}$$

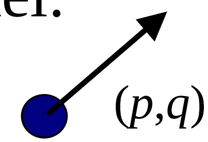
$$\ddot{\xi} = p, \;\; \|p\| \le p_{max},$$

$$\ddot{\eta} = q, \;\; \|q\| \le q_{max},$$

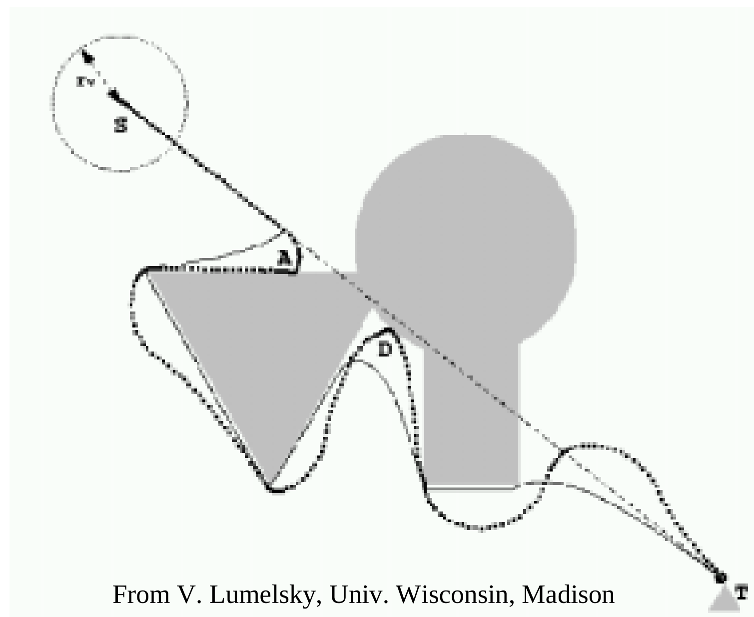
$$\xi(0) = \xi_0, \;\; \eta(0) = \eta_0, \;\; \dot{\xi}(0) = \dot{\xi}_0, \;\; \dot{\eta}(0) = \dot{\eta}_0,$$

$$\eta(t_f) = \eta(t_f) = \dot{\xi}(t_f) = \dot{\eta}(t_f) = 0$$

Point mass model:



Constraint optimization problem of finding an optimal path



From V. Lumelsky, Univ. Wisconsin, Madison

Future Combat Systems (FCS)

- Ground and air robotics vehicles
- Potential application of robotics research
- Path planning and optimization are important