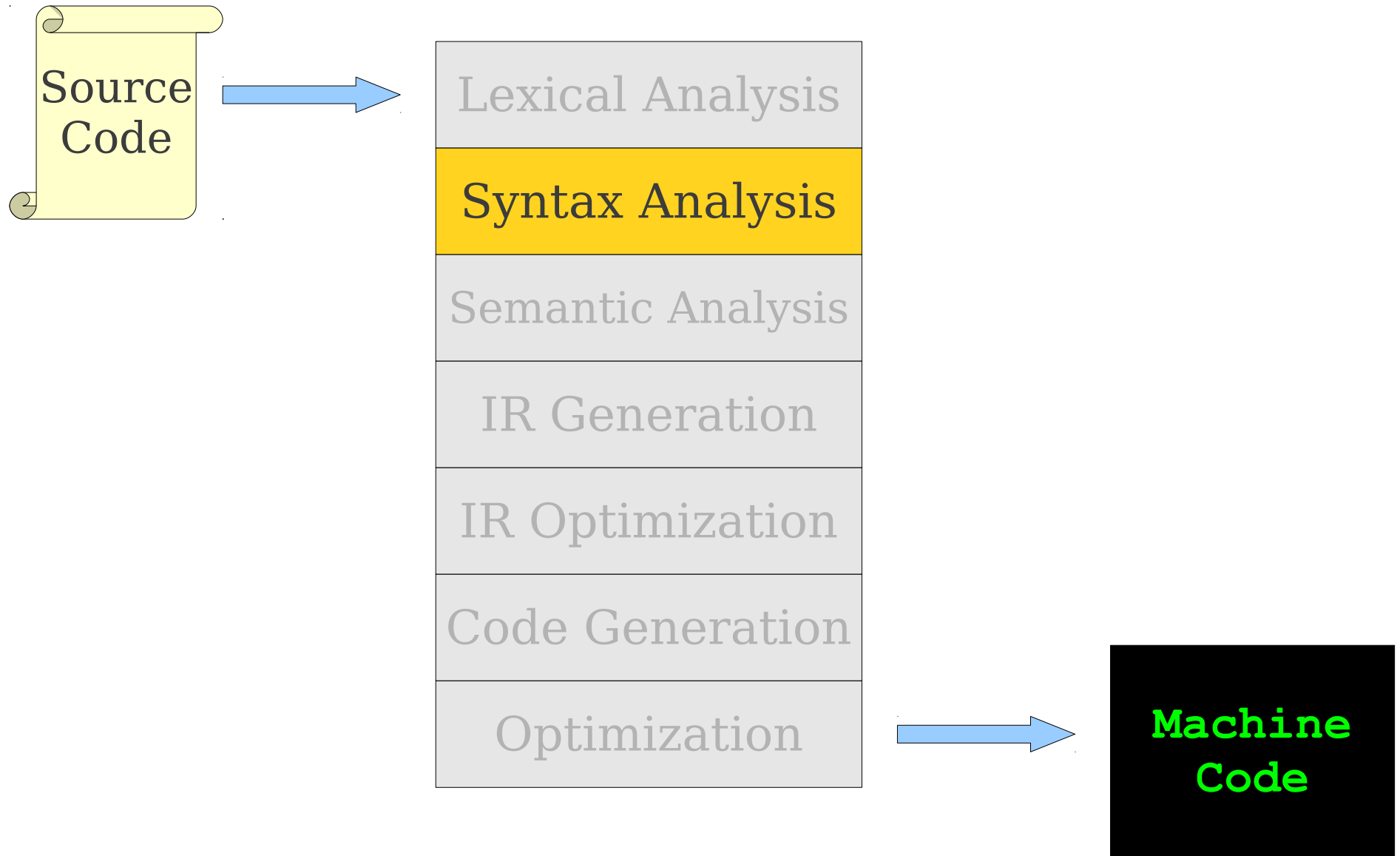


# Top-Down Parsing II

# Announcements

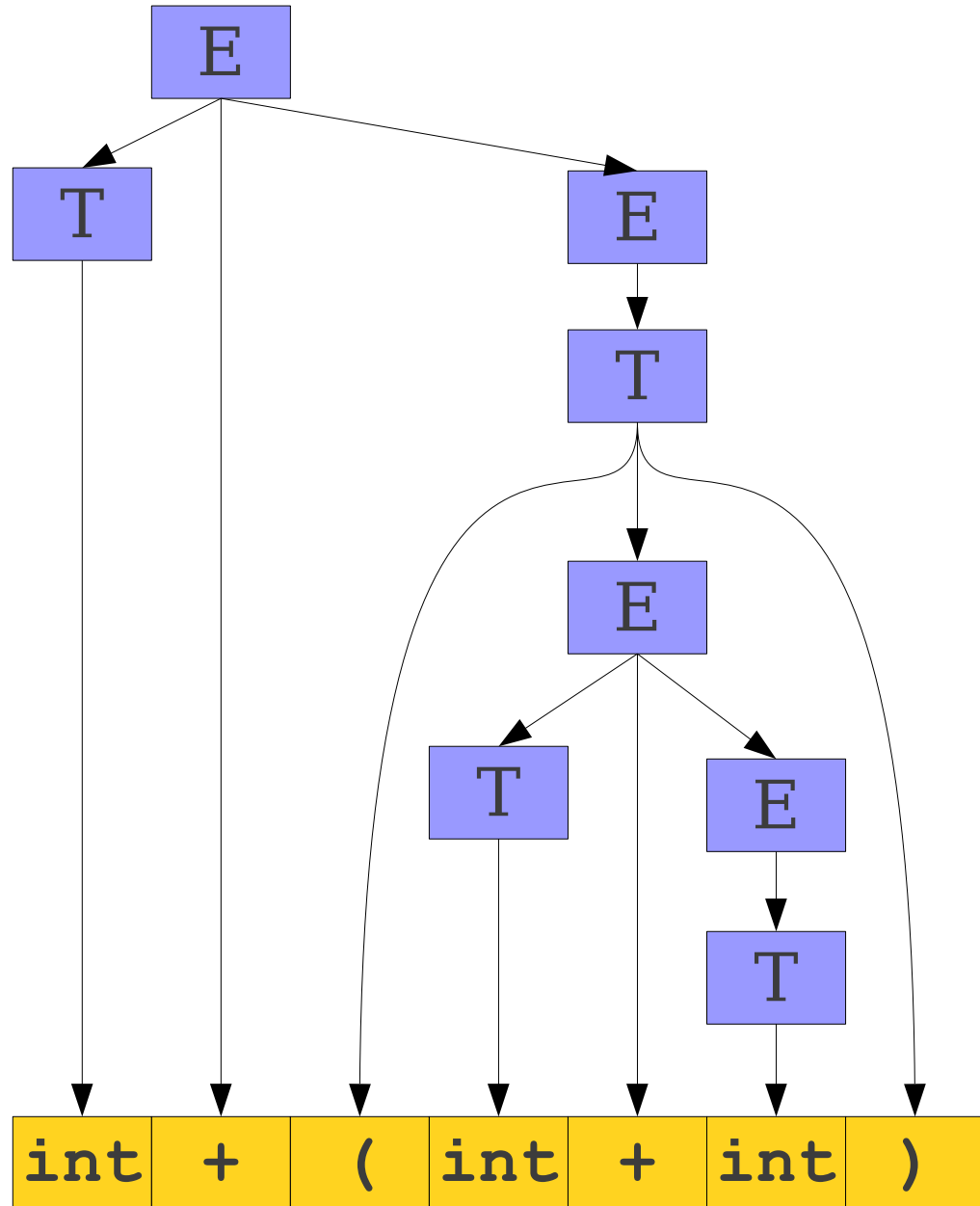
- Written Assignment 1 due this afternoon at 5PM.
  - Can submit electronically by emailing us at **[cs143-sum1112-staff@lists.stanford.edu](mailto:cs143-sum1112-staff@lists.stanford.edu)** with [WA1] somewhere in the subject line.
  - Can submit hard copies to the drop-off box in Gates (details in the problem set).
- C++ review session next Monday, time and place TBA.

# Where We Are



# Top-Down Parsing

$E \rightarrow T$   
 $E \rightarrow T + E$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$



# LL(1) Parse Tables

**E** → **int**

**E** → **(E Op E)**

**Op** → **+**

**Op** → **\***

	int	(	)	+	*
E	int	(E Op E)			
Op				+	*

# FIRST Sets

- We want to tell if a particular nonterminal **A** derives a string starting with a particular nonterminal **t**.
- We can formalize this with **FIRST sets**.

$$\text{FIRST}(\mathbf{A}) = \{ \mathbf{t} \mid \mathbf{A} \Rightarrow^* \mathbf{t}\omega \text{ for some } \omega \}$$

- We also include **ε** in  $\text{FIRST}(\mathbf{A})$  if A can produce the empty string.
- Intuitively,  $\text{FIRST}(\mathbf{A})$  is the set of terminals that can be at the start of a string produced by **A**.
- We can generalize FIRST to strings with  $\text{FIRST}^*(\omega)$  being the set of all terminals (or **ε**) that can appear at the start of a string derived from **ω**.

# FIRST Computation with $\epsilon$

- Initially, for all nonterminals  $A$ , set
$$\text{FIRST}(A) = \{ t \mid A \rightarrow t\omega \text{ for some } \omega \}$$
- For all nonterminals  $A$  where  $A \rightarrow \epsilon$  is a production, add  $\epsilon$  to  $\text{FIRST}(A)$ .
- Repeat the following until no changes occur:
  - For each production  $A \rightarrow \alpha$ , set
$$\text{FIRST}(A) = \text{FIRST}(A) \cup \text{FIRST}^*(\alpha)$$

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**  
**Sign** → **+** | **-** |  **$\epsilon$**   
**Digits** → **Digit More**  
**More** → **Digits** |  **$\epsilon$**   
**Digit** → **0** | **1** | ... | **9**



# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+** | **-** |  **$\epsilon$**

**Digits** → **Digit More**

**More** → **Digits** |  **$\epsilon$**

**Digit** → **0** | **1** | ... | **9**

	+	-	#	\$
Num				
Sign				
Digits				
More				
Digit				

# LL(1) Tables with $\epsilon$

- Num** → **Sign Digits**
- Sign** → **+ | - |  $\epsilon$**
- Digits** → **Digit More**
- More** → **Digits |  $\epsilon$**
- Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9								$\epsilon$

	+	-	#	\$
Num				
Sign				
Digits				
More				
Digit				

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num				
Sign				
Digits				
More				
Digit				

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits		
Sign				
Digits				
More				
Digit				

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits		
Sign				
Digits				
More				
Digit				

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9								$\epsilon$

	+	-	#	\$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits				
More				
Digit				

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
<b>+</b>	<b>-</b>	<b>+</b>	<b>-</b>	<b>0</b>	<b>5</b>	<b>0</b>	<b>5</b>	<b>0</b>	<b>5</b>
<b>0</b>	<b>5</b>		<b><math>\epsilon</math></b>	<b>1</b>	<b>6</b>	<b>1</b>	<b>6</b>	<b>1</b>	<b>6</b>
<b>1</b>	<b>6</b>			<b>2</b>	<b>7</b>	<b>2</b>	<b>7</b>	<b>2</b>	<b>7</b>
<b>2</b>	<b>7</b>			<b>3</b>	<b>8</b>	<b>3</b>	<b>8</b>	<b>3</b>	<b>8</b>
<b>3</b>	<b>8</b>			<b>4</b>	<b>9</b>	<b>4</b>	<b>9</b>	<b>4</b>	<b>9</b>
<b>4</b>	<b>9</b>								<b><math>\epsilon</math></b>

	<b>+</b>	<b>-</b>	<b>#</b>	<b>\$</b>
Num	<b>Sign Digits</b>	<b>Sign Digits</b>		
Sign	<b>+</b>	<b>-</b>		
Digits				
More				
Digit				

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → + | - |  $\epsilon$

**Digits** → **Digit More**

**More** → **Digits** |  $\epsilon$

**Digit** → 0 | 1 | ... | 9

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	<b>Sign Digits</b>	<b>Sign Digits</b>		
Sign	+	-		
Digits			<b>Digits More</b>	
More				
Digit				



# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**  
**Sign** → **+ | - |  $\epsilon$**   
**Digits** → **Digit More**  
**More** → **Digits |  $\epsilon$**   
**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits			Digits More	
More				
Digit				

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**  
**Sign** → **+ | - |  $\epsilon$**   
**Digits** → **Digit More**  
**More** → **Digits |  $\epsilon$**   
**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5		$\epsilon$	1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9								$\epsilon$

	+	-	#	\$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit				

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit				

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**  
**Sign** → **+ | - |  $\epsilon$**   
**Digits** → **Digit More**  
**More** → **Digits |  $\epsilon$**   
**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits		
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	



# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	$\epsilon$	
Digits			Digits More	
More			Digits	
Digit			#	

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
<b>+</b>	<b>-</b>	<b>+</b>	<b>-</b>	<b>0</b>	<b>5</b>	<b>0</b>	<b>5</b>	<b>0</b>	<b>5</b>
<b>0</b>	<b>5</b>		<b><math>\epsilon</math></b>	<b>1</b>	<b>6</b>	<b>1</b>	<b>6</b>	<b>1</b>	<b>6</b>
<b>1</b>	<b>6</b>			<b>2</b>	<b>7</b>	<b>2</b>	<b>7</b>	<b>2</b>	<b>7</b>
<b>2</b>	<b>7</b>			<b>3</b>	<b>8</b>	<b>3</b>	<b>8</b>	<b>3</b>	<b>8</b>
<b>3</b>	<b>8</b>			<b>4</b>	<b>9</b>	<b>4</b>	<b>9</b>	<b>4</b>	<b>9</b>
<b>4</b>	<b>9</b>								<b><math>\epsilon</math></b>

	<b>+</b>	<b>-</b>	<b>#</b>	<b>\$</b>
Num	<b>Sign Digits</b>	<b>Sign Digits</b>	<b>Sign Digits</b>	
Sign	<b>+</b>	<b>-</b>	<b><math>\epsilon</math></b>	
Digits			<b>Digits More</b>	
More			<b>Digits</b>	
Digit			<b>#</b>	

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**  
**Sign** → **+ | - |  $\epsilon$**   
**Digits** → **Digit More**  
**More** → **Digits |  $\epsilon$**   
**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5		$\epsilon$	1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9								$\epsilon$

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	$\epsilon$	
Digits			Digits More	
More			Digits	
Digit			#	

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**  
**Sign** → **+ | - |  $\epsilon$**   
**Digits** → **Digit More**  
**More** → **Digits |  $\epsilon$**   
**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5		$\epsilon$	1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9								$\epsilon$

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	$\epsilon$	
Digits			Digits More	
More			Digits	$\epsilon$
Digit			#	

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**

**Sign** → **+ | - |  $\epsilon$**

**Digits** → **Digit More**

**More** → **Digits |  $\epsilon$**

**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	$\epsilon$	
Digits			Digits More	
More			Digits	$\epsilon$
Digit			#	

# LL(1) Tables with $\epsilon$

**Num** → **Sign Digits**  
**Sign** → **+ | - |  $\epsilon$**   
**Digits** → **Digit More**  
**More** → **Digits |  $\epsilon$**   
**Digit** → **0 | 1 | ... | 9**

Num		Sign		Digit		Digits		More	
+	-	+	-	0	5	0	5	0	5
0	5	$\epsilon$		1	6	1	6	1	6
1	6			2	7	2	7	2	7
2	7			3	8	3	8	3	8
3	8			4	9	4	9	4	9
4	9							$\epsilon$	

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	$\epsilon$	
Digits			Digits More	
More			Digits	$\epsilon$
Digit			#	

# FOLLOW Sets

- With  $\varepsilon$ -productions in the grammar, we may have to “look past” the current nonterminal to what can come after it.
- The **FOLLOW set** represents the set of terminals that might come after a given nonterminal.
- Formally:

$$\text{FOLLOW}(\mathbf{A}) = \{ \mathbf{t} \mid \mathbf{S} \Rightarrow^* \alpha \mathbf{A} \mathbf{t} \omega \text{ for some } \alpha, \omega \}$$

where **S** is the start symbol of the grammar.

- Informally, every nonterminal that can ever come after **A** in a derivation.

# Computation of FOLLOW Sets

- Initially, for each nonterminal **A**, set
$$\text{FOLLOW}(\mathbf{A}) = \{ \mathbf{t} \mid \mathbf{B} \rightarrow \alpha \mathbf{A} \mathbf{t} \omega \text{ is a production} \}$$
- Add **\$** to FOLLOW(**S**), where **S** is the start symbol.
- Repeat the following until no changes occur:
  - If  $\mathbf{B} \rightarrow \alpha \mathbf{A} \omega$  is a production, set
$$\text{FOLLOW}(\mathbf{A}) = \text{FOLLOW}(\mathbf{A}) \cup \text{FIRST}^*(\omega) - \{ \epsilon \}.$$
  - If  $\mathbf{B} \rightarrow \alpha \mathbf{A} \omega$  is a production and  $\epsilon \in \text{FIRST}^*(\omega)$ , set
$$\text{FOLLOW}(\mathbf{A}) = \text{FOLLOW}(\mathbf{A}) \cup \text{FOLLOW}(\mathbf{B}).$$



# The Final LL(1) Table Algorithm

- Compute  $\text{FIRST}(\mathbf{A})$  and  $\text{FOLLOW}(\mathbf{A})$  for all nonterminals  $\mathbf{A}$ .
- For each rule  $\mathbf{A} \rightarrow \omega$ , for each terminal  $\mathbf{t} \in \text{FIRST}^*(\omega)$ , set  $T[\mathbf{A}, \mathbf{t}] = \omega$ .
  - Note that  $\epsilon$  is not a terminal.
- For each rule  $\mathbf{A} \rightarrow \omega$ , if  $\epsilon \in \text{FIRST}^*(\omega)$ , set  $T[\mathbf{A}, \mathbf{t}] = \omega$  for each  $\mathbf{t} \in \text{FOLLOW}(\mathbf{A})$ .

# An Egregious Abuse of Notation

- Compute  $\text{FIRST}(\mathbf{A})$  and  $\text{FOLLOW}(\mathbf{A})$  for all nonterminals  $\mathbf{A}$ .
- For each rule  $\mathbf{A} \rightarrow \omega$ , for each terminal  $\mathbf{t} \in \text{FIRST}^*(\omega\text{FOLLOW}(\mathbf{A}))$ , set  $\text{T}[\mathbf{A}, \mathbf{t}] = \omega$ .

# Example LL(1) Construction

# The Limits of LL(1)

# A Grammar that is Not LL(1)

- Consider the following (left-recursive) grammar:

$$A \rightarrow Ab \mid c$$

- $\text{FIRST}(A) = \{c\}$
- However, we cannot build an LL(1) parse table.
- **Why?**

# A Grammar that is Not LL(1)

- Consider the following (left-recursive) grammar:

$$A \rightarrow Ab \mid c$$

- $\text{FIRST}(A) = \{c\}$
- However, we cannot build an LL(1) parse table.
- **Why?**

	b	c
A		$A \rightarrow Ab$ $A \rightarrow c$

# A Grammar that is Not LL(1)

- Consider the following (left-recursive) grammar:

$$A \rightarrow Ab \mid c$$

- $\text{FIRST}(A) = \{c\}$
- However, we cannot build an LL(1) parse table.
- **Why?**

	b	c
A		$A \rightarrow Ab$ $A \rightarrow c$

- **Cannot uniquely predict production!**
- This is called a **FIRST/FIRST conflict**.

# Eliminating Left Recursion

- In general, left recursion can be converted into **right recursion** by a mechanical transformation.
- Consider the grammar

$$\mathbf{A} \rightarrow \mathbf{A}\omega \mid \alpha$$

- This will produce  $\alpha$  followed by some number of  $\omega$ 's.
- Can rewrite the grammar as

$$\mathbf{A} \rightarrow \alpha\mathbf{B}$$

$$\mathbf{B} \rightarrow \epsilon \mid \omega\mathbf{B}$$



# Another Non-LL(1) Grammar

- Consider the following grammar:

$$\mathbf{E} \rightarrow \mathbf{T}$$

$$\mathbf{E} \rightarrow \mathbf{T} + \mathbf{E}$$

$$\mathbf{T} \rightarrow \mathbf{int}$$

$$\mathbf{T} \rightarrow (\mathbf{E})$$

- $\text{FIRST}(\mathbf{E}) = \{ \mathbf{int}, ( \}$
- $\text{FIRST}(\mathbf{T}) = \{ \mathbf{int}, ( \}$
- Why is this grammar not LL(1)?

# Another Non-LL(1) Grammar

- Consider the following grammar:

$E \rightarrow T$

$E \rightarrow T + E$

$T \rightarrow \text{int}$

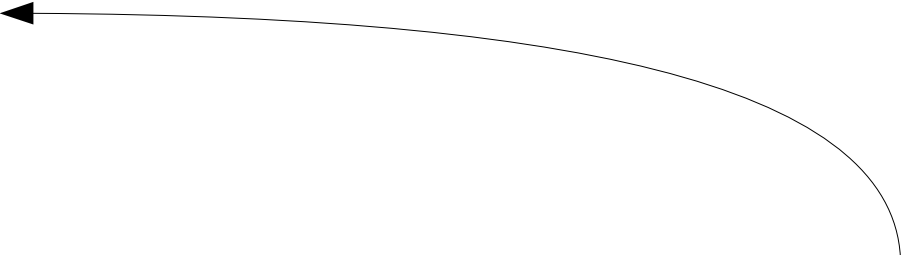
$T \rightarrow (E)$

•  $\text{FIRST}(E) = \{ \text{int}, ( \}$

•  $\text{FIRST}(T) = \{ \text{int}, ( \}$

- Why is this grammar not LL(1)?

How do you  
predict which of  
these to use?



# Left-Factoring

**E** → **T**

**E** → **T + E**

**T** → **int**

**T** → **(E)**

# Left-Factoring

**E** → **T** $\epsilon$

**E** → **T** + **E**

**T** → *int*

**T** → (**E**)

# Left-Factoring

**E** → **TY**

**T** → **int**

**T** → **(E)**

# Left-Factoring

**E** → **TY**

**T** → **int**

**T** → **(E)**

**Y** → **+ E**

**Y** → **ε**

# Left-Factoring

**E** → **TY**      **1**

**T** → **int**      **2**

**T** → **(E)**      **3**

**Y** → **+ E**      **4**

**Y** → **ε**      **5**

# Left-Factoring

<b>E</b>	→	<b>TY</b>	<b>1</b>
<b>T</b>	→	<b>int</b>	<b>2</b>
<b>T</b>	→	<b>(E)</b>	<b>3</b>
<b>Y</b>	→	<b>+ E</b>	<b>4</b>
<b>Y</b>	→	<b>ε</b>	<b>5</b>



# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
FOLLOW		
E	T	Y

# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
	int (	
FOLLOW		
E	T	Y

# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
	int (	+ ε
FOLLOW		
E	T	Y

# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
int (	int (	+ ε
FOLLOW		
E	T	Y

# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
int	int	+
(	(	ε
FOLLOW		
E	T	Y
\$		

# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
int (	int (	+ ε
FOLLOW		
E	T	Y
\$ )		

# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
int (	int (	+ ε
FOLLOW		
E	T	Y
\$ )	+	

# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
int (	int (	+ ε
FOLLOW		
E	T	Y
\$ )	+ )	\$ )



# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
int (	int (	+ ε
FOLLOW		
E	T	Y
\$ )	+ \$ )	\$ )

# Left-Factoring

<b>E</b>	→	<b>TY</b>	<b>1</b>
<b>T</b>	→	<b>int</b>	<b>2</b>
<b>T</b>	→	<b>(E)</b>	<b>3</b>
<b>Y</b>	→	<b>+ E</b>	<b>4</b>
<b>Y</b>	→	<b>ε</b>	<b>5</b>

FIRST		
E	T	Y
int	int	+
(	(	ε
FOLLOW		
E	T	Y
\$	+	\$
)	\$	)
	)	

	int	(	)	+	\$
E					
T					
Y					

# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
int	int	+
(	(	ε
FOLLOW		
E	T	Y
\$	+	\$
)	\$	)
	)	

	int	(	)	+	\$
E	<b>1</b>	<b>1</b>			
T					
Y					

# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
int	int	+
(	(	ε
FOLLOW		
E	T	Y
\$	+	\$
)	\$	)
)	)	)

	int	(	)	+	\$
E	<b>1</b>	<b>1</b>			
T	<b>2</b>	<b>3</b>			
Y					

# Left-Factoring

<b>E</b>	$\rightarrow$	<b>TY</b>	<b>1</b>
<b>T</b>	$\rightarrow$	<b>int</b>	<b>2</b>
<b>T</b>	$\rightarrow$	<b>(E)</b>	<b>3</b>
<b>Y</b>	$\rightarrow$	<b>+ E</b>	<b>4</b>
<b>Y</b>	$\rightarrow$	<b><math>\epsilon</math></b>	<b>5</b>

FIRST		
E	T	Y
int	int	+
(	(	$\epsilon$
FOLLOW		
E	T	Y
\$	+	\$
)	\$	)
	)	

	int	(	)	+	\$
E	<b>1</b>	<b>1</b>			
T	<b>2</b>	<b>3</b>			
Y				<b>4</b>	

# Left-Factoring

**E** → **TY**      **1**  
**T** → **int**      **2**  
**T** → **(E)**      **3**  
**Y** → **+ E**      **4**  
**Y** → **ε**      **5**

FIRST		
E	T	Y
int (	int (	+ ε
FOLLOW		
E	T	Y
\$ )	+ \$ )	\$ )

	int	(	)	+	\$
E	<b>1</b>	<b>1</b>			
T	<b>2</b>	<b>3</b>			
Y			<b>5</b>	<b>4</b>	<b>5</b>

# A Formal Characterization of LL(1)

- A grammar  $G$  is LL(1) iff for any productions  $\mathbf{A} \rightarrow \omega_1$  and  $\mathbf{A} \rightarrow \omega_2$ , the sets

$$\text{FIRST}(\omega_1 \text{ FOLLOW}(\mathbf{A}))$$

and

$$\text{FIRST}(\omega_2 \text{ FOLLOW}(\mathbf{A}))$$

are disjoint.

- This condition is equivalent to saying that there are no conflicts in the table.

# The Strengths of LL(1)



# LL(1) is Straightforward

- Can be implemented quickly with a table-driven design.
- Can be implemented by **recursive descent**:
  - Define a function for each nonterminal.
  - Have these functions call each other based on the lookahead token.
- See Handout #09 for more details.

# LL(1) is Fast

- Both table-driven LL(1) and recursive-descent-powered LL(1) are fast.
- Can parse in  $O(n |G|)$  time, where  $n$  is the length of the string and  $|G|$  is the size of the grammar.

# Summary

- **Top-down parsing** tries to derive the user's program from the start symbol.
- **Leftmost BFS** is one approach to top-down parsing; it is mostly of theoretical interest.
- **Leftmost DFS** is another approach to top-down parsing that is uncommon in practice.
- **LL(1)** parsing scans from left-to-right, using one token of lookahead to find a leftmost derivation.
- **FIRST sets** contain terminals that may be the first symbol of a production.
- **FOLLOW sets** contain terminals that may follow a nonterminal in a production.
- **Left recursion** and **left factorability** cause LL(1) to fail and can be mechanically eliminated in some cases.