# Programming Abstractions <br> CS106B 

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## Topics:

- Continue discussion of Binary Trees
, So far we've studied two types of Binary Trees:
- Binary Heaps (Priority Queue)
- Binary Search Trees/BSTs (Map)
, We also heard about some relatives of the BST: red-black trees, splay tress, B-Trees
- Today we're going to be talking about Huffman trees
- Misc. announcement:
, Thanks, mom!



## Getting Started on Huffman

## Encoding with Huffman Trees:

- Today we're going to be talking about your next assignment: Huffman coding
, It's a compression algorithm
, It's provably optimal (take that, Pied Piper)
, It involves binary tree data structures, yay!
, (assignment goes out Wednesday)

- But before we talk about the tree structure and algorithm, let's set the scene a bit and talk about BINARY


## In a computer, everything is numbers!

Specifically, everything is binary

- Images (gif, jpg, png): binary numbers
- Integers (int):
- Non-integer real numbers (double):
- Letters and words (ASCII, Unicode):
- Music (mp3):
- Movies (streaming):
- Doge pictures ( $\hat{c}^{-}$):
- Email messages:
binary numbers
binary numbers binary numbers binary numbers binary numbers binary numbers binary numbers

Encodings are what tell us how to translate
, "if we interpret these binary digits as an image, it would look like this"
, "if we interpret these binary digits as a song, it would sound like this"

## ASCII is an old-school encoding for characters

- The "char" type in C++ is based on ASCII
- You interacted with this a bit in WordLadder and midterm Boggle question (e.g., 'A' + 1 = 'B')
- Leftover from C in the 1970's
- Doesn't play nice with other languages, and today's software can't afford to be so America-centric, so Unicode is more common
- ASCII is simple so we use it for this assignment

| ASCll Table | DEC | OCT | HEX | BIN | Symbo | DEC | OCT | $\mathrm{H}=\mathrm{X}$ | BIN | Symbol |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 32 | 040 | 20 | 00100000 |  | 53 | 065 | 35 | 00110101 | 5 |
|  | 33 | 041 | 21 | 00100001 | ! | 54 | 066 | 36 | 00110110 | 6 |
| Notice each symbol is encoded as 8 binary digits (8 bits) | 34 | 042 | 22 | 00100010 | " | 55 | 067 | 37 | 00110111 | 7 |
|  | 35 | 043 | 23 | 00100011 | \# | 56 | 070 | 38 | 00111000 | 8 |
|  | 36 | 044 | 24 | 00100100 | \$ | 57 | 071 | 39 | 00111001 | 9 |
|  | 37 | 045 | 25 | 00100101 | \% | 58 | 072 | 3A | 00111010 |  |
|  | 38 | 046 | 26 | 00100110 | \& | 59 | 073 | 3B | 00111011 |  |
| There are 256 unique sequences of 8 bits, so | 39 | 047 | 27 | 00100111 |  | 60 | 074 | 3C | 00111100 |  |
|  | 40 | 050 | 28 | 00101000 | ( | 61 | 075 | 3D | 00111101 |  |
|  | 41 | 051 | 29 | 00101001 | ) | 62 | 076 | 3E | 00111110 | ? |
|  | 42 | 052 | 2A | 00101010 |  | 64 | 100 | 40 | 01000000 | @ |
| numbers 0-255 <br> each correspond to | 43 | 053 | 2B | 00101011 | + | 65 | 101 | 41 | 01000001 | A |
|  | 44 | 054 | 2C | 00101100 | , | 66 | 102 | 42 | 01000010 | B |
| one character <br> (this only shows 32-74) | 45 | 055 | 2D | 00101101 | - | 67 | 103 | 43 | 01000011 | C |
|  | 46 | 056 | 2E | 00101110 |  | 68 | 104 | 44 | 01000100 | D |
|  | 47 | 057 | 2F | 00101111 | 1 | 69 | 105 | 45 | 01000101 | E |
|  | 48 | 060 | 30 | 00110000 | 0 | 70 | 106 | 46 | 01000110 | F |
| $00111110=$ '<' | 49 | 061 | 31 | 00110001 | 1 | 71 | 107 | 47 | 01000111 | G |
|  | 50 | 062 | 32 | 00110010 | 2 | 72 | 110 | 48 | 01001000 | H |
|  | 51 | 063 | 33 | 00110011 | 3 | 73 | 111 | 49 | 01001001 | I |
|  | 52 | 064 | 34 | 00110100 | 4 | 74 | 112 | 4A | 01001010 | J |

## ASCII Example

| char | ASCII |
| :---: | :---: |
| h | 104 |
| a | 97 |
| p | 112 |
| y | 121 |
| i | 105 |
| 0 | 111 |
| space | 32 |

bit pattern (binary) 01101000 01100001
01110000
01111001
01101001
01101111 00100000
"happy hip hop" =
1049711211212132104105 (decimal)
Or this in binary: $a$

$\checkmark$| 01101000 | 01100001 | 01110000 | 01110000 | 01111001 | 00100000 | 01101000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 01101001 | 01110000 | 00100000 | 01101000 | 01101111 | 01110000 |  |
|  |  |  |  |  |  |  |

FAQ: Why does $104=$ 'h'?
Answer: it's arbitrary, like most encodings. Some people in the 1970s just decided to make it that way.

## [Aside] Unplugged programming: The Binary Necklace

- Choose one color to represent 0's and another color to represent 1's
- Write your name in beads by looking up each letter's ASCII encoding
- For extra bling factor, this one uses glow-inthe dark beads as delimiters between letters

| DEC | OCT | HEX | BIN | Symbol |
| :---: | :---: | :---: | :---: | :---: |
| 65 | 101 | 41 | 01000001 | A |
| 66 | 102 | 42 | 01000010 | B |
| 67 | 103 | 43 | 01000011 | C |
| 68 | 104 | 44 | 01000100 | D |
| 69 | 105 | 45 | 01000101 | E |
| 70 | 106 | 46 | 01000110 | F |
| 71 | 107 | 47 | 01000111 | G |
| 72 | 110 | 48 | 01001000 | H |
| 73 | 111 | 49 | 01001001 | I |



## ASCII

- ASCII's uniform encoding size makes it easy
, Don't really need those glow-in-the-dark beads as delimiters, because we know every $9^{\text {th }}$ bead starts a new 8-bit letter encoding
- Key insight: also a bit wasteful (ha! get it? a "bit")
, What if we took the most commonly used characters (according to Wheel of Fortune, some of these are RSTLNE) and encoded them with just 2 or 3 bits each?
, We let seldom-used characters, like \&, have encodings that are longer, say 12 bits.
, Overall, we would save a lot of space!


## Non-ASCII (variable-length) encoding example

char
h
a
p
y
i
o
space

## bit pattern

$$
\begin{aligned}
& \frac{01}{000} \\
& 10 \\
& 1111 \\
& 001 \\
& 1110 \\
& 110
\end{aligned}
$$

"happy hip hop" =

| 01 | 000 | 10 | 10 | 1111 | 110 | 01 | 001 | 10 | 110 | 01 | 1110 | 10 |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| ha $a$ | $p$ | $p$ | $y$ |  |  |  |  |  |  |  |  |  |

The variable-length encoding scheme makes a MUCH more space-efficient message than ASCII:

| 01101000 | 01100001 | 01110000 | 01110000 | 01111001 | 00100000 | 01101000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 01101001 | 01110000 | 00100000 | 01101000 | 01101111 | 01110000 |  |
|  |  |  |  |  |  |  |

## Huffman encoding

- Huffman encoding is a way of choosing which characters are encoded which ways, customized to the specific file you are using
- Example: character '\#'
, Rarely used in Shakespeare (code could be longer, say ~10 bits)
, If you wanted to encode a Twitter feed, you'd see \# a lot (maybe only ~4 bits) \#contextmatters \#thankshuffman
- We store the code translation as a tree:



## Your turn

What would be the binary encoding of "hippo" using this Huffman encoding tree?
A. 11000
B. 0101101010
C. 0100110101110
D. 0100010101111
E. Other/none/more than one


## Okay, so how do we make the tree?

1. Read your file and count how many times each character occurs
2. Make a collection of tree nodes, cach having a key = \# of occurrences and a value $=$ the character , Example: "c aaa bbb"

, For now, tree nodes are not in a tree shape
, We actually store them in a Priority Queue (yay!!) based on highest priority = LOWEST \# of occurrences
, Next:

- Dequeue two nodes and make them the two children of a new node, with no character and \# of occurrences is the sym,
- Enqueu llis rtew no

Repeat until PQ.size() $==1$



(B)

If we start with the Priority Queue above, and execute one more step, what do we get?


## Last two steps




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## Now assign codes

We interpret the tree as:

- Left child = 0
- Right child = 1

What is the code for " c "?
A. 00
B. 010
C. 101
D. Other/none

| $c$ | $a$ | $b$ |
| :---: | :---: | :---: |
| 010 | 10 | 11 |



## Key question: How do we know when one character's bits end and another's begin?

Huffman needs delimiters (like the glow-in-the-dark beads), unlike ASCII, which is always 8 bits (and didn't really need the beads).


| $c$ | $a$ | $b$ |
| :---: | :---: | :---: |
| 010 | 10 | 11 |



