

Programming Abstractions

CS106B

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Topics:

Wednesday:

- Binary Search Tree (BST)
 - › Starting with a dream: binary search in a linked list?
 - › How our dream provided the inspiration for the BST
 - Note: we do NOT actually construct BSTs using this method
 - › BST insert
 - › Big-O analysis of BST

Today:

- Binary Search Tree (BST)
 - › BST balance issues
- Traversals
 - › Pre-order
 - › In-order
 - › Post-order
 - › Breadth-first
- Applications of Traversals

BST Balance Strategies

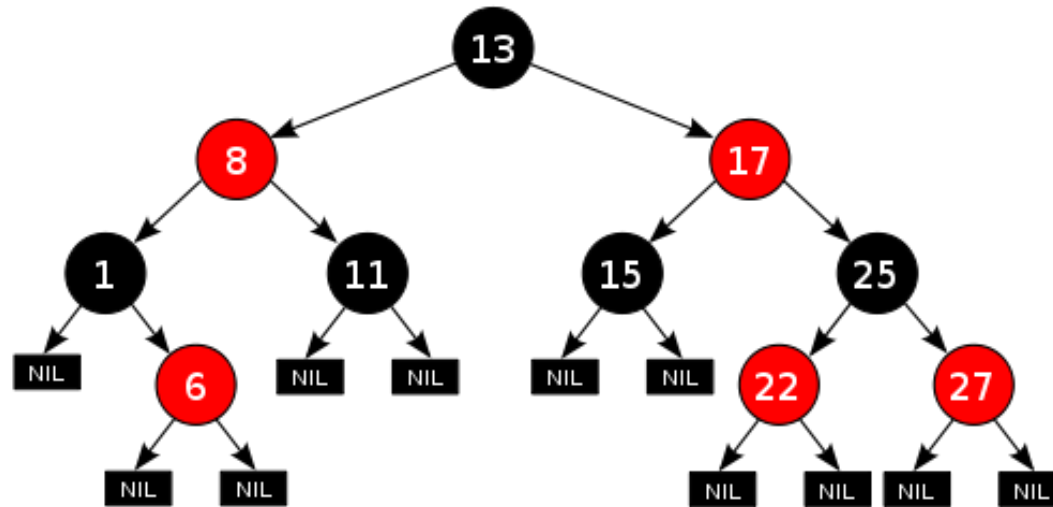
We need to balance the tree (keep $O(\log N)$ instead of $O(N)$), how can we do that if the tree structure is decided by key insert order?

Red-Black trees

One of the most famous (and most tricky) strategies for keeping a BST balanced

Not guaranteed to be perfectly balanced, but “close enough” to keep $O(\log n)$ *guarantee* on operations

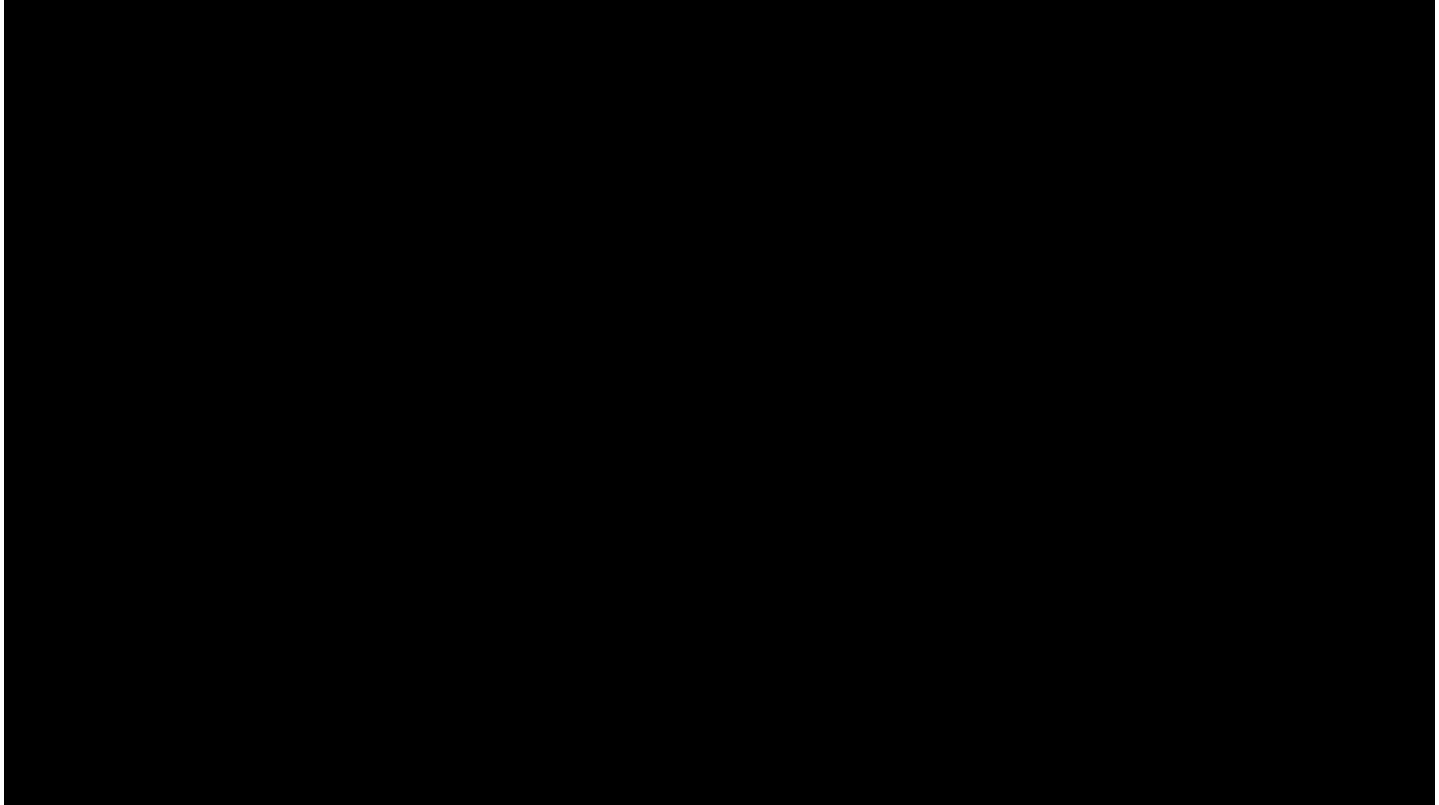
Red-Black trees



Every simple path from a given node to any of its descendant leaves contains the same number of black nodes.

- (This is what guarantees “close” to balance)

Red-Black trees insert



A few BST balance strategies

- AVL tree
- Red-Black tree
- Treap (BST + heap in one tree! What could be cooler than that, amirite? ❤️❤️)

Other fun types of **BST**

Splay tree

- Rather than only worrying about balance, Splay Tree dynamically readjusts based on **how often users search for an item**. Most commonly-searched items move to the top, saving time
 - › Example: if Google did this, “**Bieber**” would be near the root, and “**splay tree**” would be further down by the leaves

B-Tree

- Like BST, but a node can have many children, not just two
- More branching means an even “flatter” (smaller height) tree
- Used for huge databases

BST and Heap quick recap/cheat sheet

BST and Heap Facts (cheat sheet)

Heap (Priority Queue)

- **Structure:** must be “complete”
- **Order:** parent priority must be \leq both children
 - › This is for min-heap, opposite is true for max-heap
 - › No rule about whether left child is $>$ or $<$ the right child
- **Big-O:** guaranteed $\log(n)$ enqueue and dequeue
- **Operations:** always add to end of array and then “bubble up”; for dequeue do “trickle down”

BST (Map)

- **Structure:** any valid binary tree
- **Order:** $\text{leftchild.key} < \text{self.key} < \text{rightchild.key}$
 - › No duplicate keys
 - › Because it's a Map, values go along for the ride w/keys
- **Big-O:** $\log(n)$ if balanced, but might not be balanced, then $O(n)$
- **Operations:** recursively repeat: start at root and go left if $\text{key} < \text{root}$, go right if $\text{key} > \text{root}$

Tree Traversals!

These are for any binary trees, but we often do them on BSTs

What does this print?

(assume we call traverse on the root node to start)

pre-order traversal

ABDECF

```
void traverse(Node *node) {  
    if (node != NULL) {  
        cout << node->key << " ";  
        traverse(node->left);  
        traverse(node->right);  
    }  
}
```

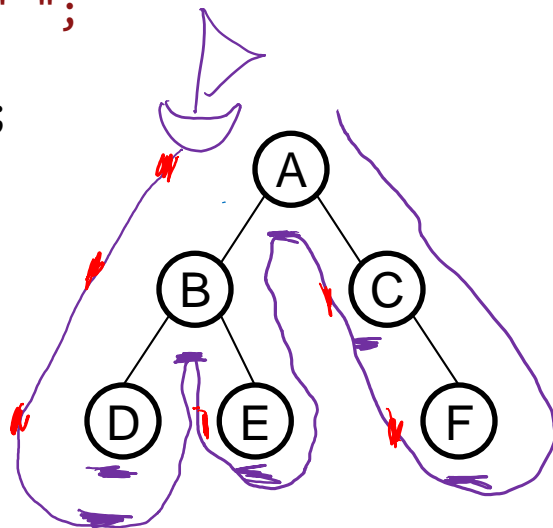
A. ABCDEF

B. ABDECF

C. DBEFC A

D. DEBFCA

E. Other/none/more



DBEACF

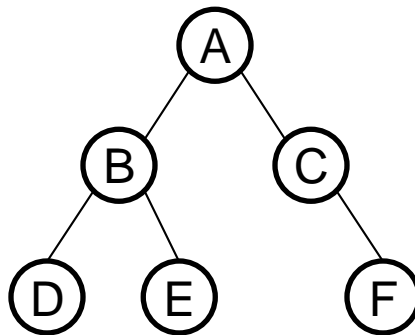
What does this print?

(assume we call traverse on the root node to start)

post-order traversal

```
void traverse(Node *node) {  
    if (node != NULL) {  
        ✓ traverse(node->left);  
        ✓ traverse(node->right);  
        ✓ cout << node->key << " ";  
    }  
}
```

DEBFCA



A. ABCDEF

B. ABDECF

C. DBEFCA

D. DEBFCA

E. Other/none/more

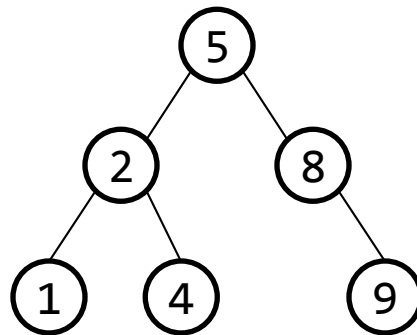
What does this print?

(assume we call traverse on the root node to start)

in-order traversal

```
void traverse(Node *node) {  
    if (node != NULL) {  
        traverse(node->left);  
        → cout << node->key << " ";  
        traverse(node->right);  
    }  
}
```

1 2 4 5 8 9



A. 1 2 4 5 8 9

B. 1 4 2 9 8 5

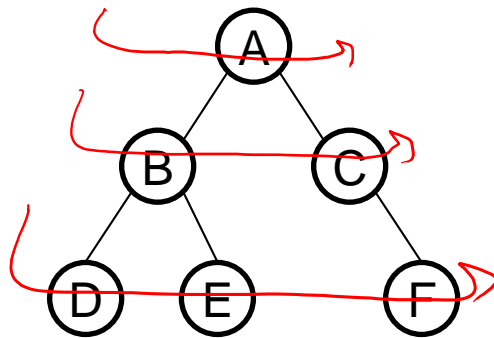
C. 5 2 1 4 8 9

D. 5 2 8 1 4 9

E. Other/none/more

How can we get code to print our ABCs in order as shown? (note: not BST order)

```
void traverse(Node *node) {  
    if (node != NULL) {  
        ?? cout << node->key << " ";  
        traverse(node->left);  
        traverse(node->right);  
    }  
}
```



You can't do it by using this code and moving around the cout—we already tried moving the cout to all 3 possible places and it didn't print in order

- You can but you use a **queue** instead of recursion
- **“Breadth-first”** search
- *Again we see this key theme of BFS vs DFS!*

Applications of Tree Traversals

Beautiful little things from an algorithms/theory standpoint, but they have a practical side too!

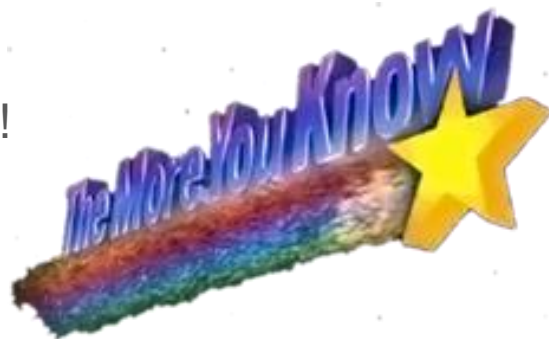
Traversals a very commonly-used tool in your CS toolkit

```
void traverse(Node *node) {  
    if (node != NULL) {  
        traverse(node->left);  
        // "do something"  
        traverse(node->right);  
    }  
}
```

- Customize and move the “do something,” and that’s the basis for dozens of algorithms and applications

Map interface implemented with BST

- Remember how when you iterate over the Stanford library Map you get the keys in sorted order?
 - (we used this for the word occurrence counting code example in class)
- ```
void printMap(const Map<string, int>& theMap) {
 for (string s : theMap) {
 cout << s << endl; // printed in sorted order
 }
}
```
- Now you know why it can do that in  $O(N)$  time!
    - “In-order” traversal



## Applications of the traversals

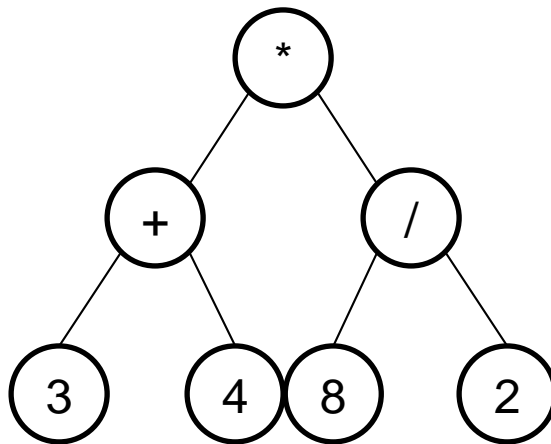
- You have a tree that represents evaluation of an arithmetic expression. Which traversal would form the foundation of your evaluation algorithm?

A. Pre-order

B. In-order

C. Post-order

D. Breadth-first



$$(3 + 4) * (8 / 2)$$

## Applications of the traversals

- You are writing the **destructor** for a BST class. Given a pointer to the root, it needs to free each node. Which traversal would form the foundation of your destructor algorithm?

- A. Pre-order
- B. In-order
- C. Post-order
- D. Breadth-first

