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CS106B

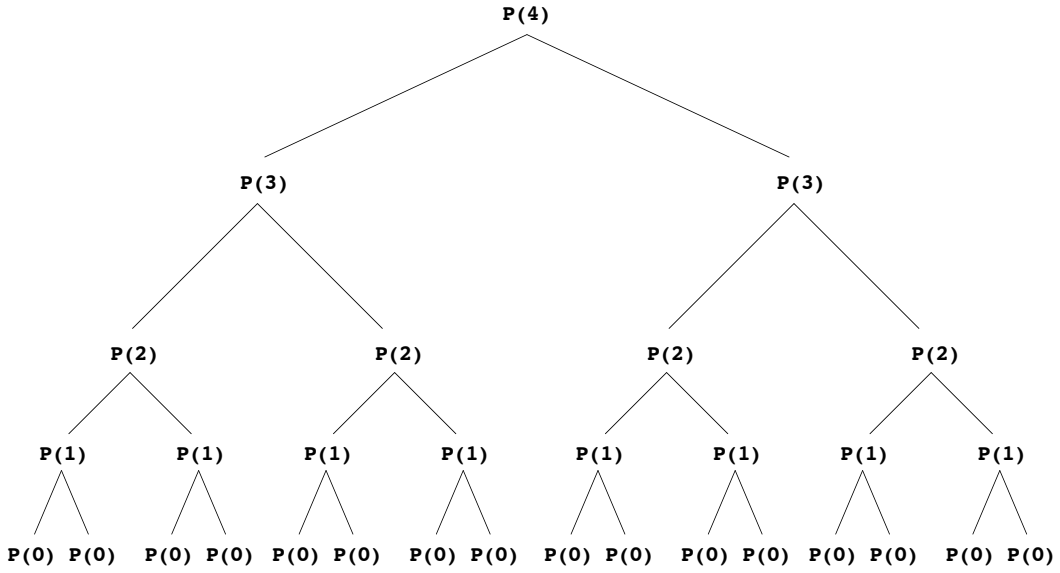
Handout #25A
February 1, 2013

Answers to Practice Midterm Exam #2

Review session: Sunday, February 3, 7:00–9:00 P.M., Hewlett 201 (next door)
Midterm #1: Tuesday, February 5, 3:15–5:15 P.M., Braun Auditorium (Chemistry)
Midterm #2: Tuesday, February 5, 7:00–9:00 P.M., CEMEX Auditorium (GSB)

Problem 1: Tracing C++ programs and big-O (10 points)

If you think about what's happening in the `puzzle` function, it should be clear that the function computes the number of moves required to solve the Tower of Hanoi problem. Thus, the value of `puzzle(4)` is 15. To understand the complexity order of the computation, it helps to draw a tree of the computations involved, which (after abbreviating `puzzle` to `P` to save space) looks like this for `puzzle(4)`:



Each new level doubles the amount of work, so the total amount of work must be $O(2^N)$. Another way to obtain this same result is that the calculation of `puzzle(`*N*`)` requires twice as many additions as the original Tower of Hanoi puzzle requires moves to solve the problem for *N* disks. If Tower of Hanoi is exponential, this function must be as well.

Note that the efficiency is a property of the implementation and not of the underlying mathematical function. If the implementation of `puzzle` were changed to

```
int puzzle(int n) {
   if (n == 0) {
      return 0;
   } else {
      return 2 * puzzle(n - 1) + 1;
   }
}
```

the complexity would be $O(N)$, even though it computes the same value.

Problem 2: Vectors, grids, stacks, and queues (10 points)

```
/*
 * Function: extract3x3Subsquare
 * Usage: Vector<int> subsquare = extract3x3Subsquare(grid, bigRow, bigCol);
 * ------------------------------------------------------------------------
 * Returns a vector containing the nine elements in the 3x3 subsquare
 * indicated by bigRow and bigCol.  Subsquares are numbered in row-major
 * order starting from the upper left corner.
 */

Vector<int> extract3x3Subsquare(Grid<int> & grid, int bigRow, int bigCol) {
   Vector<int> result;
   int r0 = 3 * bigRow;
   int c0 = 3 * bigCol;
   for (int i = 0; i < 3; i++) {
      for (int j = 0; j < 3; j++) {
         result.add(grid[r0 + i][c0 + j]);
      }
   }
   return result;
}
```

Problem 3: Lexicons, maps, and iterators (15 points)

There are several strategies that you could use to implement this problem. I believe that the simplest strategy is to calculate the result recursively, as follows:

```
/*
 * Function: countHailstoneSteps
 * Usage: int nSteps = countHailstoneSteps(n, cache);
 * --------------------------------------------------
 * Returns the number of steps in the hailstone sequence beginning
 * at n.  The cache parameter is a map that stores all previously
 * calculated chain lengths and is used to speed up the computation.
 * If the computation ever encounters a number it has seen before,
 * it simply returns the value from the cache.
 */

int countHailstoneSteps(int n, Map<int,int> & cache) {
   if (cache.containsKey(n)) return cache[n];
   if (n == 1) {
      return 0;
   } else {
      int count;
      if (n % 2 == 0) {
         count = 1 + countHailstoneSteps(n / 2, cache);
      } else {
         count = 1 + countHailstoneSteps(3 * n + 1, cache);
      }
      cache[n] = count;
      return count;
   }
}
```

You can, however, also code the solution iteratively using a stack (or a vector) to keep track of the values you need to insert into the cache:

```
int countHailstoneSteps(int n, Map<int,int> & cache) {
   Stack<int> path;
   while (n != 1 && !cache.containsKey(n)) {
      path.push(n);
      if (n % 2 == 0) {
         n /= 2;
      } else {
         n = 3 * n + 1;
      }
   }
   int count = (n == 1) ? 0 : cache[n];
   while (!path.isEmpty()) {
      count++;
      cache[path.pop()] = count;
   }
   return count;
}
```

Problem 4: Recursive functions (10 points)

```
/*
 * Function: removeDoubledLetters
 * Usage: string shorter = removeDoubledLetters(str);
 * --------------------------------------------------
 * Removes all but the first of a sequence of identical letters from str.
 */

string removeDoubledLetters(string str) {
   if (str.length() <= 1) {
      return str;
   } else if (str[0] == str[1]) {
      return removeDoubledLetters(str.substr(1));
   } else {
      return str[0] + removeDoubledLetters(str.substr(1));
   }
}
```

Problem 5: Recursive procedures (15 points)

```
/*
 * Function: tryAllOperators
 * Usage: tryAllOperators(exp, target);
 *        tryAllOperators(prefix, rest, target);
 * ----------------------------------------------
 * Recursively replaces every ? in the expression by each of the
 * primary arithmetic operators (+, -, *, /).  If the resulting
 * expression evaluates to the target integer, the function
 * prints out the expression string that generated it.  The first
 * version of the function is a simple wrapper for the second,
 * which divides up the string one character at a time, keeping
 * track of the previously considered characters in prefix.
 */

void tryAllOperators(string exp, int target) {
   tryAllOperators("", exp, target);
}

void tryAllOperators(string prefix, string rest, int target) {
   if (rest == "") {
      if (evaluateExpression(prefix) == target) {
         cout << prefix << endl;
      }
   } else if (rest[0] == '?') {
      rest = rest.substr(1);
      tryAllOperators(prefix + "+", rest, target);
      tryAllOperators(prefix + "-", rest, target);
      tryAllOperators(prefix + "*", rest, target);
      tryAllOperators(prefix + "/", rest, target);
   } else {
      tryAllOperators(prefix + rest[0], rest.substr(1), target);
   }
}
```