Recognizers and Recognizability

- A TM M is called a ***recognizer*** for a language L over Σ if the following statement is true:

$$\forall w \in \Sigma^*. (w \in L \leftrightarrow M \text{ accepts } w)$$

- A language L is called ***recognizable*** if there is a recognizer for it.
- If you are absolutely certain that $w \in L$, then running a recognizer for L on w will (eventually) confirm this.
  - Eventually, M will accept w.
- If you don't know whether $w \in L$, running M on w may never tell you anything.
  - M might loop on w – but you can't differentiate between "it'll accept if you wait longer" and "it will never come back with an answer."
- Does this feel like "solving a problem" to you?