Mathematics Worth Knowing, Resources Worth Growing, Research Worth Noting: A Response to the National Mathematics Advisory Panel Report

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The authors praise *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (2008) for focusing on the mathematics within mathematics education. They critique the Panel for (a) constraining its analysis to two traditional school courses, (b) isolating independent factors and undervaluing integrated approaches, and (c) overlooking recent insights on mathematics learning. The authors urge others to seek deeper analysis of "mathematics worth knowing," to integrate multiple resources into instructional approaches, and to delve more deeply into recent learning research.

Keywords: learning; mathematics; teaching; technology

Foundations for Success: The Final Report of the National Mathematics Advisory Panel (NMAP; 2008) joins a set of high-profile documents calling for improvements to American mathematics education and especially for a focus on improving students' performance in algebra (Augustine, 2005; Gardner, 1983; National Science Board, 2006). Distinctively, the Panel focuses on the nature of mathematics, the challenges in teaching and learning mathematics, and some of the most significant debates within communities of researchers and reformers.

The sharper focus of this report is important for the field and the nation. The report acknowledges the unique role of mathematics content for understanding and improving mathematics teaching and learning, for example:

- The importance of particular topics (e.g., rational numbers)
- The properties of focus, coherence, and closure in mathematics curricula
- The required integration of concepts and procedures
- The need to boost teachers' mathematical knowledge

Significantly, the report brought together mathematicians, researchers, and educators from various sides of the "Math Wars" (Schoenfeld, 2004) to talk about the actual content of mathematics education and to dissolve false and regressive dichotomies. For example, the report provides a balanced resolution for the incessant, unproductive debates on teacher-centered versus student-centered instruction and on the need to emphasize algorithms versus the need to emphasize concepts.

Although we recognize these strengths we also found three strategic moves by the Panel that limit the scope of its report.

First, the Panel chose to operationalize its charge from U.S. President George W. Bush as a mandate to define the content of two courses and to specify the learning experiences prerequisite to success in those courses. The president asked the Panel to examine "the critical skills and skill progressions for students to acquire competence in algebra and readiness for higher levels of mathematics" (NMAP, 2008, p. 71). Conventionally, algebra with a lowercase *a* is a central domain of mathematics, whereas Algebra I and Algebra II are particular courses, usually taken in high school. The Panel chose not to broadly and critically examine the relevance of (lowercase) algebra to modern economic life, its role in scientific activities, and its function in a highly technological world. Instead, the Panel defined Algebra I and Algebra II in the most conservative way (based on the intersection of the content of international curricula for such courses) and then further restricted its focus to "addressing the teaching and learning of mathematics from preschool to Grade 8 or so" (p. 8). We argue that the research and practical basis for this strategic move is weak and that the narrow definition adopted by the Panel limits the scope and value of high school mathematics. We submit that a response to the president's request that takes seriously the possibility of alternative mathematical pathways through and beyond algebra would better serve the nation.

Second, the Panel emphasized the contributions of isolated and individual resources or factors, such as teachers' content knowledge, classroom technologies, and instructional approaches. Not surprisingly, the report identified weak evidence for each factor and called for more research on each factor. The complexity of mathematics teaching and learning requires approaches that integrate many individual resources and capabilities, systemically, to produce profound mathematical experiences (Ball & Forzani, 2007). If continued research on isolated factors is conducted, we suggest that this psychological view of learning must be complemented by much more and better implementation research so that consumers of a portfolio of research findings will be able to consider the impact of uncontrolled or unexamined factors on the findings about the impacts of isolated variables.

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Third, operationally, the Panel broke into task groups that were largely populated by members with similar knowledge and interests. Unfortunately, the impact of each of these groups on the final report was significantly diluted (indeed, one can observe ideological shifts in the reporting of research from the task group working documents through to the report). Most particularly, we note that the work of the Task Group on Learning Processes was not integrated with the core question of what mathematics is worth knowing. Had recent advances in scientific research on learning been taken more seriously, the Panel could have benefited from "conceptual collisions" of great importance. The resulting deliberations might have led to creative synergies between models of mathematics and theories of learning.

Within space limitations, we sketch our arguments, below. We begin by briefly sharing our review process and the work that informs our perspective.

Our Review Process and Our Perspective

In our review, we read both the final report and the Panel's draft task group reports (published at http://www.ed.gov/about/bdscomm/list/mathpanel/index.html). Researchers will be pleased to learn that although the final report reflects the difficulty of reaching political consensus, the task group reports contain some bold and solid scholarship, revealing significant nuances that were lost in the final report. The data and arguments of the task group reports help clarify many of the recommendations made in the final report and are worth exploring in detail.

The lenses we bring to the report were formed in part by our own work for two programs of research: one with the National Science Foundation's Learning in Informal and Formal Education (LIFE) Science of Learning Center and the other with a Scaling Up project named SimCalc. Both programs are interdisciplinary, drawing scholars from multiple institutions to tackle challenges of teaching, learning, and meaningful and informative assessment.

LIFE draws together brain scientists, experts in informal learning, and leading theorists of the learning sciences to develop a deeper social view of lifelong and lifewide learning (Bransford et al., 2006). The work of the LIFE Center shows that each setting or context of learning (such as the school and the home) involves different constellations of social factors, including different ways of judging outcomes to be successes or failures (e.g., Banks et al., 2007). For example, one LIFE study documents the case of a young girl viewed by the schools as weak in chemistry but who avidly and systematically mixes chemicals at home to make perfumes, and plans to grow up to be a scientist (Bell, Zimmerman, Bricker, & Lee, 2008). Our research leads us to consider learning not just as mastering content but also as developing *identity.* How do students come to identify as capable and motivated mathematics learners? Another relevant component of our work rethinks transfer and assessment as *preparation for future learning* (Schwartz, Bransford, & Sears, 2005). Crucially, it is possible not only to measure current skills and knowledge but also to measure how ready the student is for future learning in the same domain. This finding speaks directly to the president's request to consider readiness for future mathematics learning, beyond high school algebra courses.

The Scaling Up SimCalc project, which spans 12 years of development, draws together mathematics educators, experimentally minded evaluators, and developers of a technology-rich curriculum to investigate whether new materials can democratize access to more advanced mathematical proficiencies (Roschelle, Tatar, Shechtman, & Knudsen, 2008). The SimCalc curriculum critically questions and redesigns the traditional list of topics in school mathematics at certain grade levels. For example, as early as seventh grade, we focus on the linkage between rate and proportionality, with extensive emphasis on the connections across multiple representations: graphing, equations, tables, and situation models (Roschelle at al., 2007). We introduce piecewise linear functions in seventh grade because of their utility in analyzing not only complex mathematical curves but many real-world situations as well. In two rigorous randomized control trials we found that an *integration* of technology, teacher professional development, and curriculum produced a large, statistically significant effect across varied demographic settings in learning this and other advanced mathematical topics (Roschelle, Tatar, Shechtman, Hegedus, et al., 2008; Roschelle et al., 2007). We see technology as an infrastructure for an expanded practice of mathematics teaching and learning, not as an isolatable silver bullet that might directly produce profound learning experiences; this is a point that we can constructively address in the section on compound resources that follows. Regrettably, the Panel's methodological filter results in a treatment of technology use in mathematics learning that is weak and dated despite the availability of some quite strong evidence, including our own, regarding the potential of technology to support learning.

Mathematics Worth Knowing

The report made an advance by focusing national attention on the content of mathematics. Now, the next step is to reconsider what kinds of mathematics are really worth knowing in response to economic challenges and our desire to enhance students' life opportunities.

The report uncritically accepted the "Educational Gospel" (Grubb & Lazerson, 2004) linking (a) academic gains in cross-sectional mathematics scores at some particular grade level to (b) later economic benefits to individuals and society. To our knowledge, there are no randomized experiments linking improvements in student performance in algebra courses to students' life outcomes or to a society's economic gains.[1] Such associations are supported only by correlational and anecdotal data. Ironically, the Panel accepted a premise grounded in weak correlational research as a strategic frame for analysis. As Thompson notes in his article in this issue of *Educational Researcher* (pp. 582-587), pervasive subjective judgment in the report contradicts a review process that self-consciously prided itself on attending only to scientific experiments. More important, such associations conflate the role of high school algebra as a *credential* with the role of a deep and meaningful understanding of algebra as *preparation for future learning.*

Two contrasting situations highlight the problems of interpreting the economic meaning of international test score comparison data:

- Singapore's mathematics scores lead in many international comparisons, but its education ministry and researchers are deeply concerned about whether the country's intensely assessment-driven culture may produce high math scores in school but squelch the creativity and innovation needed by future scientists to produce scientific and business breakthroughs (e.g., Luke, Freebody, Shun, & Gopinathan, 2005).

- The United States is more consistent in innovating and increasing productivity across many sectors of its economy than any other country (most other countries increase productivity in only a few sectors)—despite long-lived fears of declining mathematical competence among its school-aged students (Lewis, 2004).

Economic research repeatedly finds that students' progression to more and deeper mathematics is a much better predictor of economic outcomes than test scores at any cross-sectional level (Ramirez, Luo, Schofer, & Meyer, 2006). This is only common sense. The population of students who will use algebra seriously in their professional lives will relearn it with greater sophistication at the university level. For those who will use algebra only tangentially in later life, we have seen no randomized experiments showing that intervening to boost students' scores today will increase their life opportunities a decade or more later. It may be much more important that they leave school algebra with a productive disposition toward further engagement in mathematics (Grubb, 2008). Thus the economic data more strongly support a recommendation to work on learning progressions through and beyond algebra than a recommendation to work on defining a particular course and making sure students pass it. A focus on learning progressions will also steer us in the research community to deepen our understanding of the social and cultural conditions—inside and outside school contexts—that provide productive learning pathways to enable all learners to engage in mathematics over many years.

The preceding arguments may seem counterintuitive because it has become so commonplace to talk about the "gatekeeper" role of algebra for college admissions and career success. But few define exactly how today's school algebra supports students' continued progression at the university level. It could easily be the case that Algebra I and II function as important admission credentials (U.S. Department of Education, 1999) and that alternative ways of defining the mathematics curriculum could lead students through algebra in a way that better prepares students for future learning in university science, technology, engineering, and mathematics (STEM) settings and in the workplace.

Contrary to the Panel's emphasis on defining achievement at a point in time, based on Preparation for Future Learning assessments (Schwartz, Lindgren, & Lewis, in press; Schwartz & Martin, 2004), schools could simultaneously measure the specific skills taught during instruction *and* examine students' ability to learn a related new topic during the test. We can and should measure not just specific topics and skills but also readiness to use algebra in future learning, across the lifespan.

In defining high school algebra, the NMAP seems to have focused exclusively on the first two strands of the more comprehensive definition of mathematical proficiency used by the National Research Council (Kilpatrick, Swafford, & Findell, 2001). The Panel focuses on conceptual understanding and procedural fluency. This focus is laudatory because it eliminates false dichotomies between learning basic skills or concepts; we agree that students need both. However, the Panel does not include the other aspects of proficiency described by the National Research Council: strategic competence, adaptive reasoning, and productive disposition. Given the likely value of these three aspects of competency to economic competitiveness, we argue that, at a minimum, a working definition of school algebra ought to be more than a list of mathematical topics. The definition should bring to life how students will learn to use algebra with strategic competence, develop capabilities for adaptive reasoning, and perhaps most important, develop the proclivity to use mathematics appropriately and powerfully in everyday situations.

We also worry about the particular topic list that the report recommends. Modern learning research and technology, particularly those technologies that enable dynamic representation and simulation,[2] support radically reconceptualizing mathematics curricula and pedagogy. New learning pathways become possible as we learn more about learning and consider the power of new technologies. For example, with dynamic mathematics technologies, students can be introduced to piecewise linear equations (normally an upper-level university topic) beginning in seventh and eighth grades. Piecewise linear equations are powerful tools for modeling and analyzing nonlinear phenomena (such as changing rates of growth) as a series of linear approximations to intervals of the growth curve. In addition, students can learn how rates (derivatives) relate to accumulations (integrals), which is highly relevant to understanding the difference between a recession and a falling rate of economic growth.

As of this writing, we have no data from rigorous experiments or other methods to answer vital questions such as these:

- Will our society be more economically competitive if students spend more time on piecewise linear functions or radical (e.g., square root) expressions?
- Will students be more likely to succeed in college STEM degrees if, as eighth-grade students, they develop strategic and adaptive reasoning with a shorter list of key concepts—or if they develop greater speed and accuracy in the longer list of paper-and-pencil symbol manipulations that conventional tests can easily measure?
- Will more students be motivated to progress through the STEM pipeline to become world-class scientists and innovators if their middle school mathematics experience is more narrowly constrained to traditional topics—or if it expands to include relevant 21st-century mathematics, including more attention to nonlinearity, complexity, iterative systems, graph theory, and other drivers of today's intellectual and entrepreneurial progress?

A bolder report by the NMAP could have acknowledged both (a) that Algebra I and II are important credentials in the selection of students for further study, *and* (b) that—as most people intuitively know—a narrow conventional definition of algebra disconnects it from the demands of everyday economic life. There are many possible learning progressions into, through, and beyond algebra that can be equally mathematically rigorous. Some of these could better prepare students for a world in which more science depends on piecewise, iterative approximations and less depends on finding elegant, closed-form solutions. Some could compel more students to persist longer and more deeply in advanced mathematics. And some could be more strongly activated by the social and contextual factors that we will discuss shortly, and offer students much more profound learning experiences, opening their eyes to the power and beauty of mathematics. We owe our future generations a vibrant, ongoing debate about which learning progressions are most fertile.

Resources Worth Growing: Integrating Technology, Teacher Knowledge, and Teaching Practices

Despite its insistence on the centrality of randomized clinical trials in scientific progress, most of the Panel's concerns are better addressed by longitudinal methods—that is, methods that are suited to answering research questions about how we can help more students prepare for and succeed in algebra in such a way that algebra eventually contributes to economic well-being for the students as individuals and for society as a whole. One valuable source of longitudinal data is NELS88 because it tracks a large population of students across many years of development. Analyzing these data led Grubb (2008) to emphasize the difference between "simple" resources (e.g., giving a school more money) and "compound" or "complex" resources that require integrating more than one factor and often must be constructed (at least in part) by local leaders and teachers. Grubb argues that to make a meaningful difference in mathematics learning in the lives of students, we need to invest in complex or compound resources—not pin false hopes on single-factor solutions.

Regrettably, the Panel analyzed the effects of teachers and technology in isolation. We will look at technology first.

The Panel cited the study by Dynarski et al. (2007), which reported on the effectiveness of educational technology interventions. This large-scale randomized experiment compared several commercial products for improving mathematics learning in business-as-usual conditions. The study has been widely reported as showing that technology has no effect, yet other interpretations are warranted. Most important, it found enormous variation in the effect size of technology between schools. Although these effects summed to zero, a more accurate conclusion is that the effectiveness of technology depends on how teachers and schools integrate it into their practices, including planning, instruction, assessment, and reflection. An intervention that paid systematic attention to such integration could have achieved better results (the Dynarski et al. study design expected teachers who were new to technology and in Title I schools to do most of the integration work themselves). Indeed, in our own Scaling Up SimCalc work, we provided teachers with integrated teacher professional development, curriculum materials, and the aligned technology-based tools and found in two rigorous randomized controlled trials that the *integration* produced a large, statistically significant effect size (Roschelle et al., 2007; Roschelle, Tatar, Shechtman, Hegedus, et al., 2008).[3]

Another key problem in the study by Dynarski et al. has to do with measurement. In its examination of products that might improve algebra learning, the Dynarski team used a measure that was very insensitive to changes in student learning; in the control and treatment groups, the average student was able to answer only one or two additional questions correctly after a *whole year* of instruction (Dynarski et al., 2007, p. 71). Use of an insensitive measure is an important alternative explanation for the failure to detect differences. Ironically, the Panel frequently complains in its report that too much of the available research measured only students' ability to perform calculations and not their ability to use concepts. Given the Panel's strong message about the necessary integration of procedures and concepts in teaching and learning, we must be careful about generalizing findings from research (such as the Dynarski et al. study) that measures only a portion of mathematics achievement. Indeed, the Panel's Task Group on Instructional Practices noted:

> The direct implications of the Dynarski study are serious cautions to anyone who believes merely introducing technology will raise students' scores. . . . Thus, educators must consider not only empirical evidence of effectiveness of a particular software package, but also issues of scale-up, including integration with the extant curriculum, fidelity of implementation, including amount of use, and technological and pedagogical support. (p. 4-138)

In short, we see technology as an *inappropriate* unit of intervention. Technology is infrastructural (Kaput, Hegedus, & Lesh, 2007) in the same ways that paper and pencil or blackboards are infrastructural. Infrastructural components need to be treated as part of *compound* or *complex* interventions (using Grubb's terms), which integrate many factors and may need to be partially constructed by schools in situ. For example, the intervention in the Scaling Up SimCalc research integrated teacher professional development, paper curriculum, and a category of technology not considered by the Panel (but related to graphing calculators)—interactively dynamic, representational technology. We find it noteworthy that one day of professional development time was dedicated to supporting the implementing teachers in planning the detailed integration of the intervention into their local school context—and that these integrations were allowed to vary by school (in small ways unrelated to the coherence of the intervention but deeply related to how specific schools structured the use of time and physical space).

Continuing on the theme of compound resources, in a move unsupported by theory, the Panel decided to put teacher knowledge in its own task group and teacher practices in its own task group, resulting in an isolation of factors that research says must be integrated.

In our view, the Panel stretched rigorous research well beyond reasonable limits of interpretation to reach the conclusion that we must intervene to enhance teachers' mathematical knowledge per se. Scaling Up SimCalc studies have found trends linking teachers' mathematical knowledge to student achievement but found much stronger correlations between specific teaching practices and student achievement (Pierson, 2008).

Mathematical knowledge for teaching is implicated in teachers' ability to communicate relationships between ideas in classroom experiences, interpret and evaluate students' mathematical thinking on the fly, explain why students' strategies are or are not effective, and understand the longitudinal trajectories along which students can effectively learn mathematics (Grossman & Schoenfeld, 2005; Ma, 1999; Shulman, 1986). As an example, Ma found that whereas American teachers were satisfied with using "borrowing" as an explanation for subtraction while regrouping, Chinese teachers explained this procedure by talking about "decomposing a higher value unit." This instructional contrast exemplifies the overall finding that American teachers typically focus on procedures, and their knowledge is generally rule-bound and fragmented. Chinese teachers demonstrate both algorithmic competence and conceptual understanding, and accordingly, their knowledge is typically more conceptual and interconnected. These differences are evident despite the fact that the teachers spend roughly an equivalent number of years in mathematics preparation. The TIMSS international video comparison studies (Stigler & Hiebert, 1997) likewise show that American teachers tend to minimize the cognitive complexity of the tasks students work on, whereas teachers in cultures with stronger mathematical traditions require students to engage the full range of cognitive challenges present in advanced mathematics.

A contemporaneous scholarly work took a more integrated approach to the same topic. Hiebert and Grouws (2007) described the real complexity of teaching and the corresponding low probability that single-factor analyses will yield effective policy recommendations. They then noted that different teaching practices are likely to be effective for different instructional outcomes: A goal of efficient, accurate skill calls for one set of teaching approaches; a goal of complex knowledge generation calls for others. Finally, they summarized two teaching recommendations that have emerged as sound for achieving more conceptual outcomes: explicitly focusing on concepts in instruction and allowing students to struggle with challenging mathematical problems. Both of these strategies draw upon teachers' mathematical knowledge, but they also relate to the integrative processes involved in classroom enactment. In our view, the synthesis by Hiebert and Grouws is particularly satisfying because it acknowledges the real complexities of the question, situates its claims in the interactions between teachers and students (not in teachers' backgrounds or in their heads), and draws upon strong bodies of scientific research without narrowing its focus to a single methodology. Overall, we see syntheses that point to the compound nature of effective interventions as more useful than syntheses that try to reduce interventions to single factors. In our experience, mathematics teaching and learning are too complex to be amenable to single-factor interventions.

Research Worth Noting: Broadening Our Perspective on Learning

After reading the collection of public documents created by the Panel, we recommend paying particular attention to the report of the Task Group on Learning Processes (Geary et al., 2008) because of its high quality and utility. This particular report has several useful parts. It presents an overview of information processing theory, the role of mental representation, and a survey of social, motivational, and affective influences on learning. These broad, scholarly primers on their respective topics are likely to be useful to mathematics educators for reconnecting with the basic science of cognition. Another substantive section surveys what is known about particular developmental levels and mathematics topics; this handbook-like section should be useful as background for teachers and researchers who plan an investigation on a particular topic.

We focus our discussion on a smaller section, subtitled "Potential Sources of the Achievement Gap," because this section strongly demonstrates the power of good scholarship to shed light on a vexing societal issue. The task group (Geary et al., 2008) begins:

> The conventional explanations for poor math performance for Black and Hispanic students center on inadequate social experiences and learning opportunities linked to low socioeconomic status. Because Black and Hispanic children are disproportionately poor, and because poor children perform less well, this then identifies the root cause of such performance deficiencies. (p. 2-144)

But the task group then highlights how the pattern of results defies easy explanation. It boldly states a desire to go beyond simply reducing differential outcomes to poverty-based explanations by investigating actual mediating factors and processes. Further, the task group distinctly avoids a disempowering focus on poverty by looking at ways in which the structures and processes of schools and teaching can mitigate at least some of the ill effects of economic and social circumstance. This relatively compact section vitally pulls together scholarship from a variety of sources to unpack socioeconomic status into a view of learning that is (a) considerably broader than most cognitive accounts, and (b) more amenable to intervention.

To provide further research-based guidance, the task group systematically considered seven potential processes that could address the achievement gap. These processes are (1) stereotype threat; (2) cognitive load; (3) engagement, effort, and efficacy; (4) strategy use; (5) constructive and supportive academic interactions; (6) collaborative learning; and (7) culturally and socially meaningful learning contexts (we would also add meaningful assessment contexts; e.g., see Schwartz et al., 2005). Indeed, important studies using a variety of scientific methodologies are now asking when, how, and why knowledge about the presence of other people affects comprehension, memory, and communication; reduces cognitive load; and helps unlock student interests and motivation to tackle difficult concepts in mathematics and other topics (e.g., Davis, Lee, Vye, & Bransford, 2006; Lee, Davis, Vye, & Bransford, 2008). In their discussions, the task group members explicitly noted that the research they were drawing upon was not based solely on data from randomized experiments; they often justified the quality of the research on its own merits. The research cited has a tremendous amount of detail, and we lack the space here to summarize it all. We do call attention to the broader views of *learning* that the task group found to be necessary in examining ways to reduce the achievement gap.

One important set of factors in this broader view concerns stereotype threat, mathematics anxiety, and self-efficacy beliefs. A study surveying students' racial, mathematical, and academic identities, as well as their mathematics achievement, revealed that race is highly salient for students, particularly for Black students. Moreover, stereotype talk (i.e., that "Blacks aren't smart"; Hernstein & Murray, 1994) emerged when students were worried about their performance in mathematics (Steele, 1997). Research found that students in all racial/ethnic groups endorse some stereotypical and some nonstereotypical definitions of their racial/ethnic group and that these definitions are related to achievement and identity.

The work sketched above foregrounds the perspective that academic performance in STEM disciplines is, in part, created in social environments rather than inherent to individuals. For example, the Panel reports that Black students perform better in mathematics learning environments stressing communal values. Black students appear to increase performance particularly when their teachers are caring, want them to do well, and take a personal interest in them. Research has found that 71% of Black students as compared with 30% of White students appear to look to their teachers for caring and support. In a related finding, emotional feedback (e.g., giving praise, working to reduce anxiety) appears to be more important to Hispanic than to White students. Collaborative and peer-assisted social learning strategies also seem to be particularly useful for students on the wrong side of the achievement gap.

Finally, the task group points to the relevance of "socially and culturally meaningful learning contexts" (Geary et al., 2008, p. 4-104). Overall, we see this emphasis on social learning as long overdue. Students' beliefs that their interactions are socially meaningful enhance learning above and beyond the information transferred among participants in a social interaction (e.g., Okita, Bailensen, & Schwartz, 2008).

A comprehensive and systemic approach to research like this should inform both our determination of what belongs in the core mathematics curriculum for all students and our vision for excellence in mathematics teaching and learning. Unfortunately, because of the committee structure used by the Panel, this crucial research was isolated from the work defining the content of the mathematics, the work examining what teachers should know, and the work evaluating instructional practices worthy of wider adoption. Hence, the Panel missed opportunities to ask groundbreaking questions such as the following:

- How can mathematicians and communities work together to define mathematics that is rigorous and that will draw more effort, engagement, and self-efficacy from parents and their children? (Cf. Moses & Cobb, 2001.)
- How can the research community theorize about teachers' content knowledge in ways that reflect a view of mathematics as a social practice and a culture with aesthetic values, metacognitive strategies, and social norms? (Cf. Nasir & Cobb, 2006.)
- How can challenging aspects of algebra learning be addressed using classroom network technologies to overlay social and contextual meanings with mathematical meanings? (Cf. Hegedus & Penuel, 2008; Stroup, Ares, & Hurford, 2005.)

Conclusion

The NMAP report accomplished much by moving discussion of the content of mathematics to the center of the debate on improving mathematics achievement. Further, the Panel attempted to resolve false dichotomies between skills and concepts and between student-centered and teacher-centered instruction. Panel task groups collected and summarized much research, and some of their reports have high-quality literature reviews that should be of great service to researchers.

Yet, because of strategic choices made by the Panel, the report must be seen as a positive step but not as a complete framework for future research and reform efforts. Although putative arguments can be made to support instruction in algebra, links to improved economic benefits are correlational and anecdotal. We should revisit the president's charge to the Panel and present compelling pathways through algebra that prepare students for future learning and for the actual mathematical demands of their adult lives. Consistent with the National Research Council report *Adding It Up* (Kilpatrick et al., 2001), these pathways should draw upon the strategic competence, adaptive reasoning, and productive dispositions developed in school mathematics, in addition to mastery of particular skills and concepts learned in Grades K–8. We should also look seriously at the mathematics that matters in 21st-century science and ask whether we can afford to wait to introduce students to powerful ideas that could serve them well, even if this would reduce attention to topics such as balancing equations.

Education is a complex endeavor, and all available evidence suggests that compound interventions that draw on multiple resources are likely to be most effective. We need more attention to implementation research (which can include randomized experiments) that examines how multiple factors can be integrated to produce profound mathematical experiences (Schneider & McDonald, 2007). If leaders, educators, and parents want students to achieve more in school and be prepared for a full and rewarding life that builds on their growing mathematical proficiency, they will need to focus on how communities, schools, and teachers cook the stew, not only on the quality of the ingredients that went into the pot.

Finally, we see learning research as having advanced rapidly in the last quarter century and as likely to continue advancing rapidly in the near future. The broader view of learning that is emerging, which includes more attention to social, cultural, and out-of-school factors, deserves more attention in planning curricula, specifying requirements for teachers' mathematical knowledge, and designing innovations that increase the opportunities for all students to progress toward advanced mathematics.

NOTES

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[1]Some analyses link test scores to economic growth (Hanushek & Woessmann, 2007), but there are numerous problems in interpreting these data, as they are highly influenced by a small number of "Asian Tiger" economies that enacted educational and economic reforms simultaneously. Under such circumstances, causal interpretations are difficult; the relationship between education and a growing economy may flow in either direction (i.e., new wealth may cause parents to invest tremendous energy in their children's learning to solidify their initially precarious economic gains). When researchers have considered a more inclusive

data set, the correlation disappears in the short term in models with country-specific effects; only with a time lag of 15 years do math scores predict economic growth differentially across countries (Appleton, Atherton, & Bleaney, 2004).

[2]The Panel paid little attention to these categories of technology because few randomized experiments have been conducted to date.

[3]Incidentally, our randomized controlled trials were dismissed in the task group report and not considered in the final report because they used *volunteer* teachers in their sample—a reason that seems to us to dispense capriciously with ecological validity in favor of internal validity.

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