CME305 Sample Midterm II

1. **Matchings and Vertex Covers**

   (a) Define what a matching in G is.

   (b) Define what a vertex cover of G is.

   (c) Let M be a maximum matching and C a minimum vertex cover. Show that $|M| \leq |C| \leq 2|M|$.

2. **Traveling Salesman Problem**

   Assume that deciding whether a graph has a Hamiltonian cycle is NP-Complete. Prove that the Traveling Salesman Problem is NP-Hard.

3. **Lecture Attendance Planning**

   A group of students want to minimize their lecture attendance by sending only one of the group to each of the n lectures. We have the following constraints:

   - Each of the n lectures should be covered.
   - Lecture i starts at time a_i and ends at time b_i.
   - It takes r_{ij} time to commute from lecture i to lecture j.
   - Assume all times r_{ij} as well as the duration of the lectures are in minutes and integers.

   Minimize the number of students that will attend lectures i.e. develop a flow based algorithm to identify the minimum number of students needed to cover all n lectures.