While analyzing the NodeIterator algorithm, we want to bound the following expression:

$$\sum_{v \in V} \deg^*(v)^2$$

Consider those nodes for which \(\deg^*(v) < t\). Then for that partial sum we have:

$$\sum_{v \in V, \deg^*(v) < t} \deg^*(v)^2 \leq \sum_{v \in V, \deg^*(v) < t} t \deg^*(v) = t \sum_{v \in V, \deg^*(v) < t} \deg^*(v) \leq 2mt$$

Now consider those nodes for which \(\deg^*(v) \geq t\). There are at most \(2m/t\) such nodes. Note that \(\sum_{v \in V, \deg^*(v) \geq t} \deg^*(v)^2\) is upper bounded by the number of triangles between high-degree nodes, and since there are at most \(2m/t\) such nodes, we have

$$\sum_{v \in V, \deg^*(v) \geq t} \deg^*(v)^2 \leq (2m/t)^3$$

The total sum is bounded by the sum of the two partial sums we analyzed:

$$\sum_{v \in V} \deg^*(v)^2 \leq (2m/t)^3 + 2mt$$

We can set \(t = \sqrt{m}\) to minimize the above expression. Doing so, it becomes bounded by \(O(m^{3/2})\)