Equal Rights to Trade and Mediate

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Abstract: For economies with a fixed finite set of traders, few results characterize Walrasian equilibria by their social choice properties. Pareto efficient allocations typically require lump-sum transfers. Other characterizations based on the core or strategyproofness apply only when, as in continuum economies, agents cannot influence prices strategically. Or the results concern social choice with a variable number of agents. This paper considers allocations granting agents equal rights to choose net trade vectors within a convex cone and, in order to exclude autarky, an additional right to mediate mutually beneficial transactions. Under standard assumptions, these properties characterize Walrasian equilibria without transfers.

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1. Introduction: Characterizing Walrasian Equilibria

Walrasian equilibrium is known to exist for a broad class of economic environments, described by endowments, consumption sets, and preference profiles. The mapping from such environments to the set of Walrasian equilibria is an obvious example of a social choice rule. As is well known, such a rule selects Pareto efficient allocations satisfying a number of appealing properties such as individual rationality, anonymity, envy-free net trades, belonging to the core, etc.

Despite the prominence of this "Walrasian" social choice rule, there is still no completely satisfactory characterization of this rule by its social choice properties, at least for economies with a fixed finite number of agents.[1] Under standard assumptions, "non-oligarchic" Pareto efficient allocations can be characterized as Walrasian equilibria, but with lump-sum transfers — see Hammond (1998a), which uses ideas due to Arrow (1951) and McKenzie (1959, 1961) in particular. Allocations in the core have similar properties. Walrasian equilibria *without* lump-sum transfers, however, are characterized by being Pareto efficient only in single consumer economies, unless further restrictions are imposed.

In economies with a continuum of agents, there are several characterizations of Walrasian equilibrium without transfers. Of these, the best known is the core (Aumann, 1964), which is related to the notion of the bargaining set (Mas-Colell, 1989). There are also the value allocations considered by Aumann and Shapley (1974). With smooth preferences, an alternative characterization combines Pareto efficiency, and either anonymity or individual rationality, together with strategyproofness or envy-free net trades — see Hammond (1979), Champsaur and Laroque (1981, 1982). Walrasian equilibrium allocations without lump-sum transfers can also be characterized by multilateral strategyproofness (Hammond, 1987, 1999), or as f-core allocations (Hammond, Kaneko and Wooders, 1989; Hammond, 1995b). In the presence of "widespread externalities" the f-core property even characterizes Nash–Walrasian equilibria which are only constrained Pareto efficient given the extent of the externalities (Hammond, 1995a).

[1] See Maniquet (1999) for a recent alternative characterization — apart from Gevers (1986) — which does apply to a fixed finite set of agents.

All these characterizations of Walrasian equilibria without transfers rely on individual agents lacking any power to influence prices, as is the case in a continuum economy. With a fixed finite set of agents, on the other hand, Makowski, Ostroy and Segal (1999) show how strategyproof mechanisms which are individually rational or anonymous must give Walrasian equilibria without transfers in a special class of environments where agents lack the power to influence prices, at least locally. This is because at least one consumer has a locally flat indifference surface, treating all goods as perfect substitutes, in effect.[2]

Other characterizations of Walrasian equilibrium without transfers involve a finite but variable set of agents. The first and best know example is the Debreu/Scarf (1963) limit theorem on the core of a replica economy — or more precisely, the result that Walrasian equilibria without transfers are the only allocations that remain in the core no matter how often the economy is replicated. Less well known are the interesting results due to Sonnenschein (1974), Jordan (1982) and Thomson (1988) in particular — the latter paper also ends with a useful survey of numerous important results. In this connection, note that characterizations of Walrasian equilibrium consumption allocations with "equal split" — i.e., when aggregate endowments are shared equally — are easily adapted to characterizations of Walrasian equilibrium allocations of net trades, where everybody is effectively endowed with a zero net trade vector before trade starts. For example, compare the results of Hurwicz (1979) and Thomson (1979).

[2]A similar conclusion would hold in some special economies with production as well as exchange. For example, suppose that there is a single scarce factor of production (such as labour), and that every commodity except the primary factor is produced under constant returns to scale, without any joint production. These assumptions allow one to apply the non-substitution theorem that was originally proved in the contributions by Georgescu-Roegen, Samuelson, and Arrow to Koopmans (1951), then more generally by Karlin (1959, Section 8.5) and Mirrlees (1969). This result states that each commodity except the primary factor has a price proportional to the amount of the single scarce factor used directly or indirectly in its production — as in the labour theory of value. So all price ratios are independent of demand. Simpler still is the case of a small open economy with every good traded at a fixed world price. Even if some goods are not traded or there are several primary factors, a generalized form of factor price equalization known as "domestic price invariance" might still hold — see, for example, Bhagwati and Wan (1979), Diewert (1983), and Hammond (1986).

When prices are subject to strategic manipulation, strategyproofness generally conflicts with Pareto efficiency. Implementation in Nash equilibrium may not, however, especially when preferences can be represented by continuously differentiable utility functions, as in Schmeidler (1980). In this connection, a particularly simple characterization of Walrasian equilibrium without transfers is due to Gevers (1986, Theorem 1.3, p. 104). He shows that this Walrasian social choice rule is the most selective satisfying three conditions: (1) welfarism, in the sense that if all individuals are indifferent between any two allocations, then neither of these allocations can be selected without the other; (2) individual rationality, relative to the autarky allocation; (3) a weakening of the Maskin (1999) monotonocity axiom. Of course, Maskin monotonocity is a necessary condition for a social choice rule to be Nash implementable and, in combination with no veto power, is also sufficient. Nevertheless, as Gevers himself readily admits, his result falls short of characterizing the Walrasian social choice rule as welfarist, individually rational, and Nash implementable. Indeed, this result would be false because it is easy to see that a Nash implementable social choice rule must satisfy Arrow's independence of irrelevant alternatives condition. Yet this condition is violated by the Walrasian social choice rule — at least when preferences may correspond to kinked indifference curves.

This paper presents an alternative characterization which is close to the no envy criterion for net trades introduced by Schmeidler and Vind (1972) — see also the related results by Vind (1977). This no envy criterion can be interpreted as giving all agents equal rights to trade. When rights to trade include the rights to multiple proportional trades, there must be a linear subspace which serves as a common budget set that decentralizes the resulting allocation, as shown in Section 3. However, this subspace could be of low dimension — even zero, for the autarky allocation, which grants such equal rights trivially.

In Section 4, it is shown that Pareto efficiency combined with equal rights to trade characterizes Walrasian equilibrium without transfers when either: (i) there are only two goods; or (ii) at least one consumer has a unique hyperplane supporting the indifference curve at the chosen allocation. Section 5 provides a counterexample showing that this characterization does require one of the two extra conditions to hold.

Next, Section 6 introduces an additional individual right to mediate which, when combined with equal rights to trade, characterizes Walrasian equilibrium without transfers even without directly postulating Pareto efficiency. Section 7 offers some concluding remarks.

2. Preliminaries

Let N be a finite set of consumers, and G a finite set of physical commodities. Let $\mathbb{R}^G$ denote the $\#G$-dimensional commodity space of net trade vectors $t = (t_g)_{g \in G}$. Assume that each consumer $i \in N$ has a closed convex set $T^i \subset \mathbb{R}^G$ of feasible net trade vectors which has 0 as an interior point. Let T^N denote the Cartesian product set $\prod_{i \in N} T^i$. Then the set of feasible allocations is

$$A := \{ t^N \in T^N | \sum\nolimits_{i \in N} t^i = 0 \}$$

which is assumed to be bounded and so, because it is obviously a closed subset of the finite dimensional space $\mathbb{R}^{NG}$, must be compact. Finally, assume that each consumer $i \in N$ has a (complete and transitive) preference ordering $\succsim^i$ on T^i satisfying local non-satiation, convexity and continuity.

For each $i \in N$ and $\bar{t}^i \in T^i$, let

$$V^i(\bar{t}^i) := \{ v \in \mathbb{R}^G \mid \bar{t}^i + v \succ^i \bar{t}^i \}$$

denote the set of preferred incremental net trades v which improve the net trade vector $\bar{t}^i$.

For each possible set of feasible allocations A and each preference profile $\succsim^N = \langle \succsim^i \rangle_{i \in N}$, let $W(A, \succsim^N)$ denote the set of *Walrasian equilibrium allocations* $\bar{t}^N$ for which there exists an *equilibrium price vector* $p \in \mathbb{R}^G \setminus \{0\}$ with the property that for all $i \in N$, one has $p\,\bar{t}^i \leq 0$ and also $\bar{t}^i \succsim^i t^i$ whenever $t^i \in T^i$ with $p\,t^i \leq 0$. Equivalently, $p\,\bar{t}^i \leq 0$ and also $v^i \in V^i(\bar{t}^i) \Longrightarrow p\,v^i > 0$. Our assumptions are well known to guarantee that $W(A, \succsim^N)$ is non-empty.

Let $i \in N$ be any consumer, and $\bar{t}^i \in T^i$ any feasible net trade vector. Say that $p \neq 0$ is a *supporting price vector* at $\bar{t}^i$ if $\bar{t}^i$ has the cost minimizing property that $p\,t^i \geq p\,\bar{t}^i$ whenever $t^i \in T^i$ with $t^i \succsim^i \bar{t}^i$. Under the standard assumptions set out above, if $\bar{t}^N \in A$ is a Pareto efficient allocation, then there exists a common supporting price vector $p \neq 0$ for each trader $i \in N$ at $\bar{t}^i \in T^i$. Moreover, for any $i \in N$ such that $\bar{t}^i$ is an interior point of T^i, then $\bar{t}^i$ also has the preference maximizing property at the supporting price vector $p \neq 0$ that $\bar{t}^i \succsim^i t^i$ whenever $t^i \in T^i$ with $p\, t^i \leq p\, \bar{t}^i$. Indeed, as is well known, the same is true even if $\bar{t}^i$ is not an interior point of T^i, provided that there is a *cheaper point* $\underline{t}^i \in T^i$ with $p\, \underline{t}^i < p\, \bar{t}^i$.

Given an allocation $\bar{t}^N \in A \subset \mathbb{R}^{NG}$, let $L(\bar{t}^N)$ denote the linear subspace of $\mathbb{R}^G$ spanned by the associated set $\{\, \bar{t}^i \mid i \in N \,\}$ of net trade vectors. This subspace plays a key role in our later analysis.

3. Equal Rights to Multiple Proportional Trade

Say that the feasible allocation $\bar{t}^N \in A$ offers *equal rights to trade* if there exists a common *trading set* $B \subset \mathbb{R}^G$ such that, for all $i \in N$, both $\bar{t}^i \in B$ and also $t^i \in B \cap T^i \Longrightarrow \bar{t}^i \succsim^i t^i$. This gives agents the right to choose any other trader's net trade vector instead of their own. Clearly, equal rights to trade imply that each $\bar{t}^h \in B$, so $\bar{t}^N$ is envy free in the sense that $\bar{t}^i \succsim^i \bar{t}^h$ for all $h, i \in N$.

Say that $\bar{t}^N \in A$ offers *equal rights to multiple trade* if the set B is closed under addition — i.e., $B + B \subset B$. Thus, as in Schmeidler and Vind (1972), agents are allowed combinations of others' net trade vectors, provided they add supplies as well as demands. Also, say that there are *equal rights to proportional trade* if the set B is closed under multiplication by any non-negative scalar — i.e., $\lambda B \subset B$ for all $\lambda \geq 0$, implying that B is a cone. This offers each agent the right to any multiple of an allowable net trade vector, with both supplies and demands re-scaled in the same proportion. This extension is related to, but somewhat different from Schmeidler and Vind's (1972) divisibility condition. Finally, say that there are *equal rights to multiple proportional trade* if both these properties are satisfied, implying that $\lambda B + \mu B \subset B$ whenever $\lambda, \mu \geq 0$, so B must be a convex cone.

Obviously, if $\bar{t}^N \in W(A, \succsim^N)$, with $p \neq 0$ as the associated equilibrium price vector, there are equal rights to multiple proportional trade within the *Walrasian budget subspace* $B_p := \{\, t \in \mathbb{R}^G \mid p\, t = 0 \,\}$.

The following two simple results will be useful in later sections:

Lemma 1. *Suppose the feasible allocation $\bar{t}^N \in A$ offers equal rights to multiple proportional trade within the convex cone B. Then the linear subspace $L(\bar{t}^N) \subset B$, and $L(\bar{t}^N)$ is of dimension $\#G - 1$ at most.*

Proof. By definition of equal rights, one has $\bar{t}^i \in B$ for all $i \in N$. Also, because $\sum_{i \in N} \bar{t}^i = 0$, it follows that $-\bar{t}^i = \sum_{h \in N \setminus \{i\}} \bar{t}^h$. But B is a convex cone, so this implies that $-\bar{t}^i \in B$ for all $i \in N$. Hence, B must contain every linear combination $\sum_{i \in N} \lambda^i \bar{t}^i$ of the set $\{ \bar{t}^i \mid i \in N \}$ of net trade vectors, no matter what the sign of each scalar $\lambda^i \in \mathbb{R}$ may be. This proves that $L(\bar{t}^N) \subset B$.

Next, because $\succsim^i$ is locally non-satiated, there exists $\tilde{t}^i \in T^i$ with $\tilde{t}^i \succ^i t^i$. But $t^i \in B \cap T^i \Longrightarrow \bar{t}^i \succsim^i t^i$, so the set B cannot include $\tilde{t}^i$. Nor therefore can the subset $L(\bar{t}^N)$. This proves that $L(\bar{t}^N)$ must be of dimension less than $\#G$. □

Lemma 2. *Suppose that the feasible allocation $\bar{t}^N \in A$ offers equal rights to multiple proportional trade within a linear subspace $L \subset \mathbb{R}^G$ of dimension $\#G - 1$. Then $\bar{t}^N \in W(A, \succsim^N)$.*

Proof. If L is a subspace of dimension $\#G - 1$, it is a hyperplane through the origin with normal $p \neq 0$ — i.e., $L = B_p := \{ t \in \mathbb{R}^G \mid p\, t = 0 \}$ for some $p \neq 0$. Because each individual $i \in N$ has the right to trade within B_p, it follows that $t^i \in B_p \cap T^i \Longrightarrow \bar{t}^i \succsim^i t^i$. Because $\bar{t}^N \in A$, this implies that $\bar{t}^N \in W(A, \succsim^N)$. □

4. Pareto Efficiency

So far, equal rights to multiple proportional trade are consistent with the common budget space B being a low dimensional subspace — in fact, even $B = \{0\}$ is possible, with enforced autarky. Supplementing equal rights to trade with Pareto efficiency avoids this trivial case, unless autarky happens to be Pareto efficient anyway. This leads to our first two characterization results.

Theorem 1. *Suppose there are two only goods, while the feasible allocation $\bar{t}^N \in A$ is Pareto efficient and offers equal rights to multiple proportional trade. Then $\bar{t}^N \in W(A, \succsim^N)$.*

Proof. The first case is when $L(\bar{t}^N)$ has dimension zero. Then $\bar{t}^N$ is the autarky allocation 0^N. By hypothesis, $\bar{t}^N = 0^N$ is Pareto efficient. Because of the assumption that 0 belongs to the interior of each T^i, the standard argument discussed in Section 2 establishes the existence of a price vector $p \neq 0$ such that, for each $i \in N$, one has $\bar{t}^i = 0 \succsim^i t^i$ whenever $t^i \in T^i$ with $p\, t^i \leq 0$. So $\bar{t}^N = 0^N \in W(A, \succsim^N)$ in this case.

By Lemma 1, the only possible alternative is that $L(\bar{t}^N)$ has dimension $1 = \#G - 1$. Then Lemma 2 implies that $\bar{t}^N \in W(A, \succsim^N)$. $\square$

Suppose that the feasible net trade vector $\bar{t}^i \in T^i$ has a supporting price vector $p \neq 0$ with the property that any other supporting price vector $\tilde{p} \neq 0$ must be a positive multiple of p. That is, suppose the supporting price vector is unique up to multiplication by a positive scalar. In this case, say that consumer $i \in N$ has *unique supporting prices*. Otherwise, say that consumer $i \in N$ has *multiple supporting prices*.

Obviously, if an agent's preferences are not only convex but represented by a continuously differentiable utility function, at any interior Pareto efficient allocation there are unique supporting prices proportional to the gradient vector of the utility function.

Lemma 3. (See Madden, 1978, p. 281.) *Suppose that $p \neq 0$ is a supporting price vector at $\bar{t}^i \in T^i$. Suppose that there exists $\bar{v} \in \mathbb{R}^G$ with $p\,\bar{v} > 0$ such that $\bar{t}^i \succsim^i \bar{t}^i + \lambda\,\bar{v}$ for all $\lambda > 0$. Then there are multiple supporting prices.*

Proof. Let L denote the line consisting of scalar multiples of the non-zero vector $\bar{v}$. Of course, $p\,(\bar{t}^i + \lambda\,\bar{v}) < p\,\bar{t}^i$ for all $\lambda < 0$, implying that $\bar{t}^i \succ^i \bar{t}^i + \lambda\,\bar{v}$ because p is a supporting price vector. Together with the hypotheses of the Lemma and the definition of $V^i(\bar{t}^i)$ in Section 2, this implies that L and $V^i(\bar{t}^i)$ are disjoint sets. Since both sets are non-empty and convex, there exists a separating hyperplane $\tilde{p}\,v = c$ with $\tilde{p} \neq 0$ such that $\tilde{p}\,v \leq c$ for all $v \in L$ and $\tilde{p}\,v \geq c$ for all $v \in V^i(\bar{t}^i)$. Because L is a linear subspace, one also has $\tilde{p}\,v \geq c$ for all $v \in L$ and, in addition, $c = 0$ because $0 \in L$. Hence, $\tilde{p}\,v = 0$ for all $v \in L$, including $\bar{v}$, so $\tilde{p}\,\bar{v} = 0$. But $p\,\bar{v} > 0$, so $\tilde{p}$ cannot be a scalar multiple of p.

Furthermore, $\tilde{p}\,v \geq 0$ for all $v \in V^i(\bar{t}^i)$. By local non-satiation and the definition of $V^i(\bar{t}^i)$, it follows that $\tilde{p}$ is a supporting price vector at $\bar{t}^i \in T^i$. That is, there must be multiple supporting prices. $\square$

Theorem 2. *Suppose that at the Pareto efficient allocation $\bar{t}^N \in A$, at least one consumer has unique supporting prices, and that there are also equal rights to multiple proportional trade. Then $\bar{t}^N \in W(A, \succsim^N)$.*

Proof. Suppose that there is at least one consumer $h \in N$ who has unique supporting prices $p \neq 0$ at the allocation $\bar{t}^N \in A$, as well as equal rights to multiple proportional trade. Given any $v \in L(\bar{t}^N)$, one has $\bar{t}^h + \lambda v \in L(\bar{t}^N)$ for all $\lambda > 0$ because $\bar{t}^h \in L(\bar{t}^N)$ and $L(\bar{t}^N)$ is a linear subspace. By Lemma 1, equal rights to multiple proportional trade within the convex cone B imply that $L(\bar{t}^N) \subset B$. So $\bar{t}^h + \lambda v \in B$ for all $\lambda > 0$, from which it follows that $\bar{t}^h \succsim^h \bar{t}^h + \lambda v$. For this to be consistent with the unique supporting price property, Lemma 3 implies that $p\, v \leq 0$. Because this is true for all v in the linear subspace $L(\bar{t}^N)$, in fact $p\, v = 0$ for all $v \in L(\bar{t}^N)$, as in the proof of Lemma 3. In particular, $p\, \bar{t}^i = 0$ for all $i \in N$.

As discussed in Section 2, all consumers, including h, must share a common supporting price vector $\bar{p} \neq 0$ at the Pareto efficient allocation $\bar{t}^N \in A$. Because h has unique supporting prices p, the common supporting price vector $\bar{p}$ must be a positive multiple of p, so can be replaced by p.

Finally, because 0 is an interior point of T^i, there exists a cheaper point $\underline{t}^i \in T^i$ with $p\, \underline{t}^i < 0$. Then, because $p \neq 0$ is a common supporting price vector for each $i \in N$, a standard argument shows that $\bar{t}^i \succsim^i t^i$ for all $t^i \in T^i$ with $p\, t^i \leq 0$, as discussed in Section 2. Because $\bar{t}^N \in A$, it follows that $\bar{t}^N \in W(A, \succsim^N)$. □

5. A Counterexample

The following example demonstrates the need for extra hypotheses of the kind used in the last two results. That is, if $\#G \geq 3$ and there are multiple supporting prices for all consumers, then adding Pareto efficiency to equal rights to multiple proportional trade may not be enough to ensure that the allocation is a Walrasian equilibrium without transfers.

The example has two consumers, so $N = \{1, 2\}$, and three commodities, so $G = \{1, 2, 3\}$. Each consumer's feasible set of net trade vectors is given by

$$T^1 = T^2 = \{-(1,1,1)\} + \mathbb{R}^3_+ = \{ t \in \mathbb{R}^3 \mid t \geqq -(1,1,1) \}$$

as in a pure exchange economy with the same endowment vector $(1, 1, 1)$ for both consumers. Their preferences are assumed to be represented on this common feasible set by the respective utility functions

$$u^1(t^1) = \min\{ t^1_1 + 1, t^1_2 + t^1_3 + 2 \} \quad \text{and} \quad u^2(t^2) = \min\{ t^2_1 + t^2_2 + 2, t^2_3 + 1 \}$$

There are Pareto efficient allocations characterized by the two equations

$$t^1_1 + 1 = t^1_2 + t^1_3 + 2 \quad \text{and} \quad t^2_1 + t^2_2 + 2 = t^2_3 + 1$$

as well as the vector equation $t^1 + t^2 = 0$ in $\mathbb{R}^3$. This gives a total of 5 simultaneous linear equations in 6 unknowns, so there is at least one degree of freedom. Adding the first two equations and imposing $t^1+t^2 = 0$ leads to $t^2_2+3 = t^1_2+3$, implying that $t^1_2 = t^2_2 = 0$. In fact, given that $t^1_1 = \gamma$ for some arbitrary constant γ, the 5 equations have a unique solution which is easily calculated to be $\bar{t}^1 = -\bar{t}^2 = (\gamma, 0, \gamma-1)$. This solution describes a feasible allocation with $\bar{t}^1$ and $\bar{t}^2$ both $\geqq -(1, 1, 1)$ provided that $0 \leq \gamma \leq 1$. The corresponding utilities are $\bar{u}^1 = \gamma + 1$ and $\bar{u}^2 = 2 - \gamma$.

Because goods 2 and 3 are perfect substitutes for consumer 1, and goods 1 and 2 are for consumer 2, the only possible supporting price vectors are positive scalar multiples of $p = (1, 1, 1)$. Indeed, if $p_2 > p_3$ for example, then consumer 1 can gain by deviating from $\bar{t}^1$ to the alternative net trade vector $\tilde{t}^1 = (\gamma + \alpha, -1, \gamma + \alpha)$ at which utility is $\gamma + \alpha + 1$, where $\alpha := (p_2 - p_3)/(p_1 + p_3) > 0$ in order to ensure that $p\,\tilde{t}^i - p\,\bar{t}^i = p_1\,\alpha - p_2 + p_3\,(\alpha + 1) = 0$. Now, at prices $p = (1, 1, 1)$, the net trade vectors $\bar{t}^1$ and $\bar{t}^2$ have values $p\,\bar{t}^1 = 2\gamma - 1 = -p\,\bar{t}^2$. It follows that the Pareto efficient allocation $(\bar{t}^1, \bar{t}^2)$ is a Walrasian equilibrium without transfers only in the special case when $\gamma = 1/2$.

Consider next the one-dimensional linear subspace L spanned by the pair of vectors $\{\,\bar{t}^1, \bar{t}^2\,\}$. It is a line consisting of all scalar multiples of the non-zero vector $(\gamma, 0, \gamma - 1)$. Now, if $t = \delta\,(\gamma, 0, \gamma - 1)$ where $\gamma \in [0, 1]$, then

$$u^1(t) = \min\{\,\delta\gamma + 1, \delta\,(\gamma - 1) + 2\,\} \leq u^1(\bar{t}^1) = \gamma + 1$$

provided that $\delta \leq 1$ or $\delta\,(\gamma - 1) + 1 \leq \gamma$. The latter inequality is equivalent to $(1 - \delta)\,(1 - \gamma) \leq 0$ which, because $\gamma \leq 1$, is true when $\delta \geq 1$. Hence, $u^1(t) \leq u^1(\bar{t}^1)$ whether $\delta \leq 1$ or $\delta \geq 1$. Similarly,

$$u^2(t) = \min\{\,\delta\gamma + 2, \delta\,(\gamma - 1) + 1\,\} \leq u^2(\bar{t}^2) = 2 - \gamma$$

provided that $\delta\gamma \leq -\gamma$ or $(\delta + 1)\,(\gamma - 1) \leq 0$. Given that $0 \leq \gamma \leq 1$, the first inequality is satisfied iff $\delta \leq -1$ and the second iff $\delta \geq -1$, so $u^2(t) \leq u^2(\bar{t}^2)$ in both cases. This proves that neither agent prefers any net trade vector in the subspace L.

To conclude, even though the allocation $(\bar{t}^1, \bar{t}^2)$ with $\bar{t}^1 = -\bar{t}^2 = (\gamma, 0, \gamma - 1)$ and $0 \leq \gamma \leq 1$ is Pareto efficient and respects equal rights to multiple proportional trade along the line L spanned by $\{ \bar{t}^1, \bar{t}^2 \}$, it is not a Walrasian equilibrium without transfers except in the special case when $\gamma = 1/2$.

6. Rights to Mediate

Given the feasible allocation $\bar{t}^N \in A$, say that individual $i \in N$ has the *right to mediate*, as well as the right to trade within the set $B^i \subset \mathbb{R}^G$, provided that there is no combination of a $v \in B^i$, a set $K \subset N \setminus \{i\}$ of *coalition partners* for i, and preferred increments $v^h \in V^h(\bar{t}^h)$ (all $h \in K$) such that $v - \sum_{h \in K} v^h \in V^i(\bar{t}^i)$. Thus, agent i cannot gain by trading the vector $v \in B^i$, and then using v to mediate incremental transactions $v^h \in V^h(\bar{t}^h)$ with coalition partners $h \in K$ in a way that leaves every agent in $K \cup \{i\}$ better off than at $\bar{t}^N$. Equivalently, B^i must be disjoint from $\sum_{h \in K \cup \{i\}} V^h(\bar{t}^h)$.

In the special case when $v = 0$, such mediation is evidently equivalent to the coalition $K \cup \{i\}$ being able to block the allocation $\bar{t}^N$. Here, however, the blocking coalition is organized by one particular mediating agent i who also has access to the set B^i before arranging mutually beneficial transactions.

At first it might seem hard to reconcile equal rights to trade with these rather strong rights to mediate. Yet the following two results show that these combined rights offer a precise characterization of Walrasian equilibrium without transfers.

Theorem 3. *Any Walrasian equilibrium without transfers gives individuals equal rights to multiple proportional trade and rights to mediate.*

Proof. Suppose $\bar{t}^N \in W(A, \succsim^N)$, with $p \neq 0$ as the Walrasian equilibrium price vector. Then there are equal rights to multiple proportional trade in the linear subspace $B_p := \{ t \in \mathbb{R}^G \mid p\, t = 0 \}$.

Suppose that trader $i \in N$, the set $K \subset N \setminus \{i\}$ of i's coalition partners, and the preferred increments $v^h \in V^h(\bar{t}^h)$ (all $h \in K$) together satisfy $v - \sum_{h \in K} V^h \in V^i(\bar{t}^i)$ for some $v \in \mathbb{R}^G$. Because $\bar{t}^N$ is a Walrasian equilibrium allocation at prices p, it follows that $p\, v^h > 0$ for all $h \in K$, and also that $p \left(v - \sum_{h \in K} v^h \right) > 0$. Hence $p\, v > p \sum_{h \in K} v^h > 0$, implying that $v \notin B_p$. So no trader $i \in N$ is able to gain from combining trade in B_p with mediation. $\square$

Theorem 4. *Suppose the feasible allocation $\bar{t}^N \in A$ grants all agents equal rights to multiple proportional trade and rights to mediate. Then $\bar{t}^N \in W(A, \succsim^N)$.*

Proof. By Lemma 1, agents must have the equal right to trade in a set B which includes $L(\bar{t}^N)$, the linear space spanned by $\{\, \bar{t}^i \mid i \in N \,\}$. Because agents also have the right to mediate, one has $B \cap \sum_{h \in K} V^h(\bar{t}^h) = \emptyset$ for all non-empty $K \subset N$. In particular, $L(\bar{t}^N)$ and $\sum_{i \in N} V^i(\bar{t}^i)$ must be disjoint sets, which are also non-empty and convex. So there exists a separating hyperplane $p\,v = c$ with $p \neq 0$ such that $p\,v \geq c$ for all $v \in \sum_{i \in N} V^i(\bar{t}^i)$ and $p\,v \leq c$ for all $v \in L(\bar{t}^N)$.

Because $L(\bar{t}^N)$ is a linear subspace, one has $c = 0$ and also $p\,v = 0$ for all $v \in L(\bar{t}^N)$, as in the proof of Lemma 3. In particular, $p\,\bar{t}^i = 0$ for all $i \in N$.

Furthermore, $p\,v \geq 0$ for all $v \in \sum_{i \in N} V^i(\bar{t}^i)$. By local non-satiation, the vector 0 is a boundary point of $V^h(\bar{t}^h)$ for all $h \in N \setminus \{i\}$. It follows that any $v^i \in V^i(\bar{t}^i)$ must belong to the closure of $\sum_{i \in N} V^i(\bar{t}^i)$. So for each $i \in N$ it must be true that $p\,v^i \geq 0$ for all $v^i \in V^i(\bar{t}^i)$. Because 0 is an interior point of T^i, there exists a cheaper point $\underline{t}^i \in T^i$ with $p\,\underline{t}^i < 0$. Then the standard argument discussed in Section 2 establishes that $\bar{t}^i \succsim^i t^i$ for all $t^i \in T^i$ with $p\,t^i \leq 0$. Hence, $\bar{t}^N$ is a Walrasian equilibrium. □

7. Concluding Remarks

Equal rights to trade clearly imply an envy free allocation of net trade vectors. Equal rights to multiple proportional trade, as defined in Section 3, imply that no agent can find a preferred net trade vector in the linear space spanned by all agents' net trade vectors. By themselves such rights do not preclude autarky. Pareto efficiency, however, clearly does (except if autarky happens to be Pareto efficient anyway), and Section 4 gave sufficient conditions for a Pareto efficient allocation with equal rights to multiple proportional trade to be a Walrasian equilibrium without transfers. These conditions are that there are only two goods, or alternatively, that there are unique supporting prices for the Pareto efficient allocation. An example in Section 5 shows that conditions like these are required for a valid result.

Pareto efficiency, however, results from the group of all agents as a whole exercising rights cooperatively. Section 6 has proposed supplementing equal individual rights to multiple proportional trade with an additional individual right to mediate, instead of with Pareto efficiency. This extra right allows each individual to arrange additional mutually beneficial transactions with any subset of agents. It is shown that this combination of equal rights to multiple proportional trade with rights to mediate gives an exact characterization of Walrasian equilibrium without transfers — at least for "classical" economies with continuous convex preferences, and with closed and convex feasible sets which include the zero net trade vector as an interior point.

The paper has concentrated on characterizing Walrasian equilibrium without transfers. The results apply to economies in which agents are endowed with (closed and convex) individual production sets, as well as to those where each agent's endowment is a fixed commodity vector. But the results can easily be adapted to characterize Walrasian equilibrium "with equal split", where agents start with equal fixed endowment vectors before trade. The equal rights to trade defined previously should be replaced by equal rights to choose from a common linear subspace of consumption vectors. Rights to mediate can be defined as before.

A final comment. Neither economists nor social choice theorists have paid much attention to the right to exchange.[3] Yet respect for this right is sometimes claimed as the key justification for free markets. Whether ethically attractive or not, rights of this kind, when suitably defined, do at least help to characterize Walrasian equilibrium without transfers for an exchange economy with a fixed finite set of agents.

References

[1] Arrow, K.J. (1951) "An Extension of the Basic Theorems of Classical Welfare Economics" in J. Neyman (ed.) *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability* (Berkeley: University of California Press), pp. 507–532; reprinted in *Collected Papers of Kenneth J. Arrow, Vol. 2: General Equilibrium* (Cambridge, Mass.: Belknap Press of Harvard University Press, 1983).

[2] Aumann, R.J. (1964) "Markets with a Continuum of Traders," *Econometrica* **32**: 39–50.

[3] Aumann, R.J. and L.S. Shapley (1974) *Values of Non-Atomic Games* (Princeton: Princeton University Press).

[4] Bhagwati, J.N. and H. Wan (1979), "The 'Stationarity' of Shadow Prices of Factors in Project Evaluation, with and without Distortions," *American Economic Review* **69**: 261–273.

[3] For some exceptions, see the introduction to Hammond (1998b).

[5] Champsaur, P. and G. Laroque (1981), "Fair allocations in large economies," *Journal of Economic Theory* **25**: 269–282.

[6] Champsaur, P. and G. Laroque (1982), "A note on incentives in large economies," *Review of Economic Studies* **49**: 627–635.

[7] Debreu, G. and H.E. Scarf (1963) "A Limit Theorem on the Core of an Economy," *International Economic Review* **4**, 235–246; reprinted in G. Debreu, *Mathematical Economics: Twenty Papers of Gerard Debreu* (Cambridge: Cambridge University Press, 1983).

[8] Diewert, W.E. (1983), "Cost–Benefit Analysis and Project Evaluation: A Comparison of Alternative Approaches," *Journal of Public Economics* **22**: 265–302.

[9] Gevers, L. (1986) "Walrasian Social Choice: Some Simple Axiomatic Approaches," in W.P. Heller, R.M. Starr and D.A. Starrett (eds.), *Social Choice and Public Decision Making: Essays in Honor of Kenneth J. Arrow, Vol. I* (Cambridge: Cambridge University Press), ch. 5, pp. 97–114.

[10] Hammond, P.J. (1979) "Straightforward Individual Incentive Compatibility in Large Economies," *Review of Economic Studies* **46**: 263–282.

[11] Hammond, P.J. (1986) "Project Evaluation by Potential Tax Reform," *Journal of Public Economics* **30**: 1–36.

[12] Hammond, P.J. (1987) "Markets as Constraints: Multilateral Incentive Compatibility in a Continuum Economy," *Review of Economic Studies* **54**: 399–412.

[13] Hammond, P.J. (1995a) "Four Characterizations of Constrained Pareto Efficiency in Continuum Economies with Widespread Externalities," *Japanese Economic Review*, **46**: 103–124.

[14] Hammond, P.J. (1995b) "On f-Core Equivalence in a Continuum Economy with General Widespread Externalities," Stanford University Department of Economics Working Paper No. 95-004; revised version to appear in *Journal of Mathematical Economics*.

[15] Hammond, P.J. (1997) "Multilaterally Strategy-Proof Mechanisms in Random Aumann–Hildenbrand Macroeconomies," Stanford University Department of Economics Working Paper No. 97-022; revised version to appear in M. Wooders (ed.) *Topics in Game Theory and Mathematical Economics* (American Mathematical Society).

[16] Hammond, P.J. (1998a) "The Efficiency Theorems and Market Failure," in A.P. Kirman (ed.) *Elements of General Equilibrium Analysis* (Oxford: Basil Blackwell), ch. 6, pp. 211–260.

[17] Hammond, P.J. (1998b) "Rights, Free Exchange, and Widespread Externalities," in J.-F. Laslier, M. Fleurbaey, N. Gravel and A. Trannoy (eds.) *Freedom in Economics: New Perspectives in Normative Analysis* (London and New York: Routledge), ch. 11, pp. 139–157.

[18] Hammond, P.J., M. Kaneko and M.H. Wooders (1989) "Continuum Economies with Finite Coalitions: Core, Equilibrium and Widespread Externalities," *Journal of Economic Theory*, **49**: 113–134.

[19] Hurwicz, L. (1979) "On Allocations Attainable through Nash Equilibria," *Journal of Economic Theory*, **21**: 140-165; also in Laffont (1979), pp. 397–419.

[20] Jordan, J, (1982) "The Competitive Allocation Process is Informationally Efficient Uniquely," *Journal of Economic Theory*, **28**: 1–18.

[21] Karlin, S. (1959) *Mathematical Methods and Theory in Games, Programming, and Economics* (Reading, Mass.: Addison-Wesley).

[22] Koopmans, T.C. (ed.) (1951) *Activity Analysis of Production and Allocation* (New York).

[23] Laffont, J.-J. (ed.) (1979) *Aggregation and Revelation of Preferences* (Amsterdam: North-Holland).

[24] Madden, P. (1978) "Why the Edgeworth Process Assumption Isn't That Bad," *Review of Economic Studies* **45**: 279–283.

[25] Makowski, L., J.M. Ostroy and U. Segal (1999) "Efficient Incentive Compatible Economies Are Perfectly Competitive," *Journal of Economic Theory*, **85**: 169–225.

[26] Maniquet, F. (1999) "Decomposable Exchange Rules," preprint, Facultés Universitaires Notre-Dame de la Paix, Namur.

[27] Mas-Colell, A. (1989) "An Equivalence Theorem for a Bargaining Set," *Journal of Mathematical Economics*, **18**: 129–39.

[28] Maskin, E.S. (1999) "Nash Equilibrium and Welfare Optimality," *Review of Economic Studies*, **66**: 23–38.

[29] McKenzie, L.W. (1959, 1961) "On the Existence of General Equilibrium for a Competitive Market," and "———: Some Corrections," *Econometrica*, **27**: 54–71 and **29**: 247–8.

[30] Mirrlees, J.A. (1969) "The Dynamic Nonsubstitution Theorem," *Review of Economic Studies* **36**: 67–76.

[31] Schmeidler, D. (1980) "Walrasian Analysis via Strategic Outcome Functions," *Econometrica* **48**: 1585–1593.

[32] Schmeidler, D. and K. Vind (1972) "Fair Net Trades," *Econometrica* **40**: 637–42.

[33] Sonnenschein, H. (1974) "An Axiomatic Characterization of the Price Mechanism," *Econometrica*, **42**: 425–434.

[34] Thomson, W. (1979) "Comment," in Laffont (ed.) (1979), pp. 420–431.

[35] Thomson, W. (1988) "A Study of Choice Correspondences in Economies with a Variable Number of Agents," *Journal of Economic Theory*, **46**: 237–254.

[36] Vind, K. (1977) "Equilibrium with Respect to a Simple Market," in G. Schwödiauer (ed.) *Equilibrium and Disequilibrium in Economic Theory* (Dordrecht: D. Reidel), pp. 3–6.