ON THE IMPOSSIBILITY OF PERFECT CAPITAL MARKETS

Peter J. Hammond

Department of Economics, European University Institute,

Badia Fiesolana, 50016 S. Domenico (FI), Italy

and Stanford University, CA 94305–6072, U.S.A.

Final version: May 1990; footnotes removed: February 1991.

Abstract

Perfect capital markets require linear budget constraints, without credit rationing creating any tight borrowing constraints before the end of agents' economic lifetimes. Yet lifetime linear budget constraints are totally unenforceable. This paper considers what allocations can be enforced through monitoring in a simple two period economy when agents have private information regarding their endowments. Then default may not become apparent soon enough for any economic penalty to be an effective deterrent. Instead, borrowing constraints must be imposed to control fraud (moral hazard). Adverse selection often implies that some borrowing constraints must bind, creating inevitable capital market imperfections.

ACKNOWLEDGEMENTS

This paper is a drastically revised version of Hammond (1987b), which was originally produced with the support of U.S. National Science Foundation Grant SES-85-20666 to the Institute for Mathematical Studies in the Social Sciences, Stanford University. It has evolved through presentations at C.O.R.E., at Western Ontario, Southern Methodist, Tulane, Stanford, Cambridge, Mannheim, Southampton, Oxford and Bonn Universities, at the Ecole Normale Supérieure in Paris, the University of California at San Diego, and to the European Meeting of the Econometric Society in Budapest. My gratitude to all the participants for their encouragement, to Frank Hahn, Mordecai Kurz, and Joe Stiglitz for extensive discussions, as well as to an anonymous referee, Ed Green and Oliver Hart for useful suggestions.

IMPERFECT CAPITAL MARKETS

Neither a borrower nor a lender be;
For loan oft loses both itself and friend,
And borrowing dulls the edge of husbandry.

— Hamlet (Act I, Scene 3, 75–77)

But where a system of borrowing and lending exists, by which I mean the granting of loans with a margin of real or personal security, a second type of risk is relevant which we may call the lender's risk. This may due either to moral hazard, i.e., voluntary default or other means of escape, possibly lawful, from the fulfilment of the obligation, or to the possible insufficiency of the margin of security, i.e., involuntary default due to the disappointment of expectation.

— Keynes (1936, p. 144)

But contracts which encourage the dishonest select adversely.

— Hahn (1988, p. 970)

A lot of money got put into people's pockets and they've rat-holed it somewhere. Some of it is in artwork, fancy homes, fancy airplanes and Rolls-Royces. Some of it went to Rolex watches, lizard shoes, hunting parties and yachts.

— The New York Times (January 10, 1989: quoted from a statement by H. Joe Selby, former chief regulator for the Federal Home Loan Bank of Dallas)

1. Introduction

1.1. Unenforceable Budget Constraints

The general equilibrium theory developed by Walras, Arrow, Debreu, Radner and others, and expounded in Arrow and Hahn (1971), involves linear budget constraints and market clearing prices. For intertemporal environments, this is what is required for "perfect" capital markets, in which agents are free to borrow and lend at the same market rate of interest for all loans of the same maturity, and the rates of interest adjust to match the plans of borrowers and lenders. This theory is for an economy of honourable agents, who always satisfy their intertemporal budget constraints. Agents never expose themselves deliberately to the risk of default. With perfect foresight, no default would ever occur. Without perfect foresight, of course, some agents may be unable to avoid bankruptcy *ex post*, as was realized by Green (1974) and Bliss (1976), amongst others.

Yet in the theory of temporary equilibrium, as surveyed recently by Grandmont (1982, 1988), agents still arrange their affairs so that, according to their own expectations, they can fulfill their budget constraints with probability one. As Milne (1980) has pointed out, this is consistent with two traders making a contingent contract which each is sure that he himself can honour, and yet is sure that the other cannot! By contrast, the world is full of less honourable agents who, to the extent that they find it profitable, will knowingly incur debts which they may find themselves unable to honour *ex post*.

1.2. The Need for Credit Rationing

Jaffee and Russell (1976) showed the need for non-linear pricing of credit in a model where some borrowers would undertake investment projects leading to a risk of default on their loans, while others would arrange always to honour their budget constraint, but where the two types were indistinguishable *ex ante*. Later work by Stiglitz and Weiss (1981, 1983) demonstrated the need, in a somewhat more elaborate model, not only for non-linear pricing, but also for a ceiling on borrowing and also, in some cases, for asymmetric treatment of borrowers who are identical *ex ante*. Apart from being restricted to somewhat special economic environments, this work did not point out how what Keynes (1936, p. 144) called "moral hazard, i.e., voluntary default," affects even economies without uncertainty. Nor did it explain why some agents would choose to repay their loans, or why others would choose to expose themselves to the risk of becoming defaulters. Without drastic non-economic penalties, some agents will gain by deliberately planning to violate their Walrasian budget constraints. There has been some recognition of this by Allen (1981, 1983) and by Eaton and Gersovitz (1981). Yet their work includes production, or international debt, in a way which may obscure somewhat the fact that enforcement is a problem even in the simplest of intertemporal exchange economies.

In fact the essential difficulty in having perfect capital markets turns out to be the incredibility of an intertemporal Walrasian budget constraint. This actually places no limit whatsoever upon what an agent is allowed to borrow. In a two period economy, an honourable agent will not borrow more than he anticipates being able to repay, with interest. But a dishonourable agent can borrow an arbitrarily large amount, and face the consequences of default later on. For some dishourable consumers who do not care much about the future — perhaps because they are so ill that their survival is even in doubt — no bankruptcy penalty will be able to deter them from deliberate default, so the Walrasian budget constraint fails completely to limit their borrowing.

Real economies may have few consumers with such preferences. Yet real economies do have firms whose owners enjoy limited liability, and legal restrictions are clearly needed to prevent the owners having the firm borrow indefinite amounts which are then paid out as dividends to the owners, leaving the firm bankrupt. The usual Walrasian models typically presume honourable agents who, when they die, leave estates large enough to discharge any debts. Such budget constraints are especially impossible to enforce because agents have to die before they can be declared in default. One could have all liabilities as well as assets inherited by descendants (if there are any). But, with perfect capital markets, the descendants themselves can borrow to discharge the debts they have just inherited, so this is no deterrent either. To put the matter at its simplest, budget constraints can only be enforced if there is a credit ceiling or borrowing constraint at some date in the future, otherwise debts of any size can be rolled over and allowed to grow indefinitely. Such credit ceilings may appear to create imperfections in the capital market.

1.3. Feasible Allocation Mechanisms

Credit ceilings may not, however, make capital markets imperfect by themselves. Suppose that we were in an Arrow-Debreu economy in which any information that becomes available to one agent becomes available to all simultaneously. Thus all information would be public, and there would be a publicly known event tree (cf. Debreu, 1959). Then all agents would know what contingent contracts other agents could honour. In other words, at each event of the publicly known tree, it would be possible to limit each agent's total net short sales of Arrow contingent securities and/or Debreu contingent commodity contracts to what that agent was known to be able to repay for sure after that event has occurred. This form of credit rationing is mentioned by Foley and Hellwig (1975). It makes the Arrow-Debreu budget constraint enforceable by replacing it with an entire sequence of short-selling constraints preventing the agent from becoming excessively indebted in any possible event. Yet any allocation which is achievable in Arrow-Debreu complete markets is still achievable with these "perfect" short-selling constraints. These constraints only prevent default — they do not prevent agents from making any short sales which can actually be honoured for sure. Equilibrium allocations with such perfect constraints are *identical* to those in perfect capital markets, and no loss of efficency or welfare will result.

The above discussion makes it clear then that credit rationing only creates capital market imperfections when we move outside the standard Arrow-Debreu framework of a publicly known event tree. Even so, there are some forms of asymmetric information that still fail to create problems for Walras-Arrow-Debreu perfect capital market allocations. For example, if there is a continuum economy with private information regarding only agents' preferences, then the results of Hammond (1979) for static continuum economies are easily integrated with those of Gale (1980, 1982), Harris and Townsend (1981), and Harris (1987) for resource allocation mechanisms in sequence economies. The point is that private information concerning preferences does not destroy the publicly known limits on what each agent in each event is able to borrow without any risk of later default.

Although the literature on incentive compatibility concentrates on the special case when only preferences are private information, this is, of course, very special. Real economic systems must deal with private information concerning labour skills (cf. Mirrlees, 1971; Dasgupta and Hammond, 1980; Maskin, 1980) or other endowments (Postlewaite, 1979; Hurwicz, Maskin and Postlewaite, 1979; Maskin, 1980; Postlewaite, 1985). This work, however, assumes that individuals are allowed only to understate their true skill or to understate or perhaps even destroy their endowments. This is the kind of manipulation to which Walrasian equilibrium allocation mechanisms are often vulnerable. As Hurwicz, Maskin and Postlewaite in particular have pointed out, if individuals can overstate their true endowments instead, and if their consumption sets are bounded below, then there will be a problem in ensuring that an allocation mechanism actually produces feasible outcomes.

In addition, Green (1987) has considered an infinite horizon economy with a continuum of agents in which each individual's endowment stream is private information. But there is a single commodity and utility is both additively separable and negative exponential. The possibility of an infeasible net trade vector never arises, because agents are modelled as being always able to repay any arbitrarily large sum which they may have borrowed in the past, even if this requires negative consumption in some periods. Of course, in Green's model it might well take a very long run of unluckily low endowments before a debtor was forced to suffer negative consumption. Nevertheless, the feasibility issue with which I shall specifically be concerned is not addressed.

My own past work (Hammond, 1979, 1987a) and that of Gale (1980, 1982) concentrated on allocation mechanisms or game forms which determine net trade rather than consumption vectors. This implicitly allowed agents' endowments and feasible sets (consumption sets, together with any domestic production possibilities) to be private information. It did not really deal with the feasibility issue satisfactorily, however, because it assumed explicitly that agents in any game form would never choose strategies which could result in net trade vectors that are infeasible for them.

If the economy only lasts for a single period, this last assumption can often be justified by suitable monitoring of defaulters. For if an agent in a game form plays a strategy which would result in an infeasible net trade vector, presumably this fact is discovered before the allocation has been determined irreversibly. After all, even if consumption demand falls below subsistence, that is typically still feasible for an agent who is not supplying anything, as discussed in Coles and Hammond (1986). So an infeasible net trade vector only arises, one may assume, when a "defaulting" agent fails to supply something that has been promised. Typically this will be detected, if only because some agent who was expecting to receive at least a part of this supply will be disappointed and can be relied upon to complain. Then, in a single period economy, it is not too late to change the allocation by requiring the defaulting agent to amend his net trade vector to one that really is feasible. Indeed, it may be possible to monitor any defaulter's endowments, if necessary, and to confiscate enough of them to deter any such defaults.

In a sequence economy, however, the situation can be quite different. Suppose for simplicity that the economy lasts for just two periods. Then a consumer may be able to play the game form in a way which appears in the first period to ensure feasibility, but involves commitments which cannot possibly be honoured in the second period. More concretely, an agent may be able to consume excessively in the first period by acting in the economic system as would someone whose endowments were much larger, in effect borrowing more than can ever be repaid. If the agent has been careful, this inability to repay will not manifest itself until the second period. By then it may be too late to make appropriate rearrangements in order to enforce the intertemporal allocation mechanism which is incentive compatible in the usual sense. It may even be too late to punish the defaulter severely enough to deter such default — the defaulter may have disappeared without trace, or even died. Excessive consumption of this kind, of course, is precisely what fraudulent borrowers achieve in actual economies. In real economies defaulters have the additional advantage of being able to seek the protection of bankruptcy laws. Such deliberate default may be criminally fraudulent, but it does occur.

In any case, potential fraud of this kind introduces a form of moral hazard to add to the obvious problems of adverse selection arising from private information about future endowments. The consequences of default really need to be specified within the game form. This was originally pointed out by Shubik (1973, 1974) in the context of a very specific trading game, but only with a finite number of players. Later, Shubik and Wilson (1977), as well as Dubey and Shubik (1979), did consider continuum economies, but they also introduced non-economic penalties for bankruptcy. The question of whether there can ever be purely economic penalties strong enough to deter all default still needs to be carefully investigated within the framework of a game form. This paper attempts to fill the gap.

1.4. Additional Incentive Constraints

So Section 2 below considers allocation mechanisms with monitoring in order to take this additional moral hazard into account. This is done with a simple two period economy having just one consumption good. In the first period each agent has private information which tells him exactly what his second period endowment will be. Only in the second period does default become apparent, however, and only then can monitoring be used in order to acquire public information about the second period endowment. By the time the second period arrives, it is too late to reduce a defaulting borrower to autarky because the benefit from the loan taken out in the first period can no longer be taken away. Thus in sequence economies it becomes necessary to consider additional incentive or "individual feasibility" constraints, beyond those already considered in Hammond (1979, 1987a), Gale (1980, 1982), Harris and Townsend (1981), Prescott and Townsend (1984a, b), Harris (1987), Townsend (1988), etc. And to introduce credit rationing in a way which may well prevent attainment of an allocation which "perfect" capital markets would produce, if only Arrow-Debreu budget constraints could somehow be enforced.

This may appear to contradict the results of Harris and Townsend (1981) and Harris (1987, pp. 121–131) on incentive constraints in sequence economies. They were careful to consider mechanisms as extensive form games and to demonstrate the revelation principle for "perfect Bayesian equilibrium," closely related to Kreps and Wilson's (1982) concept of "sequential equilibrium." They proved that a sequential allocation mechanism is implementable in perfect Bayesian equilibrium strategies if and only if there is an equivalent direct mechanism in which each individual commits himself to a single strategy of direct revelation, and in which to do so truthfully is a Bayesian equilibrium (Theorems 1 and 2, pp. 46–7).

By contrast, in Section 3 below the equivalent direct mechanism will be required to depend not merely on the first period announcement of the second period endowment, as would be the case for the kind of equivalent direct mechanism which Harris and Townsend construct. In addition, the need for monitoring implies that in the second period the equivalent direct mechanism must also depend on what the second period endowment actually turns out to be, which will be different if a false announcement has been made in the first period. It is as though agents were allowed to revise their earlier announcements and to claim that they had previously misstated their true type. In Harris and Townsend's framework, this kind of claim is equivalent to no more than a different and inconsistent deceptive strategy in the original (indirect) game form, and never benefits the deceiver in equilibrium. But their framework incorporates the assumptions of a known common consumption possibility set (p. 40) and a known set of allocations which are "achievable" or feasible for each coalition of agents (p. 41). Since a coalition of size one can presumably achieve only its initial endowment, or some private production possibility set, this specifically rules out the crucial assumption that agents are privately informed of their own endowments. Indeed, in Harris and Townsend's framework, private information never affects what is feasible — as Harris (1987, p. 121) indeed recognizes, but without being able to suggest any resolution. Accordingly, it should not be surprising that Harris and Townsend's results need some modification in sequence economies for which there is private information regarding endowments, consumption sets, or private production possibility sets. When one constructs an equivalent direct mechanism in such a sequence economy, one cannot afford to ignore any non-null set of agents who claim that they previously mis-stated their types and now cannot supply what they are supposed to — some form of monitoring and modification of their net trade vectors seems inevitable, if the allocation mechanism really is to be both physically feasible and implementable in dominant strategies. That is why both the announced and the true endowments feature in the equivalent direct mechanism of Section 3. One can expect more complicated equivalent direct mechanisms to emerge when there are many periods in which monitoring may be triggered. And corresponding additional incentive constraints.

1.5. *Outline*

Section 2 will set out the basic assumptions of a continuum economy lasting for two periods with one good each period. It also describes the basic framework of extensive game forms in which endowment monitoring is used to ensure individual physical feasibility. Thereafter Section 3 discusses implementation in dominant strategies, as well as the incentive and individual feasibility constraints which this requires. It also describes the problem of finding incentive constrained Pareto efficient allocation mechanisms. Then Section 4 explores the conditions under which an allocation mechanism can be implemented without any monitoring occurring in equilibrium. Section 5 presents a particular example of a demand revelation game form which generates an incentive compatible allocation mechanism. An even more special example with just two possible second period endowment levels, one of which is zero, is then considered. It is shown how, when the Walrasian equilibrium allocation cannot be sustained, there will be a non-Walrasian equilibrium with binding borrowing constraints for some individuals. Moreover, the borrowing rate of interest must exceed the lending rate, even though there are no defaults on any loans.

Conclusions are set out in Section 6.

2. Two Period Extensive Game Forms with Endowment Monitoring

2.1. *A Continuum Economy*

Consider a continuum economy with a non-atomic measure space of agents $(N, \mathcal{N}, \nu)$ — in fact, one could assume that $N = [0, 1] \subset \Re$, that $\mathcal{N}$ is the usual Borel σ-algebra, and that ν is the usual Lebesgue measure. For simplicity, suppose that all agents $i \in N$ have the same known utility function $u : \Re^2_+ \mapsto \Re$ which is continuous and strictly increasing on the known common consumption set $\Re^2_+$. To keep the model simple, suppose also that all agents have the same known endowment of 1 in the first period, but that their different second period endowments $e \in \Re_+$ remain as private information in the first period, and can only be discovered through monitoring in the second period. Assume too that there is a set $E \subset \Re_+$ of possible second period endowments. Note that no trade is always possible for each consumer, even if it does not necessarily guarantee survival.

2.2. Random Endowments

Suppose that the allocation mechanism in this economy emerges from an extensive game form with the following sequence of events. At the first stage of the game, nature determines at random the measurable function $e(\cdot) : N \to \Re_+$ that specifies the endowment vector e_i of each agent $i \in N$.[1] This function determines the joint distribution $\lambda \in \Delta(N \times E)$ of agents' names and endowments given by

$$\lambda(K) = \nu(\{ i \in N \mid (i, e_i) \in K \}) \tag{1}$$

for every Borel set $K \subset N \times E$. Obviously, λ induces the marginal distribution ν on N.

Agents $i \in N$ are supposed to be informed only of their own endowments e_i, and to remain uninformed about the endowments of others, or even about the distribution λ.

2.3. Strategies

At the second stage of the game form, each agent has the same set A of possible strategies a. These can be interpreted as bargaining strategies, offers to borrow and lend, announcements of credit demand functions, or whatever. All agents $i \in N$ choose their strategy functions $a_i : E \to A$ specifying how their respective actions in the economic system depend on their endowments, which represent the differences in what they know. For simplicity it is assumed that the resulting function $a(\cdot) : N \times E \to A$ is measurable, and so induces a well defined joint distribution $\sigma \in \Delta(N \times E \times A)$ on the set of agents' names, endowments and strategies which is given by

$$\sigma(K) = \lambda(\{ (i, e) \in N \times E \mid (i, e, a_i(e)) \in K \}) = \nu(\{ i \in N \mid (i, e_i, a_i(e_i)) \in K \}) \tag{2}$$

for every Borel set $K \subset N \times E \times A$. This distribution has appropriate marginals λ on $N \times E$ and so ν on N, of course.

---

[1] For simplicity it will be assumed that this and other functions of i are measurable. This is in common with almost all the rest of the literature on continuum economies and games with a continuum of players — e.g., Green (1984). Yet it is not entirely satisfactory because, for instance, a continuum of random variables almost never produces a measurable function (cf. Gale 1979, Feldman and Gilles 1985, and Judd 1985). In fact one should probably consider distributions rather than functions — e.g., the distribution $\lambda \in \Delta(N \times E)$ described in this section — without necessarily assuming that there is a (measurable) function at all. To do so would complicate the notation even further, however.

Also, let $\alpha(\sigma) \in \Delta(N \times A)$ denote the resulting marginal joint distribution of agents' strategies and names in the game form, which is given by

$$\alpha(\sigma)(K) = \lambda(\{ (i,e) \in N \times E \mid (i, a_i(e)) \in K \}) = \nu(\{ i \in N \mid (i, a_i(e_i)) \in K \}) \tag{3}$$

for every Borel set $K \subset N \times A$. It will be assumed that all the relevant features of this distribution can be observed by all those individuals in the economy who are responsible for arranging net trades and any monitoring activities which are undertaken at later stages of the game form, to be described below.

2.4. The Provisional Allocation Mechanism

Suppose now that, at the third stage of the game form, and still in the first period of the sequence economy's two periods, a *provisional allocation* of net trades

$$\xi_i^0(a, \alpha) : N \times A \times \Delta(N \times A) \to \Re^2 \tag{4}$$

is determined for each agent i, as a jointly measurable function of i's name, of i's strategy a in the game form, as well as the entire distribution of strategies α played by all the other agents. Notice that $\xi_i^0(\cdot)$ does not depend directly upon i's endowment e_i. So the allocation mechanism is not using private information.

It is presumed that the first period part $\xi_{i1}^0(a, \alpha)$ of this provisional mechanism comes into immediate effect during this third stage of the game form, and that this is always possible because $\xi_{i1}^0(a, \alpha) + 1 \geq 0$ for all $i \in N$, all $a \in A$, and all $\alpha \in \Delta(N \times A)$. It is presumed in addition that the second period part $\xi_{i2}^0(a, \alpha)$ will also come into effect unless there is some non-null set of agents who default in the second period by failing to repay what this provisional mechanism prescribes.

2.5. Endowment Monitoring

In the second period, the game form goes on to fourth and further stages. First it is assumed that agents are given the opportunity to make payments to each other voluntarily, in an effort to settle their debts if they want to. If these voluntary payments do in fact settle (almost) all debts, so that (almost) all agents $i \in N$ have their appropriate net expenditures $\xi_{i2}^0(a, \alpha)$, then the game stops and the provisional allocation becomes the actual one. Otherwise creditors pursue defaulting debtors and set in motion processes of monitoring and debt collection, as is the standard practice in bankruptcy proceedings. It is therefore natural to suppose that monitoring signals are affected by true endowments — indeed, in the extreme case of perfect monitoring, an agent's true second period endowment will be discovered. Thus the monitoring signal $m_i \in M$ for each monitored agent $i \in N$ is supposed to be a deterministic but possibly imperfect indicator of the true endowment e_i — obviously, randomness could be introduced into a more complicated model.

In fact, it will ease notation to suppose that all agents are monitored, but that for some of them the monitoring only produces a "null signal" m_0 which can be interpreted as meaning that no real monitoring has taken place. It is therefore assumed that there is a measurable *monitoring function*

$$m_i(e, a, \sigma) : N \times E \times A \times \Delta(N \times E \times A) \to M. \tag{5}$$

The range of this is function is taken to be the fixed set M of possible monitoring signals, which has the null signal m_0 as one of its members. For technical reasaons it will be necessary to assume that M can be given a topology and so also a Borel σ-algebra of measurable sets. Of course, this is hardly a serious restriction.

Note that the monitoring signal is allowed to depend on the entire joint distribution $\sigma \in \Delta(N \times E \times A)$ of agents' names, endowments, and strategies, rather than just on the marginal distribution $\alpha(\sigma) \in \Delta(N \times A)$ of names and strategies. This is because, as discussed below, several rounds of monitoring will often be needed, with monitoring decisons at later stages depending on the results of previous monitoring.

Given the joint distribution $\sigma \in \Delta(N \times E \times A)$, this monitoring function induces a joint distribution $\mu(\sigma) \in \Delta(N \times A \times M)$ which is given by

$$\mu(\sigma)\,(K) := \sigma(\{\, (i, e, a) \in N \times E \times A \mid (i, a, m_i(e, a, \sigma)) \in K \,\}) \tag{6}$$

for every measurable subset $K \subset N \times A \times M$. It is this distribution which summarizes whatever information about individuals' endowments is available to the economic system, and how those endowments are correlated with their behaviour in that system. Let $\alpha(\mu) \in \Delta(N \times A)$ denote the corresponding marginal distribution on just names and strategies.

Notice how one can only have $m_i(e, a, \sigma) = m_0$ for almost all $i \in N$ when $\xi^0_{i2}(a, \alpha(\sigma)) + e \geq 0$ for almost all $i \in N$. Otherwise there will be a non-null set of agents who are bound to default in the second period when the economic system tries to implement the provisional allocation by calling upon them to repay their debts in full. If these defaulting agents were not monitored and punished in some way, then no borrower would ever repay his debts, and the credit allocation mechanism would collapse to autarky. So non-trivial monitoring of defaulters is usually an essential part of any well functioning credit allocation system.

In addition, if there is a non-null set of agents for whom $\xi^0_{i2}(a, \alpha) + e < 0$, their unavoidable default and the consequent changes to the allocation mechanism may trigger defaults by other agents — a familiar problem in real economies when significant numbers of agents are in financial difficulties. That is why one cannot just assume, for each separate $i \in N$, that $\xi^0_{i2}(a, \alpha(\sigma)) + e \geq 0$ implies $m_i(e, a, \sigma) = m_0$. For the same reason, several rounds of monitoring may be necessary, and the ultimate monitoring signal $m_i(e, a, \sigma)$ may depend on actual endowments through the entire distribution $\sigma \in \Delta(N \times E \times A)$ of names, endowments and actions, rather than just on the marginal distribution $\alpha(\sigma) \in \Delta(N \times A)$ of names and actions. This will be true for the special demand revelation mechanism discussed in Section 5.1 below, for instance.

2.6. *The Final Second Period Allocation Mechanism*

Here, however, I shall consider only the final outcome after as many rounds as necessary of monitoring and recontracting have been completely carried out in order to ensure feasibility. So, in the very last stage of the game form, there will be a measurable *final second period allocation function*

$$\xi^M_{i2}(a, m, \mu) : N \times A \times M \times \Delta(N \times A \times M) \to \Re. \tag{7}$$

There is also an associated *direct second period allocation function* $\xi_{i2}(a, e, \sigma)$, whose second argument is the individual's true endowment e rather than the monitoring signal m which was generated by e, and whose third argument is the joint distribution σ in the population of combinations (i, e, a). This measurable function $\xi_{i2} : N \times A \times E \times \Delta(N \times E \times A) \to \Re$ is given by the composition

$$\xi_{i2}(a, e, \sigma) \equiv \xi^M_{i2}(a, m_i(e, a, \sigma), \mu(\sigma)) \tag{8}$$

of the above allocation and monitoring functions.

2.7. Ensuring Physical Feasibility

Since the whole purpose of monitoring is to ensure physical feasibility, it will be assumed that, for all $(i, e, a, \sigma) \in N \times E \times A \times \Delta(N \times E \times A)$, the individual feasibility constraint

$$\xi_{i2}^M(a, m_i(e, a, \sigma\,), \mu(\sigma)\,) + e = \xi_{i2}(a, e, \sigma) + e \geq 0 \tag{9}$$

is satisfied. Note in particular how, for agents whose second period allocation would be individually infeasible in the absence of monitoring, the inevitable default that follows must trigger active monitoring, and lead to an adjustment of the allocation to something that is feasible given the true endowment e. The assumption (9) guarantees that a feasible allocation always results, even when individual consumers use disequilibrium strategies. This is an essential condition for an economic system to be able to function, which must always be satisfied somehow. Remember, after all, that I do *not* assume that feasibility entails survival.

To illustrate how (9) is actually quite plausible, consider first the following *perfect monitoring mechanism*. The monitoring signal space M is taken to be $\Re_+ \cup \{m_0\}$. The associated monitoring and second period allocation functions are then given by

$$\begin{aligned} m_i(e, a, \sigma) &= \begin{cases} m_0 & \text{if } \xi_{i2}^M(a, m_0, \mu(\sigma)\,) + e \geq 0; \\ e & \text{if } \xi_{i2}^M(a, m_0, \mu(\sigma)\,) + e < 0; \end{cases} \\ \xi_{i2}^M(a, m, \mu(\sigma)\,) &= -m \iff m \neq m_0. \end{aligned} \tag{10}$$

This evidently implies that

$$\xi_{i2}(a, e, \sigma) = \max\{\, \xi_{i2}^M(a, m_0, \mu(\sigma)\,), -e\,\} \tag{11}$$

and so (9) is indeed satisfied. Under this perfect monitoring mechanism, agents are monitored if and *only if* they would have to default otherwise. Also, monitoring detects all the endowment of any agent who is actively monitored. All this endowment is then confiscated, so consumption is forced down to zero in the second period. Because of our assumption that the consumption set is $\Re_+^2$, this guarantees individual feasibility. It will also deter default provided that the null monitoring consumption stream $(\,\xi_{i1}^0(a^*, \alpha(\sigma)) + 1, \xi_{i2}^M(a^*, m_0, \mu(\sigma)\,) + e\,)$ for the best strategy a^* is always at least weakly preferred to $(\,\xi_{i1}^0(a, \alpha(\sigma)) + 1, 0\,)$ for every alternative strategy $a \in A$.

Obviously punishing defaulters by reducing their consumption to zero in the second period is rather drastic, especially as there is no guarantee that the individual defaulter can then even survive. In addition, not all an agent's endowment can always be detected through monitoring — and even if it can be, it may be too costly to do so. Thus one may be forced to have "imperfect" monitoring mechanisms instead, with $m_i(e, a, \sigma) \leq e$ when $\xi_{i2}^M(a, m_0, \mu(\sigma)) + e < 0$, and $\xi_{i2}^M(a, m, \mu(\sigma)) \geq -m$ when $m \neq m_0$. Then (11) becomes replaced by the inequality $\xi_{i2}(a, e, \sigma) \geq \max\{ \xi_{i2}^M(a, m_0, \mu(\sigma)), -e \}$ and so (9) is still satisfied in this case. Such a mechanism does serve to show how individual feasibility might be guaranteed even out of equilibrium.

2.8. Resource Balance

Physical feasibility also requires aggregate resource balance constraints to be satisfied. In the second period, such constraints should reflect the resource costs of the monitoring that is required in order to bring about the specified monitoring function. Thus a reasonable formulation of such constraints is

$$\begin{aligned} \int_{N \times A} \xi_{i1}^0(a, \alpha)\ \alpha(di \times da) &\leq y_1; \\ \int_{N \times E \times A} \xi_{i2}(a, e, \sigma)\ \sigma(di \times de \times da) &\leq y_2(y_1, \sigma) \end{aligned} \tag{12}$$

for every joint distribution $\sigma \in \Delta(N \times E \times A)$ with marginal $\alpha \in \Delta(N \times A)$. Here y_1 represents the mean level in the population of net expenditure (or excess of expenditure over saving) during the first period; then $y_2(y_1, \sigma)$ is an upper bound on the mean second period net expenditure levels which are physically feasible. It reflects the non-monitoring production possibilities of the economy. The dependence of y_2 on σ, however, is a general formulation recognizing that the process of producing monitoring signals according to the function $m_i(e, a, \alpha)$ must use up some real resources. These have to be diverted away from meeting the net demands in the non-monitoring sector of the economy. Note that the costs of monitoring an agent's endowments are presumably always observable by the person doing the monitoring and so incurring the costs. For this reason they can be treated as part of the monitoring signal m, with a value of zero when $m = m_0$, the null signal.

Aggregate feasibility constraints like (12) play no role, however, in determining the incentive constraints which are our primary concern here. Thus they will largely be ignored except when constrained Pareto efficient allocation mechanisms are discussed in Section 3.4 below.

That completes the basic description of the multi-stage extensive game form for determining net trades in the simple two period sequence economy. Although its formulation may appear rather special at first sight, in fact it is hard to imagine any really different formulation of an economic system that both allows trades to be consummated without any monitoring when all individuals are indeed honouring their obligations, and also provides for the monitoring which is needed when there are defaulters.

3. Implementation in Dominant Strategies

3.1. Dominant Strategies

Suppose that each individual $i \in N$ in this game form has a dominant strategy function $a_i^*(e) : E \to A$ at the second stage which depends upon i's own endowment e, but not on the distribution α of strategies chosen by other individuals. Thus

$$\begin{aligned} a_i^*(e) &\in \arg\max_a \{\, u(\, \xi_{i1}^0(a, \alpha(\sigma)\,) + 1, \xi_{i2}^M(a, m_i(a, e, \sigma), \mu(\sigma)\,) + e\,) \mid a \in A \,\} \\ &= \arg\max_a \{\, u(\, \xi_{i1}^0(a, \alpha(\sigma)\,) + 1, \xi_{i2}(a, e, \sigma) + e\,) \mid a \in A \,\} \end{aligned} \tag{13}$$

which means that $a_i^*(e)$ is always a best response to any joint frequency distribution $\sigma \in \Delta(N \times E \times A)$ generated by the other agents' choices of strategies. Notice that there is no need to incorporate any feasibility constraint explicitly in the maximization problem (13) because of the earlier assumption (9). It will be assumed finally that the dominant strategy function $a_i^*(\cdot) : N \times E \to A$ is measurable and so, for each distribution $\lambda \in \Delta(N \times E)$ of names and endowments, induces the well defined joint distribution

$$\sigma^*(\lambda)\,(K) := \lambda(\{\,(i, e) \in N \times E \mid (i, e, a_i^*(e)\,) \in K\,\}) \in \Delta(N \times E \times A) \tag{14}$$

for every measurable subset $K \subset N \times E \times A$, whose corresponding marginal distribution is given by

$$\alpha^*(\lambda)\,(K) := \lambda\,(\{\,(i, e) \in N \times E \mid (i, a_i^*(e)) \in K\,\}) \in \Delta(N \times A) \tag{15}$$

for every measurable subset $K \subset N \times A$. In addition, the monitoring functions $m_i(e, a, \alpha\,)$ then determine the joint distribution in $\Delta(N \times A \times M)$ given by

$$\mu^*(\lambda)\,(K) := \lambda\,(\{\,(i, e) \in N \times E \mid (i, a_i^*(e), m_i(e, a_i^*(e), \alpha^*(\lambda)\,)\,) \in K\,\}) \tag{16}$$

for every measurable subset $K \subset N \times A \times M$.

3.2. An Equivalent Direct Mechanism

For each agent $i \in N$ there are now two equivalent direct mechanisms, one for each period, described by the measurable functions

$$\begin{aligned} f_{i1}(\lambda, e) &:= \xi_{i1}^0(a_i^*(e), \alpha^*(\lambda)) : \Delta(N \times E) \times E \to \Re; \\ f_{i2}^M(\lambda, e; m) &:= \xi_{i2}^M(a_i^*(e), m, \mu^*(\lambda)) : \Delta(N \times E) \times E \times M \to \Re. \end{aligned} \tag{17}$$

But also, given the game form and dominant strategy function specified above, each agent $i \in N$ has an equivalent *direct monitoring function* defined by

$$g_i(\lambda, e'; e) := m_i(e, a_i^*(e'), \alpha^*(\lambda)) : \Delta(N \times E) \times E \times E \to M \tag{18}$$

which is also measurable. Thus $g_i(\lambda, e'; e)$ is the monitoring signal generated when agent i with true endowment e is deceptive, and acts as would agent i with endowment e', when the joint distribution of all agents' names and endowments appears to be λ. There is then an even more direct second period mechanism given by

$$\begin{aligned} f_{i2}(\lambda, e'; e) &:= \xi_{i2}(a_i^*(e'), e, \sigma^*(\lambda)) \equiv \xi_{i2}^M(a_i^*(e'), m_i(e, a_i^*(e'), \alpha^*(\lambda)), \mu^*(\lambda)) \\ &\equiv \xi_{i2}^M(a_i^*(e'), g_i(\lambda, e'; e), \mu^*(\lambda)) \equiv f_{i2}^M(\lambda, e'; g_i(\lambda, e'; e)) \end{aligned} \tag{19}$$

which is a measurable function $f_{i2}(\lambda, e'; e) : \Delta(N \times E) \times E \times E \to \Re$. So the net expenditure stream $(f_{i1}(\lambda, e'), f_{i2}(\lambda, e'; e))$ results when agent i acts "deceptively," as if having second period endowment e' when it is really e, and when the joint distribution of names and endowments appears to be λ.

Note how the first period net borrowing $f_{i1}(\lambda, e')$ of somebody who acts as though their second period endowment were e' must be independent of the true endowment e, because e is unable to influence the monitoring signal $m \in M$ until the second period, and so can affect only the second period allocation. Of course $f_{i2}(\lambda, e'; e)$ does depend, in general, not only upon the apparent value e' of the agent's endowment, but also upon the true endowment e, since that ultimately influences the monitoring signal $m \in M$. But it must be independent of e for those true second period endowments satisfying $g_i(\lambda, e'; e) = m_0$, because then no monitoring occurs even in the second period.

3.3. *Incentive and Individual Feasibility Constraints*

Because of our assumption (13) that $a_i^*(e)$ is always a dominant strategy for each agent $i \in N$, one has

$$\begin{aligned} & u(\xi_{i1}^0(a_i^*(e),\, \alpha^*(\lambda)\,) + 1,\, \xi_{i2}(a_i^*(e'), e,\, \sigma^*(\lambda)\,) + e\,) \\ & \qquad \leq u(\xi_{i1}^0(a_i^*(e),\, \alpha^*(\lambda)\,) + 1,\, \xi_{i2}(a_i^*(e), e,\, \sigma^*(\lambda)\,) + e\,) \end{aligned} \tag{20}$$

for all (λ, e', e). Then it follows immediately from (17) and (19) that

$$u(\, f_{i1}(\lambda, e') + 1,\, f_{i2}(\lambda, e'; e) + e\,) \leq u(\, f_{i1}(\lambda, e) + 1,\, f_{i2}(\lambda, e; e) + e\,) \tag{21}$$

for all (λ, e', e). These are precisely the *incentive constraints* needed for truthtelling to be a dominant strategy in the direct revelation game form induced by the direct mechanism $(\, f_{i1}(\lambda, e'),\, f_{i2}(\lambda, e'; e)\,)$.

Because of (9), it must also be true that

$$\xi_{i2}(a_i^*(e'), e, \sigma^*(\lambda)\,) + e \geq 0 \tag{22}$$

for all (λ, e', e). But then (19) implies that, for all (λ, e', e), the equivalent direct mechanism must satisfy the important additional *individual feasibility constraint*

$$f_{i2}(\lambda, e'; e) + e \geq 0. \tag{23}$$

In our simple sequence economy, *both* constraints (21) and (23) must be satisfied for all possible (λ, e', e) if a direct mechanism is to be implementable in dominant strategies. Satisfying the incentive constraints (21) alone is not enough.

3.4. *Pareto Efficient Mechanisms*

Now that the physical feasibility and incentive constraints have all been specified, it is possible in principle to describe Pareto efficient mechanisms which satisfy these constraints. The choice variables are the first period allocation mechanism $f_{i1}(\lambda, e)$ and then, in the second period, the combination consisting of the space M of monitoring signals, the monitoring function $g_i(\lambda, e'; e)$, and finally the second period allocation mechanism $f_{i2}^M(\lambda, e'; m)$ based on these monitoring signals. The latter allocation mechanism gives rise to the measurable function $f_{i2}(\lambda, e'; e) \equiv f_{i2}^M(\lambda, e'; g_i(\lambda, e'; e))$.

For each possible distribution $\lambda \in \Delta(N \times E)$, a Pareto efficient mechanism will usually maximize an objective which can be expressed as a utility integral

$$W \equiv \int_{N \times E} \omega_i(\lambda, e)\, u(\, f_{i1}(\lambda, e) + 1, f_{i2}(\lambda, e; e) + e\,)\, \lambda(di \times de) \tag{24}$$

with welfare weights $\omega_i(\lambda, e)$. This should then be maximized subject to the individual feasibility constraints (23), the incentive constraints (21), and then resource balance constraints such as

$$\begin{aligned} \int_{N \times E} f_{i1}(\lambda, e)\, \lambda(di \times de) &\leq y_1 \\ \int_{N \times E} f_{i2}(\lambda, e; e)\, \lambda(di \times de) &\leq y_2(y_1) - \int_{N \times E} \gamma_i^M(\lambda, g_i(\lambda, e; e))\, \lambda(di \times de) \end{aligned} \tag{25}$$

for each of the two periods. Here, the function $\gamma_i^M(\lambda, m) : \Delta(N \times E) \times M \to \Re_+$ indicates the resource cost of monitoring, which can be assumed to satisfy the condition that $\gamma_i^M(\lambda, m_0) = 0$ — i.e., the resource cost of the null signal is zero, so that monitoring should be avoided if possible. Nevertheless, it is quite possible that monitoring will occur for some agents even when they use their (truthful) dominant strategies. Indeed, monitoring of endowments could be part of an optimal solution even if there were no problem of default: it could simply be the most cost effective incentive compatible method of arranging the redistribution needed to achieve allocations that are both distributively just and also fully Pareto efficient. After all, if monitoring of endowments were both perfect and costless, it could be used to bring about a first-best allocation in every possible economic environment. For instance, since land holdings are often quite easy to monitor, land reform is often advocated as a powerful tool for redistribution.

Although the problem of finding an incentive-constrained Pareto efficient allocation mechanism has now been formulated in a fairly simple way, its solution seems to be very far from simple, even in very special cases such as when the set E of possible endowments has only two values. Thus the characterization of such solutions has had to be left for later work. No doubt this will build on important insights such as those in Harris and Raviv (1979), Townsend (1979), Gale and Hellwig (1985), as well as more recent unpublished work by several authors.

4. When Can Monitoring Be Avoided in Equilibrium?

4.1. Incentive Constraints without Monitoring

This section will examine the conditions under which there is (almost) no monitoring in the economy when (almost) all agents use their dominant strategies in the game form. After all, if monitoring is costly, and if the second period resource balance constraint is tight even in the absence of monitoring — as it presumably should be for an optimal mechanism — then there cannot be any monitoring at all if that mechanism is indeed to be physically feasible.

To this end, first define the function

$$f^0_{i2}(\lambda, e) := f^M_{i2}(\lambda, e; m_0) : \Delta(N \times E) \times E \to \Re. \tag{26}$$

This is the second period allocation in the event of their being no monitoring. In terms of the equivalent direct mechanism, in which agents reveal their true endowments in equilibrium, there will be no monitoring in equilibrium provided that, for all $e \in E$, one has

$$g_i(\lambda, e; e) = m_0 \quad \text{and} \quad f_{i2}(\lambda, e; e) = f^M_{i2}(\,\lambda, e, m_0\,) = f^0_{i2}(\lambda, e). \tag{27}$$

Now suppose that a typical agent $i \in N$ acts as though having endowment e' even though the true endowment is e. If agent i is not monitored in the second period either when his endowment is e or when it is e', this deception can never be detected because monitoring is the only way of noticing the difference between these two endowment levels. But because of (27), an agent whose endowment appears to be either e or e' will not be monitored in dominant strategy equilibrium when those endowments are the real ones, and when $\lambda \in \Delta(N \times E)$ is the real joint distribution of names and endowments. Thus the only way in which this deception can ever be detected is if it triggers monitoring because it leads to default. In other words, if agent i acts as though his endowment were e' when it is really e, there will be no monitoring unless $f^0_{i2}(\lambda, e') + e < 0$, which is precisely when agent i will have to default and so trigger monitoring. So (27) implies that

$$\begin{aligned} g_i(\lambda, e'; e) = m_0 \text{ and } f_{i2}(\lambda, e'; e) = f^M_{i2}(\,\lambda, e', m_0\,) = f^0_{i2}(\lambda, e') \\ \iff f^0_{i2}(\lambda, e') + e \geq 0. \end{aligned} \tag{28}$$

Thus active monitoring occurs if and only if it is needed to ensure individual feasibility in the second period.

Notice how (28) implies that, if $f^0_{i2}(\lambda, e') + e \geq 0$ and the incentive constraints (21) are also satisfied, then

$$\begin{aligned} & u(\, f_{i1}(\lambda, e') + 1,\, f^0_{i2}(\lambda, e') + e\,) = u(\, f_{i1}(\lambda, e') + 1, f_{i2}(\lambda, e'; e) + e\,) \\ \leq\, & u(\, f_{i1}(\lambda, e) + 1, f_{i2}(\lambda, e; e) + e\,) = u(\, f_{i1}(\lambda, e) + 1,\, f^0_{i2}(\lambda, e) + e\,). \end{aligned} \tag{29}$$

In particular, this shows that the implication

$$\begin{gathered} f^0_{i2}(\lambda, e') + e \geq 0 \Longrightarrow \\ u(\, f_{i1}(\lambda, e') + 1, f^0_{i2}(\lambda, e') + e\,) \leq u(\, f_{i1}(\lambda, e) + 1, f^0_{i2}(\lambda, e) + e\,) \end{gathered} \tag{30}$$

must be true for all pairs $e, e' \in E$. These are "restricted" incentive constraints in the sense that they need only be satisfied for those deviations e' which result in individually feasible net expenditure streams $(\, f_{i1}(\lambda, e'), f^0_{i2}(\lambda, e')\,)$. Incentive constraints involving deviations which lead to individually infeasible allocations are simply ignored. For the case of static economies this corresponds precisely to the assumption made in Hammond (1979, 1987a), in Gale (1980, 1982), and apparently in Postlewaite and Schmeidler (1986, 1987) too.

It is therefore especially important to understand clearly the difference between (30) and the earlier combination of the incentive constraints (21) with the individual feasibility constraints (23). Because of the need to ensure individual feasibility, through monitoring if necessary, each individual's second period net expenditure $f_{i2}(\lambda, e'; e)$ in (21) depends on both e' and e (as well as on λ, of course), and the incentive constraints are required to hold for *all* possible pairs e, e'. Whereas (30) restricts attention to just those pairs e, e' for which the individual feasibility constraint $f^0_{i2}(\lambda, e') + e \geq 0$ is satisfied anyway, and does not need to make $f^0_{i2}(\lambda, e')$ depend on e as well as on e'.

Note too how, when (27) holds, then the incentive constraints (21) must implicitly include "deterrence constraints" ensuring that, if agent i defaults in the second period and so allows monitoring to be triggered, then allocating i the net expenditure level $f^M_{i2}(\lambda, e'; m)$ with $m \neq m_0$ is an effective punishment ensuring that there are no benefits to default. So, if (27) could be made true, not only would truthful revelation always be a dominant strategy, but it would also follow that monitoring is never necessary when individuals do reveal their true endowments.

4.2. *Sufficient and Necessary Conditions*

Second period consumption $f_{i2}(\lambda, e'; e) + e$ must always be non-negative, however, even if all of a defaulter's endowment is monitored and then confiscated. Since utility is strictly increasing, combining (27) with the incentive constraints (21) implies that, for all pairs $e, e' \in E$, one has

$$\begin{aligned} u(f_{i1}(\lambda, e') + 1,\, 0) &\leq u(\, f_{i1}(\lambda, e') + 1,\; f_{i2}(\lambda, e'; e) + e\,) \\ &\leq u(\, f_{i1}(\lambda, e) + 1,\; f_{i2}(\lambda, e; e) + e\,) \\ &= u(\, f_{i1}(\lambda, e) + 1,\; f^0_{i2}(\lambda, e) + e\,) \end{aligned} \tag{31}$$

as a necessary condition for feasibility to be possible without any monitoring in equilibrium.

Some unnecessarily strong sufficient conditions for this key property (31) to hold are that $f_{i1}(\lambda, e) + 1 > 0$ and $f^0_{i2}(\lambda, e) + e > 0$ for all $e \in E$, and also that

$$u(c'_1, 0) \leq u(c_1, c_2) \quad \text{whenever} \quad c'_1 \geq 0 \quad \text{and} \quad c_1, c_2 > 0. \tag{32}$$

This will be true in particular when preferences are "smooth", implying that no indifference curve intersects the $c_2 = 0$ axis. When (32) is satisfied, (31), (21) and (23) can indeed all be made true provided that any defaulter's second period consumption gets reduced to zero by means of a mechanism such as (10). In this special case, then, there will be no need for monitoring in equilibrium, when all agents do use their dominant strategies.

On the other hand, if zero consumption (or whatever other lower bound is imposed on defaulters) is not too bad, perhaps because the defaulter is no longer in a position to care very much anyway, then the necessary condition (31) for deterring all default imposes an upper bound on $f_{i1}(\lambda, e')$. For the specific case when

$$u(c_1, c_2) \equiv \sqrt{c_1} + \sqrt{c_2}, \tag{33}$$

for instance, (31) obviously implies that

$$\sup_{e' \in E} \{\, f_{i1}(\lambda, e')\,\} \leq \inf_{e \in E} \left\{ \left(\sqrt{f_{i1}(\lambda, e) + 1} + \sqrt{f^0_{i2}(\lambda, e) + e} \right)^2 - 1 \right\} \tag{34}$$

for almost all $i \in N$. This necessary condition for the absence of default when agents use their dominant strategies need not be true even for the usual kind of Walrasian mechanism, with budget sets given by the familiar

$$c_1 + (1+r)^{-1} c_2 \leq 1 + (1+r)^{-1} e \tag{35}$$

for a suitable rate of interest r. For in a pure exchange economy with utility function given by (33), the appropriate unique Walrasian equilibrium is symmetric, with net expenditures and an interest rate which are easily calculated as being given by the functions

$$f_{i1}(\lambda, e) \equiv \frac{e - \bar{e}(\lambda)}{\bar{e}(\lambda) + \sqrt{\bar{e}(\lambda)}}; \quad f^0_{i2}(\lambda, e) \equiv \frac{\bar{e}(\lambda) - e}{\sqrt{\bar{e}(\lambda)} + 1}; \quad r(\lambda) \equiv \sqrt{\bar{e}(\lambda)} - 1. \tag{36}$$

Here $\bar{e}(\lambda)$ denotes the mean second period endowment $\int_{N \times E} e\, \lambda(di \times de)$. In this case it is easy to check that (34) is satisfied if and only if

$$\frac{e^* + \sqrt{\bar{e}(\lambda)}}{e_* + \sqrt{\bar{e}(\lambda)}} \leq \left(\sqrt{\bar{e}(\lambda)} + 1 \right)^2 \tag{37}$$

where e^* and e_* denote respectively the supremum and the infimum of the set E of possible second period endowments which the allocation mechanism has to allow for. In particular, (37) is never satisfied if e is lognormally distributed.

Accordingly, it seems that one has to abandon the hypothesis that, even when individuals are privately informed about their endowments, there are always dominant strategy incentive compatible mechanisms in which all defaults can be prevented, without the need for any monitoring in equilibrium. Also, even if such mechanisms do exist, it may still be true that the incentive constraints (21) and feasibility constraints (23) are together strictly stronger than just the restricted incentive constraints (30).

5. A Special Case

5.1. A Demand Revelation Mechanism

A particular demand revelation mechanism of some interest is when each individual $i \in N$ has a strategy space A consisting of all possible net expenditure demand correspondences $D(r, \bar{x}_1) : \Re_+ \times \Re_+ \twoheadrightarrow \Re^2$ which have closed graphs and non-empty compact values, as well as satisfying the condition that

$$D(r, \bar{x}_1) \subset \{\, (x_1, x_2) \in \Re^2 \mid -1 \leq x_1 \leq \bar{x}_1 \quad \& \quad x_1 + (1+r)^{-1} x_2 \leq 0 \,\}. \tag{38}$$

The relevant parameters which determine the budget set on the right hand side of (38) are an interest rate r and a first period borrowing ceiling $\bar{x}_1$. The economy is supposed to be one of pure exchange. So there will exist functions $r(\alpha)$ and $\bar{x}_1(\alpha)$ of the distribution $\alpha \in \Delta(N \times A)$, together with selections $x_i^0(\alpha) \in D(r(\alpha), \bar{x}_1(\alpha))$, such that the resource balance constraints $\int_{N \times A} x_{it}^0(\alpha)\, \alpha(di \times da) \leq 0$ are always satisfied for both periods $t = 1, 2$.

The monitoring signal space is taken to be $\Re_+$, the set of possible endowment levels. When some agents default in the second period, and have endowments which turn out to be too low to cover their debts, there will be an aggregate deficit. For simplicity it is assumed that this is financed by means of a proportional reduction in the return to lenders, while borrowers pay no more than their nominal obligations require. If this were not true, and borrowers were also expected to contribute to the deficit, one would have to consider which additional borrowers would be forced to default because of these extra payments, how much extra deficit arises, and so on — as in the analysis by Green (1974). Thus, for each possible value of $\sigma \in \Delta(N \times E \times A)$, it is simply assumed that there exists some $\theta(\sigma) \in [0, 1]$ with the property that:

(i) in the absence of monitoring, each agent i's net expenditure level in period 2 is given by

$$\xi_{i2}^M(a, m_0, \mu(\sigma)) := \begin{cases} \theta(\sigma)\xi_{i2}^0(a, \alpha) & \text{if } \xi_{i2}^0(a, \alpha) > 0; \\ \xi_{i2}^0(a, \alpha) & \text{if } \xi_{i2}^0(a, \alpha) \leq 0; \end{cases} \tag{39}$$

(ii) the monitoring function is given by (10);

(iii) the appropriate simplified form for an exchange economy of the second period resource balance constraint in (12), namely

$$\int_{N \times E \times A} \max\{\, \xi_{i2}^M(a, m_0, \mu(\sigma)\,), -e\,\}\, \sigma(di \times de \times da) \leq y_2(\sigma), \tag{40}$$

is satisfied.

Then capital markets have just two "imperfections": first, those who want to borrow large amounts may find their credit being rationed; second, there may be some default, even though defaulters have their second period consumption reduced to zero.

5.2. Demand Behaviour

In this special case, the utility-maximizing first period net demand correspondence of an agent whose true second period endowment is e will be

$$D_1^*(r, \bar{x}_1; e) \equiv \arg\max_{x_1} \{ u(x_1 + 1, \max\{ 0, e - (1+r)\, x_1 \}) \mid -1 \leq x_1 \leq \bar{x}_1 \}. \tag{41}$$

Note that $D_1^*(r, \bar{x}_1; e)$ will be equal to $\{\bar{x}_1\}$, with the agent facing zero consumption in the second period, whenever both $e < (1+r)\, \bar{x}_1$ — so that the credit ration does not prevent the agent from ever borrowing more than can be repaid — and also the default utility level $u(\bar{x}_1 + 1, 0)$ exceeds the usual indirect utility

$$V^W(r; e) := \max_{x_1} \{ u(x_1 + 1, x_2 + e) \mid x_1 + (1+r)^{-1}\, x_2 \leq 0 \} \tag{42}$$

from optimizing subject to the Walrasian budget constraint. Of course, those agents who are induced to borrow more than they can repay at the rate of interest r must be among those who have their credit rationed, since otherwise their demands for first period consumption would be unlimited.

Some typical properties of the first period net demand correspondence D_1^* can be illustrated by taking the specific utility function (33), for which

$$D_1^*(r, \bar{x}_1; e) \equiv \arg\max_{x_1} \left\{ \sqrt{x_1 + 1} + \max\{ 0, \sqrt{e - (1+r)\, x_1} \} \mid -1 \leq x_1 \leq \bar{x}_1 \right\}. \tag{43}$$

Now, as the second period endowment e increases, with the first period endowment held fixed at the level 1, the consumer's wealth expansion path takes a form like that illustrated in Figure 1. There are three distinct cases to consider.

The first (case D) occurs for very low levels of e. Then the consumer borrows as much as possible — i.e., up to the credit ceiling $\bar{x}_1$ — and later defaults, thus choosing the point D in Figure 1 at which $D_1^*(r, \bar{x}_1; e) = \{\bar{x}_1\}$.

As e increases, this default option eventually becomes sufficiently unattractive that the individual switches discontinuously from borrowing to lending, and then starts moving up the line segment between W_1 and W_2. This is the usual Walrasian case W, in which the credit ceiling does not bind and the agent repays in full. Then $D_1^*(r, \bar{x}_1; e) = \{x_1^W\}$, where

$$x_1^W := \frac{e - (1+r)^2}{(1+r)\,(2+r)} \leq \bar{x}_1 \tag{44}$$

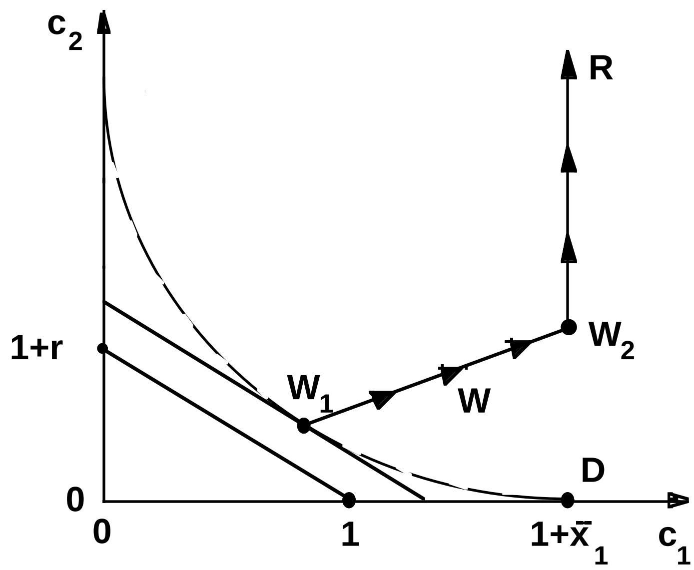

Figure 1. A Typical Wealth Expansion Path

So, once case W occurs, the consumer is a lender for values of e up to $(1+r)^2$. But he switches back to being a borrower thereafter, as e becomes large enough to make it desirable to transfer some purchasing power from the future to the present even though (unlike in case D), the loan will be repaid.

The indirect utility function for case W is

$$V^W(r;e) := \sqrt{(2+r)\,[1+(1+r)^{-1}\,e]}. \tag{45}$$

Accordingly, the borderline between cases D and W must occur when

$$e = (2+r)^{-1}\,(1+r)\,[\bar{x}_1 - (1+r)] \tag{46}$$

because then the Walrasian indirect utility $V^W(r;e)$ given by (45) is just equal to the default utility $\sqrt{\bar{x}_1+1}$. Note how (46) implies that $e < (1+r)\,\bar{x}_1$ so that, for all e below this critical value, default is indeed bound to occur after the consumer has borrowed up to the limit $\bar{x}_1$ in the first period.

Ultimately, however, at the point W_2, the credit ceiling that was necessary to control the default behaviour of those with very low values of e begins to bind even for those with very high values of e, who cannot be distinguished *ex ante*. This is true even though individuals with very high values of e have no intention of defaulting. This is the third case R, when the individual's credit is rationed, but there is no default. Then $D_1^*(r, \bar{x}_1; e) =$

$\{\bar{x}_1\}$ and $e - (1+r)\,\bar{x}_1 \geq 0$. This case occurs when

$$e > (1+r)^2 + (1+r)\,(2+r)\,\bar{x}_1 \tag{47}$$

because then (44) implies that the Walrasian net demand x_1^W exceeds the ceiling $\bar{x}_1$. Case R corresponds to the last part of the wealth expansion path shown in Figure 1, represented by the vertical line up from W_2.

It should be noted finally that the intermediate Walrasian case W will occur on a non-trivial interval of values of e whenever $\bar{x}_1 + 1 > 0$, because this is sufficient to ensure that the lower limit determined by (46) is less than the upper limit determined by (47).

5.3. A Two Type Example

Let us now restrict this special example further by assuming that the set of possible second period endowments E is just the pair $\{\,0, e^*\,\}$, and that $e^*(>0)$ is the endowment of a proportion π of the population, while a proportion $1-\pi$ have a zero endowment. Then the mean second period endowment is $\pi\,e^*$. So, according to (36), the unique Walrasian equilibrium for this distribution λ has the interest rate $r = \sqrt{\pi\,e^*} - 1$ and (in an obvious notation) the allocation

$$\begin{aligned} x_1(e^*) &= \frac{e^* - \pi\,e^*}{\pi\,e^* + \sqrt{\pi\,e^*}}; \quad & x_2(e^*) &= \frac{\pi\,e^* - e^*}{\sqrt{\pi\,e^*}+1}; \\ x_1(0) &= \frac{-\pi\,e^*}{\pi\,e^* + \sqrt{\pi\,e^*}}; \quad & x_2(0) &= \frac{\pi\,e^*}{\sqrt{\pi\,e^*}+1}. \end{aligned} \tag{48}$$

Thus those with zero second period endowment save by making loans to those with e^* so that both types of agent have positive consumption in both periods.

Also, according to (37), this unique Walrasian equilibrium is unsustainable in case $e^* + \sqrt{\pi\,e^*} > \sqrt{\pi\,e^*}\left(\sqrt{\pi\,e^*}+1\right)^2$, which is true iff $\pi^{-1} - 2 > \sqrt{\pi\,e^*}$ or iff $\pi < \frac{1}{2}$ and $e^* < (1-2\,\pi)^2\,\pi^{-3}$. Recall that the unsustainability arises because agents with zero second period endowment prefer the consumption stream which they can get by borrowing like an agent with e^* and then defaulting in the second period to the consumption stream which they can get by being "honest."

When this Walrasian allocation is unsustainable, we shall look for an alternative allocation with a borrowing rate r_B which is typically higher than the lending rate r. There will also be a credit ceiling $\bar{x}_1$ imposed to prevent those with zero second period endowment from borrowing with the intention to default. In addition, those agents who do not default in the second period will each receive a dividend d which is financed by the profits that are earned from the difference between the borrowing and lending rates of interest. Those with zero second period endowment will still be lending to those with e^*. And even though the need to allow for default prevents attainment of the Walrasian allocation, there will actually be no default in the new equilibrium. The need for differential borrowing and lending rates will also be demonstrated, in the case when the Walrasian equilibrium is unsustainable.

Since individuals with $e = 0$ are lending, the credit ceiling does not affect them at all (except to deter borrowing with intent to default), and so their net borrowing is at the Walrasian level given by

$$x_1^W(r, d; 0) = \frac{d - (1+r)^2}{(1+r)(2+r)}, \tag{49}$$

after changing (44) above to take account of the dividend d in the second period, and then putting $e = 0$. To ensure that $x_1^W(r, d; 0) < 0$, of course, requires that $d < (1+r)^2$. The net repayment of each such individual, in the event of no default, is

$$x_2^W(r, d; 0) = d - (1+r)\, x_1^W(r, d; 0) = \frac{(d+1+r)(1+r)}{2+r}. \tag{50}$$

Thus these individuals are at the point E^0 in Figure 2. The associated utility level is

$$u^W(0) = \sqrt{\frac{(d+1+r)(2+r)}{1+r}}. \tag{51}$$

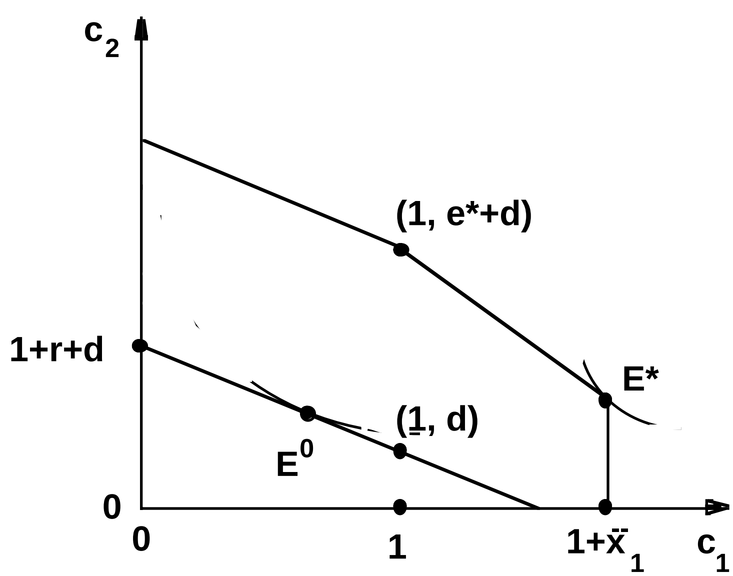

Figure 2. A Non-Walrasian Equilibrium

On the other hand, in order to investigate whether there can be a non-Walrasian equilibrium even when the borrowing and lending rates of interest are equal, it will be assumed that individuals with second period endowment e^* are credit constrained. So they must be at the point E^* in Figure 2, with $x_1(e^*) = \bar{x}_1$ and $x_2(e^*) = \bar{x}_2 = d - (1 + r_B)\,\bar{x}_1$. Then the market clearing conditions $\pi\, x_t(e^*) + (1 - \pi)\, x_t(0) = 0$ in each of the two periods $t = 1$ and $t = 2$ obviously require that

$$\bar{x}_t = -(1 - \pi)\, x_t^W(r, d; 0)/\pi. \tag{52}$$

But then the borrowing rate of interest must be given by

$$1 + r_B = \frac{d - \bar{x}_2}{\bar{x}_1} = 1 + r + \frac{(1 + r)\,(2 + r)\,d}{(1 - \pi)\,[(1 + r)^2 - d]}, \tag{53}$$

as can be shown by routine manipulation. In particular, this shows that the borrowing rate succeeds the lending rate if and only if the dividend d is positive. Also, r_B must increase whenever d increases because, as remarked above, it must be true that $d < (1 + r)^2$.

Default will be deterred provided that the (indirect) utility (51) of those with $e = 0$ does not fall below $\sqrt{\bar{x}_1 + 1}$. So we must have

$$d \geq \underline{d}(r) := \frac{(1 + r)^2\,[1 - \pi\,(3 + r)]}{1 + \pi\,(1 + r)\,(3 + r)} = \frac{(1 + r)\,(2 + r)}{1 + \pi\,(1 + r)\,(3 + r)} - (1 + r). \tag{54}$$

Note that $(1 + r)^2 > \underline{d}(r)$ and so this is consistent with allowing $x_1^W(r, d; 0) < 0$.

In order to have a non-Walrasian equilibrium, it must also be true that agents with $e = e^*$ are borrowing no more than they wish. Thus their marginal rate of substitution $(1 + \bar{x}_1)^{-\frac{1}{2}}/(e^* + \bar{x}_2)^{-\frac{1}{2}}$ at the consumption stream $(1 + \bar{x}_1,\, e^* + \bar{x}_2)$ cannot be less than the borrowing interest factor $1 + r_B$. This is true iff

$$e^* \geq (1 + r_B)^2\,(1 + \bar{x}_1) - \bar{x}_2 = (\bar{x}_1)^{-2}\,(d - \bar{x}_2)^2\,(1 + \bar{x}_1) - \bar{x}_2, \tag{55}$$

where the last equality follows from (53). Now consumption expenditure in each period is evidently a normal good, with $x_1^W(r, d; 0)$ and $x_2^W(r, d; 0)$ both increasing functions of d. Then (52) implies that $\bar{x}_1$ and $\bar{x}_2$ are both decreasing functions of d. As $\bar{x}_1 > 0$, it follows that the right hand side of (55) is an increasing function of d. So this inequality serves to define an upper bound $\bar{d}(e^*, r)$ on allowable values of d. Of course, (55) also ensures that $e^* + \bar{x}_2 \geq 0$, thus guaranteeing individual feasibility.

So all non-Walrasian equilibrium allocations of this form are sustainable provided that d satisfies $\underline{d}(r) \leq d \leq \bar{d}(e^*, r)$ for the chosen lending rate r, and provided that the credit ceiling and borrowing rate are then determined by (52) and (53) respectively. Some tedious calculation shows that, whenever the sufficient condition $\pi^4 e^* < 1 + \pi (1 - \pi)^3$ is satisfied, then such a non-Walrasian equilibrium exists, with

$$\begin{aligned} 1 + r &= \pi (1 - \pi) \sqrt{\frac{e^*}{1 + \pi (1 - \pi)^3}} \qquad (< \pi^{-1} - 1) \\ \text{and} \quad d &= \frac{(1 + r)^2 [1 - \pi (2 + r)]}{1 + \pi (1 + r)(2 + r)} \qquad (> \max\{ 0, \underline{d}(r) \}). \end{aligned} \tag{56}$$

Moreover, the above sufficient condition *is* satisfied whenever e^* is too low to allow the Walrasian equilibrium to be sustained — i.e., whenever $\pi < \frac{1}{2}$ and $e^* < (1 - 2\pi)^2 \pi^{-3}$.

It remains to be shown that, for any non-Walrasian equilibrium, the borrowing rate must succeed the lending rate in the case when the Walrasian equilibrium is unsustainable. Indeed, suppose it were true that $d = 0$ and so $r_B = r$. Then (54) clearly implies that $0 \geq 1 - \pi (3 + r)$ or that $1 + r \geq \max\{ 0, \pi^{-1} - 2 \}$. But when $d = 0$, (55) reduces to $\pi e^* \geq (1 + r)^2$, and so an equilibrium of this particular form exists for interest rates in the range satisfying

$$\max\{ 0, \pi^{-1} - 2 \} \leq 1 + r \leq \sqrt{\pi e^*}. \tag{57}$$

In the case when $\pi \geq \frac{1}{2}$, such equilibria exist for all positive values of e^*. But when $\pi < \frac{1}{2}$, such equilibria exist only if and only if $e^* \geq (1 - 2\pi)^2 \pi^{-3}$, which is precisely the condition for Walrasian equilibrium to be sustainable anyway. This proves the assertion: when the Walrasian equilibrium is unsustainable, a non-Walrasian equilibrium with credit rationing of the kind considered here can only be sustained by means of a positive gap between the borrowing and lending rates of interest.

6. Conclusion: Perfect Capital Markets are Generally Not Possible

This paper has considered borrowing and lending in the simplest of all possible models, in which capital markets have the best chance of performing perfectly because there is a continuum of agents. There are no jointly owned private producers and no uncertainty — just an exchange economy lasting for only two periods with a single consumption good, in which agents have the same utility function and the same known first period endowment. It was also assumed that all agents know in the first period what their second period endowment will be, but this is private information. Apart from the continuum of agents, none of these features of the model is important except insofar as they help to concentrate on essentials.

Of much more significance is the assumption that (32) is violated. This requires that the consumers' common utility function $u(c_1, c_2)$ for two period consumption streams satisfy the condition $u(c_1', 0) > u(c_1, c_2)$ for some c_1' which is sufficiently large. So there must be some indifference curves meeting the c_1-axis. Then the threat of imposing zero second period consumption may be ineffective. This is crucial to the specific simple model used here. But not really to the need for credit rationing, because the familiar lifetime budget constraints of more general models are clearly incredible. Indeed, even a lifetime budget constraint is just a particular borrowing constraint which purports to prohibit dying in debt.

So the model presented here illustrates rather starkly the fact that, when some information about individuals is private, "perfect capital markets" may be impossible. If agents have private information about their future endowments, then an allocation mechanism which always selects a "perfect" Walrasian equilibrium in every economic environment is generally manipulable by agents who plan deliberately to violate their Walrasian budget constraints. By contrast, in a static continuum economy, such a mechanism is not manipulable, provided only that default can be detected in time and defaulters punished by being reduced to autarky or with some other sufficient deterrent. Thus the inevitable "market failure" in these economic environments is due to the sequential nature of the economy rather than to private information *per se*.

It appears that Stigler's (1967) plea for a theory behind assumed capital market imperfections can now be answered. Or, if not, that our conception of what constitutes a perfect capital market needs changing drastically to allow incentive constrained Pareto efficient allocations which generally require credit rationing, nonlinear pricing, and regulated entry into and participation in credit markets. Indeed, in future work I would like to consider "perfected" capital markets, with option contracts somewhat similar to those introduced in Hammond (1989).

The model used here also has the virtue of bringing out the need to control fraud. It should be remembered that U.S. banks have been said to lose eighteen times as much from fraud as they do from robbery. And that the Federal Bureau of Investigation is reported to have fifty agents working on bank fraud cases in Los Angeles alone.[2]

REFERENCES

Allen, F. (1981): "The Prevention of Default," *Journal of Finance*, 36, 271-276.

Allen, F. (1983): "Credit Rationing and Payment Incentives," *Review of Economic Studies*, 50, 639–646.

Arrow, K.J. and F.H. Hahn (1971): *General Competitive Analysis*. San Francisco: Holden–Day.

Bliss, C.J. (1976): "Capital Theory in the Short Run," pp. 187-205 in *Essays in Modern Capital Theory*, ed. M. Brown, K. Sato and P. Zarembka. Amsterdam: North-Holland.

Coles, J.L. and P.J. Hammond (1986): "Walrasian Equilibrium without Survival: Existence, Efficiency, and Remedial Policy," Stanford University Institute of Mathematical Studies in the Social Sciences, Economics Technical Report No. 483.

Dasgupta, P.S. and P.J. Hammond (1980): "Fully Progressive Taxation," *Journal of Public Economics*, 13, 141–154.

Debreu, G. (1959): *Theory of Value: An Axiomatic Analysis of Economic Equilibrium.* New York: John Wiley.

Dubey, P. and M. Shubik (1979): "Bankruptcy and Optimality in a Closed Trading Mass Economy Modeled as a Non-Cooperative Game," *Journal of Mathematical Economics*, 6, 115–134.

[2] This is based on my own notes of a television documentary on the crisis in banking which was shown on the KQED Public Broadcasting Service station in the San Francisco Bay Area of California early in 1989.

Eaton, J. and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 48, 289–309.

Feldman, M. and C. Gilles (1985): "An Expository Note on Individual Risk without Aggregate Uncertainty," *Journal of Economic Theory*, 35, 26–32.

Foley, D.K. and M.F. Hellwig (1975): "A Note on the Budget Constraint in a Model of Borrowing," *Journal of Economic Theory*, 11, 305–314.

Gale, D.M. (1979): "Large Economies with Trading Uncertainty," *Review of Economic Studies*, 46, 319–338.

Gale, D.M. (1980): "Money, Information and Equilibrium in Large Economies," *Journal of Economic Theory*, 23, 28–65.

Gale, D.M. (1982): *Money: In Equilibrium*. Welwyn: James Nisbet.

Gale, D.M. and M. Hellwig (1985): "Incentive-Compatible Debt Contracts: The One-Period Problem," *Review of Economic Studies*, 52, 647–663.

Grandmont, J.-M. (1982): "Temporary General Equilibrium Theory," Ch. 19, pp. 879-922 in *Handbook of Mathematical Economics, Vol. II*, ed. K.J. Arrow and M.D. Intriligator. Amsterdam: North-Holland.

Grandmont, J.-M. (ed.) (1988): *Temporary Equilibrium: Selected Readings*. San Diego: Academic Press.

Green, E.J. (1984): "Continuum and Finite-Player Noncooperative Models of Competition," *Econometrica*, 52, 975–993.

Green, E.J. (1987): "Lending and the Smoothing of Uninsurable Income," Ch. 1, pp. 3–25 in *Contractual Arrangements for Intertemporal Trade*, ed. E.C. Prescott and N. Wallace. Minneapolis: University of Minnesota Press.

Green, J.R. (1974): "Pre-existing Contracts and Temporary General Equilibrium," Ch. 10, pp. 263–286 in *Essays on Economic Behavior Under Uncertainty*, ed. M.S. Balch, D.L. McFadden and S.Y. Wu. Amsterdam: North-Holland.

Hahn, F.H. (1988): "On Monetary Theory," *Economic Journal*, 98, 957–953.

Hammond, P.J. (1979): "Straightforward Individual Incentive Compatibility in Large Economies," *Review of Economic Studies*, 46, 263–282.

Hammond, P.J. (1987a): "Markets as Constraints: Multilateral Incentive Compatibility in a Continuum Economy," *Review of Economic Studies*, 54, 399–412.

Hammond, P.J. (1987b): "On the Impossibility of Perfect Capital Markets," Stanford University Institute of Mathematical Studies in the Social Sciences, Economics Technical Report No. 516.

Hammond, P.J. (1989): "Perfected Option Markets in Economies with Adverse Selection," European University Institute, Working Paper No. 89/426.

Harris, M. (1987): *Dynamic Economic Analysis*. New York and Oxford: Oxford University Press.

Harris, M. and A. Raviv (1979): "Optimal Incentive Contracts with Imperfect Information," *Journal of Economic Theory*, 20, 231–259.

Harris, M. and R.M. Townsend (1981): "Resource Allocation under Asymmetric Information," *Econometrica*, 49, 33–64.

Hurwicz, L., E. Maskin and A. Postlewaite (1979): "Feasible Implementation of Social Choice Correspondences by Nash Equilibrium," Paper presented to the Stanford University Institute of Mathematical Studies in the Social Sciences, Summer Workshop in Mathematical Economics.

Jaffee, D.M. and T. Russell (1976): "Imperfect Information and Credit-Rationing," *Quarterly Journal of Economics*, 90, 651–666.

Judd, K. (1985): "The Law of Large Numbers with a Continuum of Random Variables," *Journal of Economic Theory*, 35, 19–25.

Keynes, J.M. (1936): *The General Theory of Employment, Interest and Money*. London: Macmillan.

Kreps, D. and R. Wilson (1982): "Sequential Equilibria," *Econometrica*, 50, 863–894.

Maskin, E.S. (1980): "On First-Best Taxation," pp. 9–22 in *Income Distribution: The Limits to Redistribution*, ed. D. Collard, R. Lecomber, M. Slater. Bristol: John Wright and Sons Ltd..

Milne, F. (1980): "Short-Selling, Default Risk and the Existence of Equilibrium in a Securities Model," *International Economic Review*, 21, 225–267.

Mirrlees, J.A. (1971): "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 175–208.

Postlewaite, A. (1979): "Manipulation via Endowments," *Review of Economic Studies*, 46, 255–262.

Postlewaite, A. (1985): "Implementation via Nash Equilibria in Economic Environments," Ch. 7, pp. 205–228 in *Social Goals and Social Organization: Essays in Memory of Elisha Pazner*, ed. L. Hurwicz, D. Schmeidler and H. Sonnenschein. Cambridge: Cambridge University Press.

Postlewaite, A. and D. Schmeidler (1986): "Implementation in Differential Information Economies," *Journal of Economic Theory*, 39, 14–33.

Postlewaite, A. and D. Schmeidler (1987): "Differential Information and Strategic Behavior in Economic Environments: A General Equilibrium Approach," Ch. 12, pp. 330–348 in *Information, Incentives, and Economic Mechanisms: Essays in Honor of Leonid Hurwicz*, ed. T. Groves, R. Radner and S. Reiter. Minneapolis: University of Minnesota Press.

Prescott, E.C. and R.M. Townsend (1984a): "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard," *Econometrica*, 52, 21–45.

Prescott, E.C. and R.M. Townsend (1984b): "General Competitive Analysis in an Economy with Private Information," *International Economic Review*, 25, 1–20.

Shubik, M. (1973): "Commodity Money, Oligopoly, Credit and Bankruptcy in a General Equilibrium Model," *Western Economic Journal*, 11, 24-38.

Shubik, M. (1974): "Money, Trust and Equilibrium Points in Games in Extensive Form," *Zeitschrift für Nationalökonomie*, 34, 365–385.

Shubik, M. and C. Wilson (1977): "The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money," *Zeitschrift für Nationalökonomie*, 37, 337–354.

Stigler, G. (1967): "Imperfections in the Capital Market," *Journal of Political Economy*, 85, 287–292.

Stiglitz, J.E. and A. Weiss (1981): "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, 71, 393–410.

Stiglitz, J.E. and A. Weiss (1983): "Incentive Effects of Terminations: Applications to the Credit and Labor Markets," *American Economic Review*, 73, 912–927.

Townsend, R.M. (1979): "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory*, 21, 265–293.

Townsend, R.M. (1988): "Information Constrained Insurance: The Revelation Principle Extended," *Journal of Monetary Economics*, 21, 411–450.

Peter J. Hammond, "On the Impossibility of Perfect Capital Markets" — endnotes, etc.

ACKNOWLEDGEMENTS

This paper is a drastically revised version of Hammond (1987b), which was originally produced with the support of U.S. National Science Foundation Grant SES-85-20666 to the Institute for Mathematical Studies in the Social Sciences, Stanford University. It has evolved through presentations at C.O.R.E., at Western Ontario, Southern Methodist, Tulane, Stanford, Cambridge, Mannheim, Southampton, Oxford and Bonn Universities, at the Ecole Normale Supérieure in Paris, the University of California at San Diego, and to the European Meeting of the Econometric Society in Budapest. My gratitude to all the participants for their encouragement, to Frank Hahn, Mordecai Kurz, and Joe Stiglitz for extensive discussions, as well as to an anonymous referee, Ed Green and Oliver Hart for useful suggestions.

1. For simplicity it will be assumed that this and other functions of i are measurable. This is in common with almost all the rest of the literature on continuum economies and games with a continuum of players — e.g., Green (1984). Yet it is not entirely satisfactory because, for instance, a continuum of random variables almost never produces a measurable function (cf. Gale 1979, Feldman and Gilles 1985, and Judd 1985). It fact one should probably consider distributions rather than functions — e.g., the distribution $\lambda \in \Delta(N \times E)$ described in this section — without necessarily assuming that there is a (measurable) function at all. To do so would complicate the notation even further, however.

2. This is based on my own notes of a television documentary on the crisis in banking which was shown on the KQED Public Broadcasting Service station in the San Francisco Bay Area of California early in 1989.