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Consequentialist Demographic Norms and Parenting Rights*

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Abstract. This paper extends the author's recent work on dynamically consistent consequentialist social norms for an unrestricted domain of decision trees with risk to trees in which the population is a variable consequence – i.e., endogenous. Given a form of ethical liberalism and ethical irrelevance of distant ancestors, classical utilitarianism is implied (provided also that a weak continuity condition is met). The "repugnant conclusion" that having many poor people may be desirable can be avoided by denying that individuals' interests extend to the circumstances of their birth. But it is better avoided by recognizing that potential parents have legitimate interests concerning the sizes of their families.

"That action is best, which procures the greatest happiness for the greatest numbers." Francis Hutchison (1725).

"Quelle est la fin de l'association politique? C'est la conservation et la prospérité de ses membres. Et quel est le signe le plus sûr qu'ils se conservent et prospérent? C'est leur nombre et leur population. N'allez donc pas chercher ailleurs ce signe si disputé. Toute chose d'ailleurs égale, le Gouvernement sous lequel, sans moyens étrangers, sans naturalisations, sans colonies les Citoyens peuplent et multiplient davantage, est infailliblement le meilleur: celui sous lequel un peuple diminue et dépérit est le pire. Calculateurs, c'est maintenant votre affaire; comptez, mesurez, comparez." (Translation see Footnote 1 on p 128) J.-J. Rousseau: *Du contrat social* (1762), Livre III, Chapitre IX ("Des signes d'un bon gouvernment").

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"... la massima felicità divisa nel maggior numero,"[2] Cesare Beccaria (1764) *Dei delitti e delle pene* (introduzione).

"... he was the parent of a family of three daughters. This number may appear unduly small, but although in early days he had often reflected that members of a large family are more genial and bright, and often more vigorous in every way than members of a small family, it was yet true that the additional benefit which a person derives from a given stock of a thing diminishes with every increase in the stock which he already has. That is to say, that the marginal utility decreases, and the merchant had observed that the marginal utility of daughters decreases with surprising rapidity..." from Joan Robinson: "Beauty and the Beast", pp 267–268 of *Contributions to Modern Economics* (Blackwell 1979) (written as an undergraduate, in 1921–24, in collaboration with Dorothea Morison (later Braithwaite)).

1. Introduction

Following Arrow's *Social Choice and Individual Values*, almost all existing work in social choice theory treats as fixed both the number of individuals in a society and the welfare orderings of those individuals. In Hammond (1987a) I used the consequentialist approach to normative decision making in order to justify a modified form of Harsanyi's (1955) utilitarianism for the choice of risky consequences in social decision trees.

That work considered "consequentialist social norms". These prescribe ethically appropriate behaviour at each decision node of an unrestricted domain of finite decision trees which have determinate consequences at each terminal node. Such norms represent the appropriate objectives of an "ethical agent" or policy maker in *all* the (consequentialist) extensive form games with which he could conceivably be confronted. When other individuals are involved in these games, the ethical agent's behaviour will naturally be that which is prescribed given appropriate probabilistic expectations regarding these other individuals' behaviour.

Consequentialism alone does not provide a complete ethical theory, or even determine *any* correct decisions. It only lays down some necessary implications of certain appealing conditions in normative decision theory. What decisions are ethically appropriate depends, in particular, on the fundamental utility function. This expresses each individual's interests in the sense of the ethical norm which should be used whenever a single individual's personal consequences are affected by whatever decision is taken. A complete ethical theory would have to specify what these interests really are and how they are to be represented in the fundamental

[1] "What is the end of political association? The preservation and prosperity of its members. And what is the surest mark of their preservation and prosperity? Their numbers and population. Seek then nowhere else this mark that is in dispute. The rest being equal, the government under which, without external aids, without naturalization or colonies, the citizens increase and multiply most is beyond question the best. The government under which a people wanes and diminishes is the worst. Calculators, it is left for you to count, to measure, to compare." (Translation by G.D.H. Cole).

[2] "... the greatest happiness shared by the greatest number," in the introduction to Cesare Beccaria's *Crime and Punishment*. For this reference I am indebted to Mamoru Kaneko.

utility function. That is important work which I shall leave for others more qualified than myself to carry out.

In the earlier work variations in individuals' welfare orderings and other welfare relevant personal consequences were allowed, but the number of individuals remained fixed. As Parfit (1984) and Dasgupta (1985) put it, I dealt only with "Same Number Choices", and so left unexplored the "Different Number Choices" that clearly lie right at the heart of ethical issues concerning population. This paper is intended to fill the gap and to extend "fundamental utilitarianism" to variable populations. Specifically, I shall consider consequentialist social norms when the space of consequences allows the number of people to take any integer value, and allows the membership of society to be any finite subset of a countably infinite set $\bar{M}$ of potential individuals. The norms considered will be demographic in the sense that they include population as a endogenous variable consequence.

Sections 2 and 3 of the paper contain preliminaries, and briefly recapitulate the most relevant previous results on consequentialism. In particular, Sect. 3 extends the key "ethical liberalism" postulate of Hammond (1987a) to consequentialist social norms when the domain of consequences is extended to include a variable set M of individual members of the society. It then derives an extended form of utilitarianism based on von Neumann-Morgenstern utility functions.

The explicit consideration of demographic consequences really starts in Sect. 4, which begins an analysis that takes time into account. In particular, it introduces the key assumption that long dead individuals are not to be regarded as members of contemporary or future society. Now, it must be admitted that sometimes individuals do appear to alter their behaviour out of respect for distant and long dead ancestors or predecessors, and that sometimes this may even be categorized as ethical behaviour. If it is, however, this perhaps corresponds more closely to the interests and "ethical utility" of the contemporary inviduals who profess respect for their predecessors than it does to the interests of those long dead predecessors themselves. Put more simply, if practices like ancestor worship can be justified, it must be because they benefit the worshippers rather than the worshipped.

A noteworthy implication of this assumption is the exclusion of the average utility principle (Edgeworth 1925; Rawls 1972) because, as an example shows, it is dynamically inconsistent or "subgame imperfect" even under certainty.[3] From the assumptions so far, Sect. 5 then derives classical total utilitarianism, in which the optimal population is determined by the first order condition that the marginal person has zero utility. This objective is already familiar from the early work of Meade (1955) and Dasgupta (1969) – see also Sidgwick (1887) and Blackorby and Donaldson (1984). Also noteworthy is the failure to escape Parfit's (1984) "repugnant conclusion" that having a lot of people, who are all only slightly better off having been born than not, is as good as having only a few people who are much better off.

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[3] Interestingly enough, Harsanyi also defends this average utility principle; indeed, this is one matter on which Harsanyi and Rawls are in agreement. See Harsanyi's correspondence with Ng, published as an appendix to Ng (1983). Harsanyi's modification of the original position, allowing an agent to use the knowledge of his own existence, is to me not very convincing. But here I avoid original position arguments anyway.

The justification offered in Hammond (1987a) for "ethical liberalism" is that, in those one person issues which are, by definition, properly the concern of just one individual, the social norm should coincide with an "individual norm" that corresponds to the individual's "ideal utility" or "welfare". Dasgupta (1985), in criticizing the "Pareto-plus" principle which follows from the strong form of ethical liberalism considered in Sect. 3, rightly questions whether a potential individual has a right to exist just because existence is better than non-existence for that individual alone. Section 6 accordingly restricts ethical liberalism to one person issues that do not involve the circumstances of a person's birth (or conception, or parentage, or early upbringing, as one's ethics deem appropriate). This allows the social norm to be concerned with not only how many individuals there should be and what personal characteristics they should have, but also when, where and to whom they should be born, etc. Also, anonymity is compatible with attaching lower weight to the utility of those who are born at later dates. Indeed, one can even include a threshold level of utility for determining who should be born and allow it to increase for later potential births. The repugnant conclusion becomes blunted, as with Parfit's (1984, p 412) use of a "Valueless Level" of utility below which potential individuals' utility should not be allowed to fall, even though any positive utility is better for a person already alive than having not been born at all.

In Sect. 5, individuals implicitly had the *ceteris paribus* right to determine both whether they should be born at all, and also all the circumstances surrounding their birth. Section 6 denied this right to anybody. By contrast, Sect. 7 considers the implications of according to parents *ceteris paribus* rights regarding birth consequences. Like Phelps (1966, pp 179–183), Votey (1969, 1972), Mirrlees (1972), Samuelson (1975), Deardorff (1976), Lane (1975, 1977), Arthur and McNicoll (1977), Nerlove et al. (1985, 1986) and no doubt many others, this involves including the number of children in parental welfare functions. Phelps, indeed, discusses a "Golden Rule of Procreation". But this previous work has too often been based on the assumption of a "representative parent" – an analogy to the equally unacceptable concept of the "representative consumer". With the parenting rights considered here, classical utilitarianism can escape the repugnant conclusion, unless parents benefit from children sufficiently strongly to make a large number of poor people no longer repugnant. Indeed, parenting rights seem not only ethically appealing for their own sake, but a way out of many paradoxes. Section 8 contains concluding remarks.

2. Notation and Preliminaries

The notation is deliberately chosen to conform with Hammond (1987a), together with some additions because of the need to consider demographic consequences. Individuals will be given uniquely identifying labels i in a set M, the *membership* of the society, which will be a variable consequence of actions taken in the decision trees being considered. Each individual $i \in M$ has a variable *personal characteristic* θ_i in the set Θ of all possible personal characteristics. Thus a *society* is described by the pair (M, θ^M) where $\theta^M := \langle\theta_i\rangle_{i \in M}$; both M and θ^M are variable. Each individual $i \in M$ is concerned about the pair (θ_i, x_i) consisting of his/her personal characteristic and also x_i representing those other aspects of the "social state" (Arrow 1963) of relevance to i. Where several individuals are concerned about public goods, externalities, or aspects of the "public environment" in general, feasibility may require that, for any pair of individuals $h, i \in M$, the two lists x_h, x_i must have many elements in common. But I shall consider the whole *space of consequences*:

$$Y := \{(M, \theta^M, x^M) | M \subset \bar{M}, \theta_i \in \Theta, x_i \in X_i(\theta_i) \ (i \in M)\} \tag{1}$$

where $\bar{M}$ denotes the maximal set of all possible individual identifiers, x^M denotes the profile $\langle x_i \rangle_{i \in M}$, and $X_i(\theta_i)$ denotes the set of aspects of the social state that are both relevant to and feasible for individual i with personal characteristic θ_i.

I shall now consider "consequentialist ethical norms" defined for an unrestricted domain of "finite consequentialist decision trees".[4] A *finite consequentialist decision tree* is a finite tree with an initial node, and a finite set of nodes partitioned into decision, chance, and terminal modes. Positive probabilities are specified at each chance node.[5] Instead of payoffs, as in game theory or Raiffa's (1968) discussion of standard decision trees, terminal nodes of consequentialist decision trees have *consequences* in the fixed space Y.

A *norm* is a function $\beta(T, n)$ defined at every decision node n of every finite consequentialist decision tree T – the latter constitutes the *unrestricted domain assumption*. Each value of $\beta(T, n)$ is a non-empty subset of the set of nodes which immediately succeed node n in tree T.

At any node n of T there is a *continuation subtree* $T(n)$ which starts from the initial node n, contains all the other nodes of T which succeed n, and has the same tree structure on those nodes, the same probabilities at all chance nodes, and the same consequences attached to the terminal nodes. Any decision node n' of $T(n)$ is also a decision node of T, and whatever decision is taken at n' in T is also taken at n' in $T(n)$, since they are one and the same decision node. Thus it is natural to require a norm β to be dynamically *consistent* in the sense that:

$$\beta(T, n') = \beta(T(n), n') \tag{2}$$

whenever T is a finite consequentialist decision tree, $T(n)$ is a continuation subtree, and n' is a decision node in $T(n)$.

A consistent norm β is said to be *consequentialist* if, given any two consequentialist decision trees T, T' in which the ranges of possible random consequences resulting from all feasible decision strategies in each tree are equal, then there is equality between the two ranges of possible random consequences of selections from the correspondences $\beta(T, \cdot)$, $\beta(T', \cdot)$ defined at the decision nodes of T and T' respectively. Thus a consistent norm is consequentialist if and only if it "reveals" a *consequence choice function* determining the set of random consequences which can result from the norm as a function of the set of random consequences which are feasible in the given consequentialist decision tree.

In earlier work it was proved that such a consequentialist norm corresponds to maximizing a (complete, transitive) preference ordering R on the space $\Delta(Y)$ of

[4] See Hammond (1986, 1987a, and especially 1988b) for fuller discussion of the topics summarized in the rest of this section.

[5] Hammond (1987b, 1988a, b) contain discussions of the need for strictly positive probabilities, and the first paper suggests a space of extended probabilities which circumvents the problems which zero probabilities create in game theory.

discrete probability distributions on Y with finite supports. Moreover, R must satisfy Samuelson's (1952) independence axiom:

$$[\pi\lambda+(1-\pi)v]R[\pi\mu+(1-\pi)v]\Leftrightarrow\lambda R\mu \tag{3}$$

for all λ, μ, $v\in\Delta(Y)$ and all $\pi\in(0,1)$.

In view of Herstein and Milnor's (1953) simplification of earlier sets of axioms, to ensure that the preference ordering R can be represented by the expected value of a unique cardinal equivalence class of *von Neumann-Morgenstern Utility Functions* (NMUF's) v – i.e., that:

$$\lambda_1 R\lambda_2\Leftrightarrow\sum_{y\in Y}[\lambda_1(y)-\lambda_2(y)]v(y)\geq 0 \tag{4}$$

for all λ_1, $\lambda_2\in\Delta(Y)$ – it suffices to assume that the consequentialist consistent behaviour norm $\beta(T,\cdot)$ is *weakly continuous* in the sense of having a closed graph as the probabilities at each chance node of any decision tree T vary.

In the present context, consequentialism and weak continuity together imply that there is a *social* NMUF w defined on the space Y of combinations (M,θ^M,x^M).

3. Complete Ethical Liberalism

The space $X_i(\theta_i)$ of possible x_i's was deliberately constructed so that pairs (θ_i,x_i) could be regarded as *personal consequences*. Call a consequentialist decision tree T a *personal decision tree* for individual i if:

(i) there is a fixed set of individuals M, with $i\in M$;
(ii) there exist $(\bar\theta_j,\bar x_j)$ for all $j\in M\backslash\{i\}$ such that all possible consequences of all the end points of T satisfy $\theta_j=\bar\theta_j$ and $x_j=\bar x_j$.

Thus, in a personal decision tree, only one individual is affected by the consequences of each possible decision. "Ethical liberalism" will require that the norm treat these personal decision trees as if there were only one individual in the society. Formally, the social norm β is said to satisfy *ethical liberalism* if in, all such personal decision trees T, $\beta(T,\cdot)$ has consequences of the form $(M,\Lambda_i\times\{(\bar\lambda_j)_{j\in M\backslash\{i\}}\})$ where:

(i) $\bar\lambda_j(\bar\theta_j,\bar x_j)=1$ (all $j\in M\backslash\{i\}$)
(ii) $\Lambda_i\subset\Delta(\{(\theta_i,x_i)|\theta_i\in\Theta,\ x_i\in X_i(\theta_i)\})$
(iii) Λ_i is the set of possible random consequences of $\beta(T',\cdot)$, where T and T' are identical consequentialist decision trees except that at every terminal node n in T whose consequence is $(M,\theta_i(n),x_i(n),\langle\bar\theta_j,\bar x_j\rangle_{i\in M\backslash\{i\}})$, the corresponding consequence in T' is $(\{i\},\theta_i(n),x_i(n))$.

In effect, ethical liberalism is a form of separability assumption, allowing one to ignore the consequences for unconcerned individuals in any personal decision tree. One may also think of building up the social norm from *individual norms* β_i $(i\in\bar M)$ which are defined for decision trees with consequences in the subset of Y with $M=\{i\}$ – i.e. with *one-person consequences*. Indeed, *(iii)* above requires that Λ_i be the set of random consequences which result from $\beta_i(T',\cdot)$. The individual norms are just the social norms applied to appropriate one-person societies. There is an obvious counterpart in the usual Pareto condition. But the interpretation is different – the Pareto condition has become a tautology because individual "preferences" are *defined* as social preferences in one-person situations, and because separability is assumed.

Anyway, ethical liberalism implies that for every fixed set of individuals M, expected social welfare can be written in the separable form:

$$\mathbb{E}w(M, \theta^M, x^M) \equiv F(M, \langle \mathbb{E}v_i(\theta_i, x_i) \rangle_{i \in M}) \tag{5}$$

for a function $F(M, \cdot)$ which is strictly increasing on $\mathbb{R}^M$ for each fixed M, and where $\mathbb{E}v_i$ represents the individual norm for i – i.e., the social norm in societies consisting only of individual i. As shown by Harsanyi (1955) and Border (1985), $F(M, \cdot)$ must be linear on $\mathbb{R}^M$ for each fixed M, so that there are constants $\omega_i(M) > 0$ and $\delta(M)$ for which:

$$w(M, \theta^M, x^M) \equiv \sum_{i \in M} \omega_i(M) v_i(\theta_i, x_i) + \delta(M) \ . \tag{6}$$

4. The Inconsistency of Average Utilitarianism

Consequentialism has further implications for the form of the functions $\omega_i(M)$ $(i \in M)$ and $\delta(M)$ $(M \subset \bar{M})$ in the social NMUF at the end of the previous section. For in each consequence (M, θ^M, x^M) of a decision tree T, one should take as the membership M the set of all individuals who will have at least part of their lives after the beginning of the decision tree T. This does leave out the dead who perhaps should be respected in some matters – e.g. in wills or promises. So perhaps M does need to include some of the recently dead as well. The abortion issue also shows that when life begins is highly debateable, and that also affects who is counted in M. And some (e.g. Ng 1983) have argued that some animals should be included too. Let me pass by these difficult issues here and assume, as Arrow did, that who constitutes the society is well determined. Let me also note, however, that decisions governing the size and the composition of the population of a society are allowed given that M can vary between different consequences.

From now on I shall always assume that in any consequentialist decision tree T, if individual i is long enough dead by the time that node n occurs, then there exists a fixed pair $(\bar{\theta}_i(n), \bar{x}_i(n))$ such that every random consequence of decisions in the continuation tree $T(n)$ leads to the fixed personal consequence $(\bar{\theta}_i(n), \bar{x}_i(n))$ occurring for sure. Thus, no decision taken after n affects i's personal consequences. Moreover, decisions in the continuation tree $T(n)$ can be treated as though i were not a member of the society. Call this assumption *ethical irrelevance of distant ancestors.*

This assumption has major implications for the form of the social NMUF $w(M, \theta^M, x^M)$. For example, it excludes the "average utility principle" (Rawls 1972) according to which:

$$w(M, \theta^M, x^M) = \sum_{i \in M} v_i(\theta_i, x_i) / \# M \tag{7}$$

where $\#M$ denotes the number of members of the set M, so that social welfare is equal to utility per head, and increases in population are desirable if and only if the extra people enjoy utility above the average. To see why the average utility principle is excluded, consider a decision tree T as illustrated in Fig. 1 below,

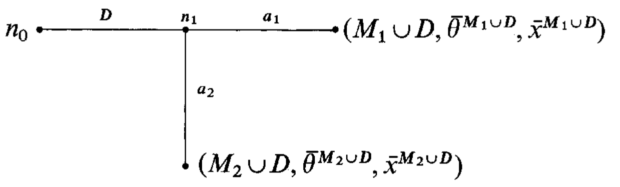

Fig. 1

in which the initial node n_0 is trivial, the set D of agents is long dead before node n_1 occurs, and there is a simple binary choice of act $a_k (k=1,2)$ at node n_1 leading to one of two alternative consequences $(M_k \cup D, \bar{\theta}^{M_k \cup D}, \bar{x}^{M_k \cup D})$ $(k=1,2)$. Then in T, act a_1 is weakly preferred to a_2 if and only if:

$$\sum_{i \in M_1 \cup D} v_i(\bar{\theta}_i, \bar{x}_i)/\#(M_1 \cup D) \geq \sum_{i \in M_2 \cup D} v_i(\bar{\theta}_i, \bar{x}_i)/\#(M_2 \cup D) \tag{8}$$

whereas act a_1 is weakly preferred to a_2 in the continuation tree $T(n_1)$ if and only if:

$$\sum_{i \in M_1} v_i(\bar{\theta}_i, \bar{x}_i)/\#M_1 \geq \sum_{i \in M_2} v_i(\bar{\theta}_i, \bar{x}_i)/\#M_2 \ . \tag{9}$$

These two are consistent, in general, only if $\#D=0$. In particular, if $\#D=1$, $\#M_1=1$, $\#M_2=2$, it is easy to construct an example of inconsistency with:

$$\sum_{i \in M \cup D} v_i(\bar{\theta}_i, \bar{x}_i)/2 > \sum_{i \in M_2 \cup D} v_i(\bar{\theta}_i, \bar{x}_i)/3$$

$$\sum_{i \in M_1} v_i(\bar{\theta}_i, \bar{x}_i) \quad < \sum_{i \in M_2} v_i(\bar{\theta}_i, \bar{x}_i)/2$$

– e.g. $M_1=\{1\}$, $M_2=\{1,2\}$, $D=\{0\}$ and $v_0(\bar{\theta}_0, \bar{x}_0)=3$, $v_1(\bar{\theta}_1, \bar{x}_1)=0$, $v_2(\bar{\theta}_2, \bar{x}_2)=1$.

5. Classical Utilitarianism

In fact a trivial decision tree like that of Fig. 1 can be used to demonstrate that consequentialism requires classical utilitarianism, in effect, with a social NMUF of the form:

$$w(M, \theta^M, x^M) \equiv \sum_{i \in M} w_i(\theta_i, x_i) \ . \tag{10}$$

For suppose the choice at n_1 is between two acts a_1, a_2 yielding as consequences the two simple probability distributions λ_1, λ_2 on the space of combinations (M, θ^M, x^M), with M disjoint from D in every possible consequence. At n_1, consequentialism requires maximizing:

$$\mathbb{E}_\lambda w(M, \theta^M, x^M) \equiv \mathbb{E}_\lambda \left[\sum_{i \in M} \omega_i(M) v_i(\theta_i, x_i) + \delta(M) \right] \tag{11}$$

over the pair $\{\lambda_1, \lambda_2\}$. At n_0, however, consequentialism requires maximizing:

$$\mathbb{E}_{\tilde{\lambda}}[w(D \cup M, \bar{\theta}^D, \theta^M, \bar{x}^D, x^M)]$$
$$\equiv \mathbb{E}_{\tilde{\lambda}}\left[\sum_{i \in D} \omega_i(D \cup M) v_i(\bar{\theta}_i, \bar{x}_i) + \sum_{i \in M} \omega_i(D \cup M) v_i(\theta_i, x_i) + \delta(D \cup M)\right] \quad (12)$$

over the pair $\{\tilde{\lambda}_1, \tilde{\lambda}_2\}$ where:

$$\tilde{\lambda}_k(D \cup M, \bar{\theta}^D, \theta^M, \bar{x}^D, x^M) = \lambda_k(M, \theta^M, x^M) \quad (13)$$

Notice that this definition of $\tilde{\lambda}_k$ $(k=1,2)$ gives:

$$\mathbb{E}_{\tilde{\lambda}_k}[w(D \cup M, \bar{\theta}^D, \theta^M, \bar{x}^D, x^M)] \equiv \sum_{i \in D} \{\mathbb{E}_{\lambda_k} \omega_i(D \cup M) v_i(\bar{\theta}_i, \bar{x}_i)\}$$
$$+ \delta(D \cup M) + \sum_{i \in M} \{\mathbb{E}_{\lambda_k} \omega_i(D \cup M) v_i(\theta_i, x_i)\} \quad (14)$$

because there is no uncertainty about $(D, \bar{\theta}^D, \bar{x}^D)$. Since dynamic consistency has been assumed, the same act – be it a_1 or a_2 – must be chosen in T as in $T(n_1)$ for all possible pairs λ_1, λ_2. So, comparing (11) with (14), this implies that, for all fixed $(D, \bar{\theta}^D, \bar{x}^D)$, the two expected utility maximands:

$$\mathbb{E}_{\lambda}\left[\sum_{i \in M} \omega_i(M) v_i(\theta_i, x_i) + \delta(M)\right] \quad (15)$$

and

$$\mathbb{E}_{\lambda}\left[\sum_{i \in D} \omega_i(D \cup M) v_i(\bar{\theta}_i, \bar{x}_i) + \delta(D \cup M) + \sum_{i \in M} \omega_i(D \cup M) v_i(\theta_i, x_i)\right] \quad (16)$$

must be ordinally equivalent functions of the lottery λ, over the space of all possible combinations (M, θ^M, x^M) with M disjoint from D. But then the Lemma in the Mathematical Appendix, applied to the case when $y_i = (\theta_i, x_i)$ for all $i \in M$ and when $\omega_i(M)$, $\delta(M)$ are independent of b^M for all sets M, implies that:

$$w(M, \theta^M, x^M) \equiv \alpha + \sum_{i \in M} [\omega_i v_i(\theta_i, x_i) + \delta_i] \ . \quad (17)$$

It is routine to verify that maximizing the expected value of this form of w is indeed consequentialist.

$$\text{Define} \quad w_i(M, \theta_i, x_i) := \begin{cases} \omega_i v_i(\theta_i, x_i) + \delta_i & (i \in M) \\ 0 & (i \notin M) \end{cases} \quad (18)$$

for all $i \in \bar{M}$, $M \subset \bar{M}$, $\theta_i \in \Theta$ and $x_i \in X_i(\theta_i)$. The one has:

$$w(M, \theta^M, x^M) \equiv \alpha + \sum_{i \in \bar{M}} w_i(M, \theta_i, x_i) \equiv \alpha + \sum_{i \in M} w_i(\theta_i, x_i) \quad (10')$$

where $w_i(\theta_i, x_i) := \omega_i v_i(\theta_i, x_i) + \delta_i$. The additive constant α in (10′) is clearly irrelevant and so can be omitted. So (10) is true. The following theorem has been proved:

Theorem A. *Suppose that a consistent consequentialist social norm is defined upon the unrestricted domain of all finite decision trees with random consequences in the form of probability distributions on the set Y of combinations (M, θ^M, x^M). Suppose that this norm is weakly continuous as the probabilities vary at the chance nodes of any tree, and that the norm also satisfies both ethical liberalism and ethical irrelevance of distant ancestors. Then in any consequentialist decision tree the norm gives rise to those random consequences which maximize the expected value of a classical utilitarian social welfare function of the form* (10) *over the set of random consequences which are feasible in the decision tree.*

The proof of this result rested crucially on the unrestricted domain assumption and particularly on the fact that trees like that illustrated in Fig. 1 can be constructed whenever D, M_1 and D, M_2 are both disjoint pairs. This assumption would be absurd if individuals' labels included any information about when they lived, because all the individuals in D are supposed to have been long dead by the time node n_1 is reached and the relevant sets of individuals are M_1, M_2. That is, all individuals in D die long before those in M_1 or M_2 live. It would also be absurd if θ_i or x_i contained date-specific information, such as dates of birth or death, because one might have θ_i (say) for $i \in D$ imply that i died after rather than long before the date of j's lifetime implied by θ_j, for some $j \in M_1 \cup M_2$.[6] So the unrestricted domain assumption apparently forces us to consider extremely counterfactual personal consequences such as readers who are alive now experiencing personal consequences equivalent to those they might have had if they had lived in the ancient world. Individuals do not actually have to travel in time, since actual dates cannot matter, but the personal consequences may have to be identical to those they would have had if they did become time-travellers into their past or their future. Such counterfactual consequences pose no paradoxes. Indeed they allow answers in principle to questions like whether a particular person was born far too early or far too late. Yet one might obviously prefer to avoid considering such consequences if at all possible. I propose, however, to leave till later the question of how restricted the domain of consequentialist decision trees can be while preserving the validity of Theorem A.

A much more compelling assumption than unrestricted domain is that only personal consequences are relevant for the ethical norm, and not individuals' labels. This assumption implies that only the size of the set M and the distribution of personal consequences (θ_i, x_i) among the members of M are relevant. So $w_i(\theta_i, x_i) = w_h(\theta_h, x_h)$ whenever $\theta_h = \theta_i$ and $x_h = x_i$ for any pair of individuals h, i. Thus there exists a *fundamental* NMUF $v(\theta, x)$, independent of i, such that $w_i(\theta_i, x_i) \equiv v(\theta_i, x_i)$. This implies

$$w(M, \theta^M, x^M) \equiv \sum_{i \in \bar{M}} v(\theta_i, x_i) \equiv \sum_{i \in M} v(\theta_i, x_i) \ . \tag{19}$$

6 This difficulty was pointed out by John Weymark.

6. Incomplete Ethical Liberalism

Non-existence can be regarded as a particular personal consequence (θ^0, x^0), and then one can normalize the fundamental NMUF v so that $v(\theta^0, x^0)=0$. The expected social welfare function takes the form:

$$w^*(\theta^{\bar{M}}, x^{\bar{M}}) \equiv \sum_{i\in\bar{M}} v(\theta_i, x_i) \tag{20}$$

because, for all $i \notin M$, $(\theta_i, x_i)=(\theta^0, x^0)$ and so $v(\theta_i, x_i)=0$. So the welfare of *all* potential individuals in $\bar{M}$ is included in the total. This implies that each individual i is implicitly given the right to choose between (θ^0, x^0) – non-existence – and all other possible personal consequences (θ, x). Thus each individual is given the *ceteris paribus* right to have their personal interests determine whether, when, to whom he/she is born, etc. Many writers, especially Dasgupta (1985), have questioned this right, which is implicit in classical utilitarianism. It leads to the "Pareto plus" principle and to the "repugnant conclusion" of Parfit (1984). Apparently the legal system is also reluctant to accord rights to potential people, in that trusts can usually only embody people who are actually alive.[7] It is also rather obviously impossible as yet to construct realistic personal decision trees which give the interests of a potential individual the right to determine their own birth without affecting the interests other individuals.

Here I examine a way of conceding the quite compelling objections of Parfit, Dasgupta and others while maintaining consequentialism, ethical liberalism regarding all personal issues after infancy, and ethical irrelevance of distant ancestors. Let b_i denote all those ethically relevant consequences concerning i's birth and infancy which ethical liberalism fails to allow i's interests the right to determine even in i's personal decision trees. Indeed, even if $i=j$, the fact that $b_i \neq b_j'$ is enough to determine a fundamentally different person, and a personal decision tree takes the *birth consequence* b_i as fixed throughout. This is *incomplete ethical liberalism.*

Arguing as in Sect. 3, incomplete ethical liberalism implies that for every fixed set of individuals M with a profile of birth consequences $b^M := \langle b_i \rangle_{i\in M}$, there is an increasing function $F(M, b^M, \cdot): \mathbb{R}^M \to \mathbb{R}$ such that:

$$\mathbb{E}w(M, b^M, \theta^M, x^M) \equiv F(M, b^M, \langle \mathbb{E}v_i(\theta_i, x_i)\rangle_{i\in M}) \ . \tag{5'}$$

Then in fact $F(M, b^M, \cdot)$ must be linear on $\mathbb{R}^M$ so that there exist constants $\omega_i(M, b^M)$ $(i\in M)$ and $\delta(M, b^M)$ for which:

$$w(M, b^M, \theta^M, x^M) \equiv \sum_{i\in M} \omega_i(M, b^M) v_i(\theta_i, x_i) + \delta(M, b^M) \ . \tag{6'}$$

Arguing as in Sect. 5, the two expected utility maximands:

$$\mathbb{E}_\lambda \left[\sum_{i\in M} \omega_i(M, b^M) v_i(\theta_i, x_i) + \delta(M, b^M) \right] \tag{15'}$$

and

[7] I owe this observation to Patrick Suppes.

$$\mathbb{E}_\lambda\left[\sum_{i\in D}\omega_i(D\cup M,\bar{b}^D,b^M)v_i(\bar{\theta}_i,\bar{x}_i)+\delta(D\cup M,\bar{b}^D,b^M)\right.$$
$$\left.+\sum_{i\in M}\omega_i(D\cup M,\bar{b}^D,b^M)v_i(\theta_i,x_i)\right] \tag{16'}$$

must be ordinally equivalent functions of the lottery λ over the space of all possible combinations (M, b^M, θ^M, x^M) with M disjoint from D. So the Lemma of the Mathematical Appendix applies, with $y_i=(\theta_i, x_i)$ for all $i\in M$, and leads to:

$$w(M,b^M,\theta^M,x^M)\equiv\alpha+\sum_{i\in M}[\omega_i(b_i)v_i(\theta_i,x_i)+\delta_i(b_i)] \tag{17'}$$

for suitable functions $\delta_i(\cdot)$. Define:

$$w_i(M,b_i,\theta_i,x_i):=\begin{cases}\omega_i(b_i)v_i(\theta_i,x_i)+\delta_i(b_i) & (i\in M)\\ 0 & (i\notin M)\end{cases} \tag{8''}$$

for all $i\in\bar{M}$, $M\subset\bar{M}$, $\theta_i\in\Theta(b_i)$, $x_i\in X_i(\theta_i,b_i)$. After dropping the irrelevant additive constant, one has:

$$w(M,b^M,\theta^M,x^M)\equiv\sum_{i\in M}w_i(b_i,\theta_i,x_i)\ . \tag{10''}$$

The following has been proved:

Theorem B. *Suppose that all the hypotheses of Theorem A of Sect. 5 are satisfied, except that ethical liberalism is replaced by incomplete ethical liberalism. Then the social norm gives rise to those random consequences which maximize the expected value of a utilitarian social welfare function of the form* (10″) *above over the set of random consequences which are feasible in the decision tree.*

If one imposes anonymity then, as in Sect. 5, (10″) becomes:

$$w(M,b^M,\theta^M,x^M)\equiv\sum_{i\in M}v^*(b_i,\theta_i,x_i) \tag{19'}$$

for a fundamental NMUF v^* which can be expressed as:

$$v^*(b_i,\theta_i,x_i)\equiv\omega(b_i)v(\theta_i,x_i)+\delta(b_i) \tag{21}$$

where v is the fundamental NMUF of Sect. 5 which represents the interests of an individual over personal consequences (θ_i, x_i).

The objective (10″) differs from classical utilitarianism because variations in w_i as b_i varies do *not* represent the welfare of any *individual* – rather, they are comparisons of *different* individuals with *differing birth consequences*. The difference from classical utilitarianism may seem only formal, yet is highly significant. In particular, the constants $\omega(b_i)$ and $\delta(b_i)$ in the b_i-individual's welfare measure (21) allow discounting of those born later (if that is thought to be desirable, which I do not) and, by making $\delta(b_i)$ negative for later births, also allow raising the threshold required for a birth to be desirable. Thus the repugnant conclusion can be avoided. Indeed, one can regard $\delta(b_i)$ as a "Valueless Level" of a person's utility (Parfit 1984, p 412).

7. Parenting Rights

Section 6 introduced birth consequences which, although in some sense personal to the individual being (or not being) born, were excluded from the personal issues over which ethical liberalism requires the individual to have rights. Yet there the only individual affected by i's birth consequences b_i was i himself. Here a rather richer and more acceptable concept of ethical liberalism is used, in which parents have rights concerning the birth consequences of their children. More extensive family ties could easily be accommodated similarly, as long as they work only forwards. Without a time machine, children cannot exercise rights over who their parents are or over their past.

To include parenting rights, let us include an extra variable consequence B_i to cover i's parenting activities. One might write:

$$B_i := (S_i, \langle C_{ij} \rangle_{j \in S_i}, \langle b_{ik} \rangle_{k \in C_i}) \tag{22}$$

where S_i is the set of i's "co-parents", C_{ij} is the set of children which i "co-parents" with j, $C_i := \bigcup_{j \in S_i} C_{ij}$ denotes i's children, and b_{ik} denotes the birth consequences of i's child k. Of course, the constraints that $C_{ij} = C_{ji}$ and that $b_{ik} = b_{jk}$ for all $k \in C_{ij} = C_{ji}$ have to be met as facts of biological life, so that parenting is never entirely a personal issue. Nevertheless, I shall specify the demographic norm as an objective which is meaningful even if this constraint could be ignored, recognizing that it will have to be imposed later. The same applies to the requirement that if $k \in C_{ij} = C_{ji}$, then it must be true that $k \in M$ and that $b_k = b_{ik} = b_{jk}$.

Indeed, given a potential population $\bar{M}$ and parenting activities $B := B^{\bar{M}} = \langle B_i \rangle_{i \in \bar{M}}$ satisfying the above constraints for all individuals who are ever parented, there exists a resultant set $M(B) := M(\bar{B}^M)$ of actual individuals with a profile $b^{M(B)} := b^M(B^{\bar{M}})$ of birth consequences. Consequentialism of a consistent norm, together with weak continuity as probabilities change, implies a norm which maximizes the expected value of a unique cardinal equivalence of social welfare functions of the form $w(M, b^M, \theta^M, x^M, B^M)$. Ethical liberalism implies the existence, for every possible (M, b^M), of an increasing function $F(M, b^M, \cdot)$ from $\mathbb{R}^M$ to $\mathbb{R}$, such that:

$$\mathbb{E} w(M, b^M, \theta^M, x^M, B^M) \equiv F(M, b^M, \langle \mathbb{E} v_i(\theta_i, x_i, B_i) \rangle_{i \in M}) \tag{5''}$$

Again, it follows that F must be linear, and that:

$$w(M, b^M, \theta^M, x^M, B^M) \equiv \sum_{i \in M} \omega_i(M, b^M) v_i(\theta_i, x_i, B_i) + \delta(M, b^M) \ . \tag{6''}$$

And, arguing as in Sect. 6, one has:

$$\begin{aligned} w(M, b^M, \theta^M, x^M, B^M) &\equiv \alpha + \sum_{i \in M} [\omega_i(b_i) v_i(\theta_i, x_i, B_i) + \delta_i(b_i)] \\ &\equiv \alpha + \sum_{i \in \bar{M}} w_i(M, b_i, \theta_i, x_i, B_i) \end{aligned} \tag{17''}$$

or, after ignoring the irrelevant additive constant α:

$$w(m, b^M, \theta^M, x^M, B^M) \equiv \sum_{i \in \bar{M}} w_i(M, b_i, \theta_i, x_i, B_i) \tag{10'''}$$

where:

$$w_i(M, b_i, \theta_i, x_i, B_i) := \begin{cases} \omega_i(b_i) v_i(\theta_i, x_i, B_i) + \delta_i(b_i) & (i \in M) \\ 0 & (i \notin M) \end{cases} . \tag{18''}$$

Indeed:

Theorem C. *Suppose that all the hypotheses of Theorem B are satisfied (with incomplete ethical liberalism). Then the social norm in any decision tree gives rise to random consequences which maximize the expected value of a utilitarian social welfare function of the form* (10‴) *over the set of random consequences which are feasible in the decision tree.*

Thus classical utilitarianism is expanded in scope to include demographic variables B_i $(i \in \bar{M})$ and $M(B^{\bar{M}})$, etc., and to concede limited parenting rights to determine family size, composition etc. as long as, for example, no externalities due to overpopulation are caused. On the other hand, it is still the case that the functions $\omega_i(b_i)$, $\delta_i(b_i)$ need not be determined by the effect of i's birth consequences on i's personal welfare. However, now they could be so determined without necessarily implying the repugnant conclusion. For one could have complete ethical liberalism and an anonymous welfare function of the classical utilitarian form:

$$w(M, b^M, \theta^M, x^M, B^M) \equiv \sum_{i \in M} v(b_i, \theta_i, x_i, B_i) . \tag{23}$$

The dependence of v on B_i allows one to express the benefits and the costs of children to their parents. Then, even though there are many extra potential children whose utility would be positive if they were born, such children may still be undesirable because the net costs to their parents outweighs the benefits of their own existence.

8. Concluding Remarks

Classical utilitarianism is thought to imply that population must expand until the marginal person's utility drops to zero, and that this implies that it is better to have many somewhat poor people – for whom, however, subsistence is better than not having been born at all – rather than fewer more prosperous people. Parfit (1984) calls this "the repugnant conclusion". Dasgupta (1985) has resorted to suggesting "incoherent" objectives, involving the kind of fundamental inconsistency explored in Hammond (1976). Here, I maintain coherent objectives and consequentialist dynamically consistent norms, even in the face of risky consequences, and derive a form of "ideal classical" utilitarianism, borrowing ideas from Harsanyi (1955) as modified in Hammond (1987a). The derivation relies on assuming, as usual, an unrestricted domain of decision trees, but also invokes an assumption that distant ancestors are ethically irrelevant. These two assumptions may be difficult to reconcile. Thus it may be possible to weaken classical utlitarianism after all. The repugnant conclusion appears not be a sufficient reason to do so, however, for one can escape it quite easily by denying that children have interests concerning their existence or their birth consequences. Rather superior, however, in my view, is to allow that children do have such interests, but to recognize that individuals have *ceteris paribus* parenting rights. One escapes the repugnant conclusion if large families are costly. Indeed, if parents' welfare functions make large families an inferior good in the usual economic sense, it is trivially optimal to have families limit their size as people become more prosperous.

Properly interpreted, then, classical utilitarianism need not entail the repugnant conclusion. If it is good for many parents to have many children, even when they are rather poor, then a large poor population is indeed prescribed by classical utilitarianism, but is no longer repugnant.

Mathematical Appendix

The following Lemma is used in Sects. 5–7 in order to derive each of the three Theorems A–C:

Lemma. *Suppose that, for every $(D, \bar{b}^D, \bar{y}^D)$, the two expected utility maximands:*

$$\mathbb{E}_\lambda w(M, b^M, y^M) \equiv \mathbb{E}_\lambda \left[\sum_{i \in M} \omega_i(M, b^M) v_i(y_i) + \delta(M, b^M) \right] \tag{A.1}$$

and:

$$\mathbb{E}_\lambda \left[\sum_{i \in D} \omega_i(D \cup M, \bar{b}^D, b^M) v_i(\bar{y}_i) + \delta(D \cup M, \bar{b}^D, b^M) \right. \\ \left. + \sum_{i \in M} \omega_i(D \cup M, \bar{b}^D, b^M) v_i(y_i) \right] \tag{A.2}$$

are ordinally equivalent functions of the lottery λ defined on the space of all possible combinations (M, b^M, y^M) of sets of individuals M disjoint from D together with their birth consequences $\langle b_i \rangle_{i \in M}$ and their other personal consequences $\langle y_i \rangle_{i \in M}$. Then there exist functions $\omega^i(b_i) := \omega_i(\{i\}, b_i)$ and $\delta_i(b_i)$ for all $i \in \bar{M}$ as well as an arbitrary constant α for which:

$$w(M, b^M, y^M) \equiv \alpha + \sum_{i \in M} [\omega_i(b_i) v_i(y_i) + \delta_i(b_i)] \ . \tag{A.3}$$

Proof. (1) If (A.1) and (A.2) are ordinally equivalent, as assumed, then they must actually be cardinally equivalent functions of λ. So there exist additive constants $\alpha(D, \bar{b}^D, \bar{y}^D)$ and multiplicative constants $\mu(D, \bar{b}^D, y^D)$ for all fixed $(D, \bar{b}^D, \bar{y}^D)$ such that:

$$\mathbb{E}_\lambda \left[\sum_{i \in D} \omega_i(D \cup M, \bar{b}^D, b^M) v_i(\bar{y}_i) + \delta(D \cup M, \bar{b}^D, b^M) \right. \\ \left. + \sum_{i \in M} \omega_i(D \cup M, \bar{b}^D, b^M) v_i(y_i) \right] \\ \equiv \alpha(D, \bar{b}^D, \bar{y}^D) + \mu(D, \bar{b}^D, \bar{y}^D) \mathbb{E}_\lambda \left[\sum_{i \in M} \omega_i(M, b^M) v_i(y_i) + \delta(M, b^M) \right] \ . \tag{A.4}$$

(2) Except in the trivial case when $v_i(y_i)$ is independent of y_i, the coefficients of v_i $(i \in M)$ must be equal in (A.4), and so:

$$\omega_i(D \cup M, \bar{b}^D, b^M) \equiv \mu(D, \bar{b}^D, \bar{y}^D)\omega_i(M, b^M) \tag{A.5}$$

for all $i \in M$ and all possible b^M. In particular, μ must be independent of $\bar{y}^D$ and so one can write $\mu^*(D, \bar{b}^D)$ instead of $\mu(D, \bar{b}^D, \bar{y}^D)$ to give:

$$\omega_i(D \cup M, \bar{b}^D, b^M) \equiv \mu^*(D, \bar{b}^D)\omega_i(M, b^M) \ . \tag{A.6}$$

Now let:

$$\omega_i(b_i) := \omega_i(\{i\}, b_i) \tag{A.7}$$

when $M = \{i\}$ and $D = \emptyset$, and let:

$$\mu_i(b_i) := \mu^*(\{i\}, b_i) \tag{A.8}$$

when $D = \{i\}$. Then, taking $M = \{i\}$ in (A.6) gives:

$$\omega_i(D \cup \{i\}, \bar{b}^D, b_i) \equiv \mu^*(D, \bar{b}^D)\omega_i(b_i) \tag{A.9}$$

whenever $i \notin D$. Taking $D = \{j\}$ in (A.9), with $i \neq j$, gives:

$$\omega_i(\{i,j\}, \bar{b}_j, b_i) \equiv \mu_j(\bar{b}_j)\omega_i(b_i) \ . \tag{A.10}$$

Going back to (A.6), when $M = \{i,j\}$ with $i \neq j$ and this M is disjoint from D, we have:

$$\omega_j(D \cup \{i,j\}, \bar{b}^D, b_i, b_j) \equiv \mu^*(D, \bar{b}^D)\mu_j(b_j)\omega_i(b_i) \tag{A.11}$$

from (A.10) and

$$\omega_i(D \cup \{i,j\}, \bar{b}^D, b_i, b_j) \equiv \mu^*(D \cup \{j\}, \bar{b}^D, b_j)\omega_i(b_i) \tag{A.12}$$

from (A.6) itself. Since $\omega_i(b_i)$ is always positive, and the right hand sides of (A.11) and (A.12) must be equal:

$$\mu^*(D \cup \{j\}, \bar{b}^D, b_j) \equiv \mu^*(D, \bar{b}^D)\mu_j(b_j) \tag{A.13}$$

whenever $j \in D$. Suppose that $D = \{j_1, j_2, \ldots, j_r\}$ and write $D_k := \{j_1, j_2, \ldots, j_k\}$ for $k = 1$ to r. Then (A.13) implies that, whenever D is non-empty:

$$\begin{aligned} \mu^*(D, b^D) = \mu^*(D_r, b^{D_r}) &= \mu^*(D_1, b^{D_1}) \prod_{k=2}^{r} [\mu^*(D_k, b^{D_k})/\mu^*(D_{k-1}, b^{D_{k-1}})] \\ &= \mu^*(\{j_1\}, b_{j_1}) \prod_{k=2}^{r} \mu_{jk}(b_{jk}) = \prod_{j \in D} \mu_j(b_j) \ . \end{aligned} \tag{A.14}$$

Then (A.9) implies that, whenever $i \in M$:

$$\begin{aligned} \omega_i(M, b^M) &\equiv \mu^*(M \backslash \{i\}, b^{M \backslash \{i\}})\omega_i(b_i) \\ &\equiv \left[\prod_{j \in M \backslash \{i\}} \mu_j(b_j) \right] \omega_i(b_i) \\ &\equiv \mu^*(M, b^M)\omega_i(b_i)/\mu_i(b_i) \ . \end{aligned} \tag{A.15}$$

(3) Since the coefficients in (A.4) of each $v_i(\cdot)$ ($i \in M$) have now been equated, the other terms must also be equal. So, after substituting for $\omega_i(D \cup M, \bar{b}^D, b^M)$ and

$\mu(D, \bar{b}^D, b^M)$ from (A.15) and (A.14) respectively, one has:

$$\mu^*(D \cup M, \bar{b}^D, b^M) \sum_{i \in D} [\omega_i(b_i) v_i(\bar{y}_i)/\mu_i(b_i)] + \delta(D \cup M, \bar{b}^D, b^M)$$
$$\equiv \alpha(D, \bar{b}^D, \bar{y}^D) + \mu^*(D, \bar{b}^D)\delta(M, b^M) \ . \tag{A.16}$$

But (A.16) holds even as $\bar{y}^D$ varies with D and $\bar{b}^D$ fixed, so whenever D and M are disjoint sets:

$$\alpha(D, \bar{b}^D, y^D) - \alpha(D, \bar{b}^D, \bar{y}^D)$$
$$\equiv \mu^*(D \cup M, \bar{b}^D, b^M) \sum_{i \in D} \{[\omega_i(b_i)/\mu_i(b_i)] [v_i(y_i) - v_i(\bar{y}_i)]\} \tag{A.17}$$

because the other terms of (A.16) cancel in the subtraction. Since the left hand side of (A.17) is independent of (M, b^M), the right hand side must be also. After ruling out the uninteresting case when the sum which multiplies $\mu^*(D \cup M, \bar{b}^D, b^M)$ in (A.17) is identically zero, it follows that $\mu^*(D \cup M, \bar{b}^D, b^M)$ is also independent of (M, b^M). But by (A.14):

$$\mu^*(D \cup M, \bar{b}^D, b^M) \equiv \left[\prod_{j \in D} \mu_j(\bar{b}_j)\right] \left[\prod_{i \in M} \mu_i(b_i)\right] \tag{A.18}$$

which is only independent of (M, b^M) for all disjoint D and M if $\mu_i(b_i) = 1$ for all $i \in \bar{M}$ and all possible b_i. So (A.14) and (A.15) together imply that, for all M:

$$\mu^*(M, b^M) = 1 \tag{A.19}$$

and also that:

$$\omega_i(M, b^M) = \omega_i(b_i) \tag{A.20}$$

whenever $i \in M$. Thus (A.1) takes the form:

$$w(M, b^M, y^M) \equiv \sum_{i \in M} \omega_i(b_i) v_i(y_i) + \delta(M, b^M) \ . \tag{A.21}$$

(4) Now, by (A.19), (A.16) becomes:

$$\sum_{i \in D} \omega_i(b_i) v_i(\bar{y}_i) + \delta(D \cup M, \bar{b}^D, b^M) \equiv \alpha(D, \bar{b}^D, \bar{y}^D) + \delta(M, b^M) \tag{A.22}$$

whenever D and M are disjoint. Thus:

$$\delta(D \cup M, \bar{b}^D, b^M) - \delta(M, b^M) \equiv \alpha(D, \bar{b}^D, \bar{y}^D) - \sum_{i \in D} \omega_i(b_i) v_i(\bar{y}_i) \tag{A.23}$$

in which the right hand side is independent of (M, b^M). So therefore is the left hand side. In particular, when $D = \{j\}$ with $j \notin M$, one has:

$$\delta(M \cup \{j\}, \bar{b}_j, b^M) - \delta(M, b^M) = : \delta_j(\bar{b}_j) \tag{A.24}$$

for some function $\delta_j(\cdot)$ which is independent of (M, b^M).

Now suppose that $M = \{i_1, i_2, \ldots, i_m\}$ and write $M_k := \{i_1, i_2, \ldots, i_k)$ for $k = 1$ to m. Then (A.24) implies that whenever M is non-empty:

$$\delta(M,b^M)=\delta(M_m,b^{M_m})=\delta(M_1,b^{M_1})+\sum_{k=2}^{m}[\delta(M_k,b^{M_k})-\delta(M_{k-1},b^{M_{k-1}})]$$

$$=\delta(\{i_1\},b_{i_1})+\sum_{k=2}^{m}\delta_{i_k}(b_{i_k})=\sum_{i\in M}\delta_i(b_i)+\delta(\{i_1\},b_{i_1})-\delta_{i_1}(b_{i_1}) \quad \text{(A.25)}$$

Since i_1 was an arbitrary member of M, when $M=\{i,j\}$, (A.25) implies that for all (i,j,b_i,b_j):

$$\delta(\{i,j\},b_i,b_j)-\delta_i(b_i)-\delta_j(b_j)\equiv\delta(\{i\},b_i)-\delta_i(b_i)\equiv\delta(\{j\},b_j)-\delta_j(b_j) \quad \text{(A.26)}$$

which implies that each is a constant α, independent of i, b_i. Therefore (A.25) becomes:

$$\delta(M,b^M)\equiv\alpha+\sum_{i\in M}\delta_i(b_i) \quad \text{(A.27)}$$

and (A.21) takes the form:

$$w(M,b^M,y^M)\equiv\alpha+\sum_{i\in M}[\omega_i(b_i)v_i(y_i)+\delta_i(b_i)]$$

which is precisely (A.3). Q.E.D.

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