Social Choice of Individual and Group Rights

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ABSTRACT

Individual rights can generally be respected if and, except in rare special cases, only if they apply to independent components of a Cartesian product space of social states, and also each individual is indifferent to how others exercise their rights. This is true whether or not the Pareto criterion is satisfied. Group rights can also be respected if they apply to the independent components for the different individual members of the group. This holds not only for social choice rules, but also for outcomes that arise when individuals and groups use equilibrium strategies in some game form. So only exceptionally is it possible to respect all rights. The paper concludes by considering different ways of including rights in the social states which are the object of individual preference and of social choice.

Social Choice of Rights

1. Introduction

It is more than twenty years since Amartya Sen set out to incorporate respect for individual rights in social choice theory. Though dictatorship is generally undesirable, there are certain private matters over which it is probably desirable for individuals' preferences to be decisive in the Arrow social welfare function that determines social preferences. Sen's decisiveness approach was soon extended to group rights by Batra and Pattanaik (1972) in their discussion of "federalism." In the case of individual rights, this approach is what Riley (1989) calls "formulation A." Section 2 explains what it means for a social choice rule to respect both individual and group rights.

Sen (1970a, b) showed how it was generally impossible to grant even just two individuals rights over a single issue each without generating a Pareto inefficient outcome. He provided an example in which individuals have certain rights to create externalities, so that exercising those rights leads to Pareto inefficiency. Gibbard (1974) had another example showing how it could be impossible to grant rights to two different individuals over two binary issues each. For instance, suppose there are two people, the first of whom wants to wear the same colour clothing as the second, while the second wants to wear a different colour from the first. Then there is no feasible choice of colours that respects both individuals' rights to choose what colour clothing to wear.

Then Section 3 argues that this kind of example can be excluded by restricting individuals' preferences to be *privately oriented*, as in Hammond (1982).[1] That is, each individual should be indifferent over any issue that some other individual or group has the right to decide. Really, this amounts to assuming that when any individual or group exercises its rights, this never creates externalities for any other individuals or groups. It is then easy to prove that having preferences be privately oriented is sufficient to ensure that *any* social choice rule can be strictly Paretian only if it respects, not just individual rights as in Coughlin (1986), but group rights as well. In particular, there is no longer any conflict between different individual and group rights, nor between rights and the Pareto

[1] Similar restrictions on preferences have also been considered by Bernholz (1974), Gibbard (1974), Blau (1975), Farrell (1976), Breyer (1978), Ferejohn (1978), Suzumura (1978), Gaertner and Krüger (1981), and Riley (1990), amongst others.

principle. Of course, Sen's and Gibbard's original examples, together with many others of interest, involve preferences which are not privately oriented.

The rest of Section 3 goes on to present necessary conditions for it to be true that, given any pair of social states, there always exist privately oriented preferences allowing somebody to express a strict preference over this pair. In fact, the effective rights of different individuals, including also those of the groups to which they belong, must be *independent*, meaning that they involve disjoint components of a Cartesian product set of social states, as in the formulation due to Bernholz (1974) and Gibbard (1974).

Many libertarians and others wanting to emphasize the value of freedom have objected to this social choice formulation of rights. They claim that society should not have any preference over personal issues, which should be settled by individual rather than social choice. This view underlies Nozick's (1974) influential work, and has been forcefully expressed by numerous other writers, including Barry (1985). It suggests that, if an individual i or a group G has a right to choose x over y, then the social system has to provide that individual or group with some way of ensuring that y never comes about when x is feasible.

Since the work of Sugden (1985a, 1985b, 1986), followed by Gaertner, Pattanaik, and Suzumura (1988), it has become common to model this approach to rights by means of a game form.[2] A "libertarian" game form is one in which individuals or groups are given the power to determine any private issue over which they should have rights. This version of rights is what Riley (1989) called "formulation B." In Section 4 it is shown that any social choice rule which selects among the relevant "strong equilibria" of the game form must respect rights. Conversely, under the assumptions of Section 3, there exists a game form that, with complete information, implements a strictly Pareto efficient outcome, and this must respect rights. But the game form is generally not libertarian. In this sense, the game form approach seems to be no more general than the classical Sen approach.

In my view, however, neither of these two approaches treats rights satisfactorily. Both

[2] Gärdenfors (1981) has often been credited with using game forms to model rights. In fact, he modelled a right as giving an individual or group the power to confine the social outcome within a specified set of outcomes. His model of rights therefore resembles the "effectivity functions" considered later by Moulin and Peleg (1982), Peleg (1984) and Kolpin (1988). Gärdenfors did also consider strategic games in which individuals and groups chould choose either to exercise or waive each of their rights — cf. Gibbard (1974). These are very particular game forms, however. Also, the approach derives a game form from rights, rather than using a game form to represent rights.

treat rights as absolutes, never to be violated. Or at least they follow Rawls (1971) in giving rights absolute priority over outcomes in a lexical social preference ordering. Yet the results of Section 3 below show how rarely is it possible to respect all individual and group rights. Also, if individuals prefer good outcomes to the chance to exercise all their rights, and if exercising some of these rights would lead to bad outcomes, why should all their rights predominate? As Gibbard (1982, p. 604) states, "liberty is a matter of norms." This suggests that, along with social states in the usual sense, both individual and group rights should themselves become the object of both individual preference and social choice. So one needs to consider a space of *rights-inclusive social states.*

Section 5 discusses three different versions of this formulation, leading up to a new way of including rights in the social state. This follows Pattanaik and Suzumura (1992) in considering the *whole* game form, and not just equilibrium strategies or the outcomes which these strategies lead to. But since it is *outcomes* and the opportunities to change those outcomes that seem to matter, I will suggest that one should represent game forms by their induced *rights structures.* These simply specify what opportunities to change the outcome of the game form are enjoyed by each individual and each group.

Section 6 contains a summary and some concluding remarks.

2. Rights-Respecting Social Choice

2.1. Preferences and Social Choice

Suppose that there is a fixed *underlying set* X of social states, and a fixed finite set of individuals N with variable *preference orderings* R_i ($i \in N$), which are complete and transitive binary relations defined on X. Let P_i and I_i ($i \in N$) denote the corresponding *strict preference* and *indifference* relations, respectively; these must also be transitive.

Write R^N for the typical *preference profile* $\langle R_i \rangle_{i \in N}$ of individual preference orderings. Then, for each such profile R^N and each non-empty $G \subset N$, let $P_G(R^N)$ and $P^*_G(R^N)$ denote the corresponding *strict* and *strongly strict group preference relations* defined for all pairs $a, b \in X$ by

$$a\ P_G(R^N)\ b \iff \forall i \in G : a\ P_i\ b$$

$$\text{and} \quad a\ P^*_G(R^N)\ b \iff \{[\forall i \in G : a\ R_i\ b]\ \&\ [\exists h \in G : a\ P_h\ b]\}.$$

Because each individual's preference relation is transitive, so are the relations $P_G(R^N)$

and $P^*_G(R^N)$. In particular, the *weak* and the *strict Pareto dominance relations* $P_N(R^N)$ and $P^*_N(R^N)$ are both transitive.

Let $\mathcal{F}(X)$ denote the set of all non-empty finite subsets of X. A *social choice rule* (or SCR) is a mapping $C(\cdot, \cdot) : \mathcal{F}(X) \times \mathcal{P}^N \to \mathcal{F}(X)$ which determines, for every *feasible set* $A \in \mathcal{F}(X)$ and every preference profile R^N in a (restricted) *domain* $\mathcal{P}^N$, a non-empty *choice set* $C(A, R^N) \subset A$. Given the SCR C and the preference profile $R^N \in \mathcal{P}^N$, define the corresponding *revealed strict preference relation* $P^C(R^N)$ so that

$$a \; P^C(R^N) \; b \iff [\forall A \in \mathcal{F}(X) : a \in A \Longrightarrow b \notin C(A, R^N)].$$

In particular, $a \; P^C(R^N) \; b$ implies that $b \notin C(\{ a, b \}, R^N)$, but the same condition imposes restrictions on choice from larger sets $A \supset \{ a, b \}$ as well. Because $C(A, R^N)$ must be non-empty whenever A is non-empty and finite, it is easy to see that the relation $P^C(R^N)$ must be *acyclic* — that is, there can be no *cycle* $c^0, c^1, c^2, \ldots, c^n$ with $c^0 = c^n$ and $c^k \; P^C(R^N) \; c^{k-1}$ for $k = 1$ to n.

Say that the SCR C is *strictly Paretian* provided that $a \; P^C(R^N) \; b$ whenever $a, b \in X$ and $R^N \in \mathcal{P}^N$ satisfy $a \; P^*_N(R^N) \; b$. In this case, for every feasible set $A \in \mathcal{F}(X)$, the SCR C will always select some of the (strictly) Pareto efficient social states in A. Recall that Sen's (1970a) *strict Pareto extension rule* is defined as the SCR $C^{\mathrm{Par}}(\cdot, \cdot)$ which, for every feasible set $A \in \mathcal{F}(X)$ and every $R^N \in \mathcal{P}^N$, has

$$C^{\mathrm{Par}}(A, R^N) = \{ a \in A \mid b \; P^*_N(R^N) \; a \Longrightarrow b \notin A \}.$$

In other words, $C^{\mathrm{Par}}(A, R^N)$ consists of those members of A which are strictly Pareto efficient given the preference profile R^N. Evidently a general SCR C is strictly Paretian if and only if $\emptyset \neq C(A, R^N) \subset C^{\mathrm{Par}}(A, R^N)$ throughout the domain $\mathcal{F}(X) \times \mathcal{P}^N$. Note especially that, because $P^*_N(R^N)$ is transitive and so acyclic, one has $C^{\mathrm{Par}}(A, R^N) \neq \emptyset$ throughout $\mathcal{F}(X) \times \mathcal{P}^N$. This implies that there is a strictly Paretian SCR — in fact, there must be many unless the preference domain $\mathcal{P}^N$ is very restricted.

Given any pair $a, b \in X$, the (non-empty) group $G \subset N$ is said to be *decisive for a over b* if, whenever the profile R^N is such that $a \; P_G(R^N) \; b$, then $a \; P^C(R^N) \; b$. This definition implies, of course, that $\{a\} = C(\{ a, b\}, R^N)$ when $a \; P_G(R^N) \; b$; thus, if G is decisive for a over b, then a is the only possible social choice from the pair $\{ a, b \}$ when all members of the group G strictly prefer a to b. In case G is decisive for b over a as well as for a over b, say that G is *decisive over* $\{ a, b\}$.

2.2. *Rights*

Sen, together with Batra and Pattanaik, regarded the rights of each individual and of each group $G \subset N$ as being represented by a (possibly empty) collection $D_G \subset X \times X$ of ordered pairs over which G is supposed to be decisive. Of course, this set D_G can be regarded as the graph of a binary preference relation; this being so, D_G can be called a *rights relation* without undue confusion. It will be assumed that D_G is irreflexive — i.e., that there is no $x \in X$ with $x \; D_G \; x$.

Let $\mathcal{G}$ denote the collection of groups G having non-trivial rights relations D_G. In other work it is often assumed that only individuals have rights, so that $\mathcal{G} = \{ \{i\} \mid i \in N \}$. But no such assumption will be needed here — groups may have rights, and some or all individuals may have no rights. Often D_i instead of $D_{\{i\}}$ will be used to indicate individual i's rights relation.

As Sen (1992) is right (and also has the right) to remind us, the purpose of his original work was to demonstrate how the Pareto principle could easily conflict with even such a minimal form of liberalism as that requiring there to be at least two pairs of social states $\{ x_i, y_i \}$, $\{ x_j, y_j \}$ (possibly overlapping, as in his example concerning which of two rather perverse individuals is to read one particular copy of the novel *Lady Chatterley's Lover*, by D.H. Lawrence), and two individuals $i, j \in N$ who are granted the right to be decisive for x_i over y_i and for x_j over y_j respectively. Nevertheless, it is still a powerful and much used model of rights for a broader class of problems.

In what follows, it will be assumed that a particular *rights profile* $D^{\mathcal{G}}$ of irreflexive rights relations $\langle D_G \rangle_{G \in \mathcal{G}}$ has been specified, for some set $\mathcal{G} \subset 2^N$ of groups (and indviduals) with rights. Though minimal rights relations need not satisfy this extra property, the results of Section 3 will require each rights relation D_G to be *symmetric* in the sense that $x \; D_G \; y \iff y \; D_G \; x$. Note that, if G' is a proper subset of G, then G will be decisive over $\{ x, y \}$ whenever G' is. In order to avoid redundancy, however, it will be assumed that if $x \; D_G \; y$, then there is no proper subset G' of G for which $x \; D_{G'} \; y$. In other words, I shall consider only *minimal* decisive groups as having rights. Thus, one should regard D_G as indicating what extra rights G has in addition to those of all its proper subgroups.

Finally, say that the SCR $C : \mathcal{F}(X) \times \mathcal{P}^N \to \mathcal{F}(X)$ *respects the rights profile* $D^{\mathcal{G}}$ if, whenever $a, b \in X$ with $a \; D_G \; b$ and $a \; P_G(R^N) \; b$, then $a \; P^C(R^N) \; b$. In other words, whenever $a, b \in X$ with $a \; D_G \; b$, group G should be decisive over $\{ a, b \}$.

3. Independent Rights

3.1. Privately Oriented Preferences

Given the rights profile $D^{\mathcal{G}} = \langle D_G \rangle_{G \in \mathcal{G}}$ together with any individual $i \in N$, let E_i denote the corresponding *no rights* relation defined on X so that

$$a \; E_i \; b \iff \exists G \in \mathcal{G} : [i \notin G \quad \text{and} \quad a \; D_G \; b].$$

The interest of this relation is that, if $a \; E_i \; b$, then it is generally impossible to allow i to have a right between x and y without contradicting some other individual's or group's right over the same pair. For obviously, if $a \; D_G \; b$ and also $b \; D_{G'} \; a$ for some disjoint pair of groups G, G', then it is impossible to respect both groups' rights whenever their members' preferences are strictly opposed, with $a \; P_G(R^N) \; b$ and also $b \; P_{G'}(R^N) \; a$.

Let $\mathcal{R}(X)$ denote the set of all logically possible preference orderings defined on the set X. And let

$$\mathcal{R}_i(X, D^{\mathcal{G}}) := \{ \, R \in \mathcal{R}(X) \mid a \; E_i \; b \Longrightarrow a \; I_i \; b \, \}$$

denote the set of *privately oriented* preference orderings for individual i relative to the rights profile $D^{\mathcal{G}}$ — namely, the set of those orderings on X that express indifference over any pair for which some group excluding i has a right. The Sen and Gibbard paradoxes arise from preferences that are not privately oriented in this way. Note how the definition extends that of Hammond (1982) not only by allowing group as well as individual rights, but also by not requiring X to be a Cartesian product space. Shortly, however, the need for such a product space will be demonstrated, under a weak additional condition on the domain of allowable preferences. Let $\mathcal{R}^N(X, D^{\mathcal{G}}) := \prod_{i \in N} \mathcal{R}_i(X, D^{\mathcal{G}})$ denote the set of all possible privately oriented preference profiles (or POPPs).

The following result shows how the Sen and Gibbard paradoxes can indeed be avoided by limiting the preference domain to POPPs; there is no need for any more severe restrictions on individuals' preferences. Actually, as Coughlin (1986) has noticed, Pareto efficiency even *requires* respect for individual rights in this case; now it will be shown that group rights must be respected as well.

Theorem 1. *Suppose that, for the given rights profile $D^{\mathcal{G}}$, the domain $\mathcal{P}^N$ of allowable preference profiles R^N is restricted to POPPs, so that $\mathcal{P}^N \subset \mathcal{R}^N(X, D^{\mathcal{G}})$. Then the social* *choice rule $C(A, R^N)$ on the domain $\mathcal{F}(X) \times \mathcal{P}^N$ satisfies the strict Pareto rule only if it respects both individual and group rights.*

PROOF: Suppose that the social states $a, b \in X$ and the group $G \in \mathcal{G}$ are such that $a\ D_G\ b$. Suppose too that the POPP $R^N \in \mathcal{R}^N(X, D^{\mathcal{G}})$ satisfies $a\ P_G(R^N)\ b$. Since R^N is a POPP, it follows that $a\ I_i\ b$ for all $i \in N \setminus G$, and so $a\ P^*_N(R^N)\ b$. If the SCR C is strictly Paretian, therefore, it must be true that $a\ P^C(R^N)\ b$, proving that G is decisive for a over b. So all rights are respected by any strictly Paretian SCR. ∎

Since a Pareto efficient SCR certainly exists, Theorem 1 assures us that when preferences are privately oriented, then all rights can be respected — indeed, they must be, by any strictly Paretian SCR. Nor need the strict Pareto criterion then be violated in respecting individual and group rights.

The converse of Theorem 1 would state that respecting individual and group rights is sufficient for Pareto efficient social choice. This is true for individual rights alone under the extra assumptions imposed by Coughlin (1986), but is not true generally. See Section 3.3 below.

3.2. Necessity of Independent Rights

Suppose that a, b are two different social states in X. Then it seems reasonable that there should be a preference profile R^N in the domain $\mathcal{P}^N$ for which at least one individual $i \in N$ has a preference ordering with a and b not indifferent. Moreover, this should be true even when preference profiles are restricted to POPPs in $\mathcal{R}^N(X, D^{\mathcal{G}})$. Call this the *rich private domain assumption.* It will now be shown that this assumption has the important implication that the underlying set X has a Cartesian product structure such as that originally considered by Bernholz (1974) and Gibbard (1974).

Indeed, say that the rights profile $D^{\mathcal{G}} = \langle D_G \rangle_{G \in \mathcal{G}}$ is *weakly independent* if X is equivalent to a subset of some Cartesian product set $X^N := \prod_{i \in N} X_i$ with the property that, for each $i \in N$ and each pair $a = \langle a_i \rangle_{i \in N}, b = \langle b_i \rangle_{i \in N} \in X$, one has $a\ D_G\ b$ only if $a_i = b_i$ for all $i \in N \setminus G$ (cf. Hammond, 1982). Thus X can be regarded as a subset of a product space with a separate component X_i for each individual $i \in N$, such that groups (including those with single individuals) have rights only to issues affecting just their members' components of the product space. Similarly, say that $D^{\mathcal{G}}$ is *strongly independent* if it is weakly independent, and if the component spaces X_i ($i \in N$) also have the property that

$$a\ D_G\ b \iff G = \{\, i \in N \mid a_i \neq b_i \,\}.$$

It might seem at first that these two definitions of independent rights exclude the possibility that the underlying set of social states takes the form of a Cartesian product $Z = \prod_{G \in \mathcal{G}_{\geq 2}} Z_G \times \prod_{i \in N} Z_i$ where, for any G in the set $\mathcal{G}_{\geq 2}$ of groups in $\mathcal{G}$ having at least two members, Z_G consists of public or club good vectors shared by all the members of group G, while each Z_i consists of i's private good consumption vectors. In fact, however, it is possible to construct a separate copy Z_{Gi} of the space Z_G whenever $G \in \mathcal{G}_{\geq 2}$, and then let $X_i := \prod_{i \in G \in \mathcal{G}_{\geq 2}} Z_{Gi} \times Z_i$ for each $i \in N$. This allows X to be defined as the subset of $X^N := \prod_{i \in N} X_i$ whose elements take the form $x^N = \langle (\langle z_{Gi} \rangle_{i \in G \in \mathcal{G}_{\geq 2}}, z_i) \rangle_{i \in N}$, with a common $z_G \in Z_G$ for which $z_{Gi} = z_G$ (all $i \in G \in \mathcal{G}_{\geq 2}$) — i.e., each such z_{Gi} is just a personalized copy of z_G. Of course, this is equivalent to the device used by Foley (1970) and Milleron (1972) to describe allocations with public goods, with Lindahl prices as the prices of personalized public goods.

Lemma 2. *Suppose that the set $\mathcal{R}^N(X, D^{\mathcal{G}})$ of POPPs satisfies the rich private domain assumption. Then rights are weakly independent. Moreover, the component spaces X_i ($i \in N$) for which $X \subset \prod_{i \in N} X_i$ have the property that $a\ I_i\ b$ whenever $a, b \in X$, $i \in N$, $a_i = b_i$, and $R^N \in \mathcal{R}^N(X, D^{\mathcal{G}})$.*

Proof: For each $i \in N$, let E_i denote i's no rights relation, which can be thought of as $\cup_{i \notin G \in \mathcal{G}} D_G$. Let E_i^* denote the *transitive completion* of E_i — i.e., the relation defined so that $a\ E_i^*\ b$ if and only if there is a finite chain $c^0, c^1, c^2, \ldots, c^m \in X$ with $c^0 = a$ and $c^m = b$ such that $c^{k-1}\ E_i\ c^k$ for $k = 1$ to m. Evidently E_i^* is symmetric and transitive, so it is an equivalence relation.

For each $i \in N$, let $Q_i := X/E_i^*$ denote the quotient set whose members are the E_i^*-equivalence classes in X. For each $x \in X$, let $[x]_i \in Q_i$ denote the unique E_i^*-equivalence class having x as a member.

Now suppose that $i \in N$ and $[a]_i = [b]_i$. By definition of the relations E_i^*, it must then be true that $a\ E_i^*\ b$. Suppose too that R^N is any POPP in $\mathcal{R}^N(X, D^{\mathcal{G}})$. Then, since $x\ E_i\ y \Longrightarrow x\ I_i\ y$, and since the indifference relation I_i is transitive, it must be true that $a\ I_i\ b$.

Next, suppose that $a, b \in X$ are such that $[a]_i = [b]_i$ for all $i \in N$. Then, for any POPP $R^N \in \mathcal{R}^N(X, D^{\mathcal{G}})$, the previous paragraph shows that $a\ I_i\ b$ (all $i \in N$). So the rich private domain assumption implies that $a = b$.

It follows that there is a one-to-one mapping $x \mapsto \langle [x]_i \rangle_{i \in N}$ from X into the Cartesian product $Q^N := \prod_{i \in N} Q_i$ of the quotient spaces. So we can identify X with the range

$$Q := \{ \langle q_i \rangle_{i \in N} \in Q^N \mid \exists x \in X : q_i = [x]_i \text{ (all } i \in N) \} \subset Q^N$$

of this one-to-one mapping. Thus, there is a one-to-one correspondence $\rho : X \to Q$ with $\rho(x) := \langle [x]_i \rangle_{i \in N}$ for all $x \in X$. After identifying X with Q, we can go on to regard X as a subset of $\prod_{i \in N} X_i$, where each X_i is just a relabelling of Q_i. Then each $x \in X$ can be expressed as $\langle x_i \rangle_{i \in N}$, where $x_i \in X_i$ is really just shorthand for $[x]_i \in Q_i$.

In future, the condensed notation x_i will be used throughout. By the result of the third paragraph above, it follows that $a \; I_i \; b$ whenever $a, b \in X$, $i \in N$, $a_i = b_i$, and $R^N \in \mathcal{R}^N(X, D^{\mathcal{G}})$.

Finally, suppose that $a \; D_G \; b$ for some pair $a, b \in X$ with $a \neq b$. Then, for all $i \in N \setminus G$, one has $a \; E_i \; b$ and so $a_i = b_i$. This completes the proof of weak independence. ▮

Though simple, Lemma 2 has powerful implications. There is a clear sense in which exercising rights creates no externalities precisely when there is a POPP. Lemma 2 says that one can have a POPP without forcing everybody always to be indifferent between some pair of social states if and only if the underlying set is a subset of a Cartesian product set in a way that makes rights weakly independent.

Even under the rich private domain assumption, it is not generally true that rights have to be strongly independent. Nevertheless, it is possible to replace the original rights profile $D^{\mathcal{G}}$ with the new strongly independent rights profile $\hat{D}^{2^N} = \langle \hat{D}_G \rangle_{G \subset N}$ having the following four properties:[3]

[3] Really, I should print $\hat{D}^{2^N \setminus \{\emptyset\}}$ instead of $\hat{D}^{2^N}$. But the empty group will be given a vacuous rights relation anyway.

(1) the domains of all POPPs relative to the two different rights profiles are equal;

(2) whenever $a\ D_G\ b$ then, though it may not be true that $a\ \hat{D}_G\ b$, there is nevertheless some subset G' of G for which $a\ \hat{D}_{G'}\ b$;

(3) any SCR which respects the new rights profile $\hat{D}^{2^N}$ will also respect $D^{\mathcal{G}}$;

(4) any SCR which is strongly Paretian on some domain of POPPs will respect the new rights profile $\hat{D}^{2^N}$ (as well as $D^{\mathcal{G}}$).

Thus $\hat{D}^{2^N}$ is virtually an extension of $D^{\mathcal{G}}$ because of properties (2) and (3). Yet, because of properties (1) and (4), the Sen and Gibbard paradoxes are avoided for the same domain of POPPs. Indeed:

THEOREM 3. *Suppose that the set $\mathcal{R}^N(X, D^{\mathcal{G}})$ of POPPs satisfies the rich private domain assumption. Then X is equivalent to a subset of $\prod_{i\in N} X_i$ such that, for the strongly independent rights profile $\hat{D}^{2^N}$ defined on that subset by*

$$a\ \hat{D}_G\ b \iff G = \{\, i \in N \mid a_i \neq b_i \,\} \text{ (all } G \subset N\text{)},$$

the four properties (1)–(4) are satisfied.

PROOF: The four properties are verified in turn as follows:

(1a) Suppose that R^N is any POPP in $\mathcal{R}^N(X, D^{\mathcal{G}})$. For any $i \in N$ and $a, b \in X$, suppose that $a\ \hat{D}_G\ b$ for some $G \not\ni i$. Then $a_i = b_i$ and so, by Lemma 2, $a\ I_i\ b$. Hence the restrictions for $R^N \in \mathcal{R}^N(X, \hat{D}^{2^N})$ are all satisfied.

(1b) Conversely, for any $i \in N$ and $a, b \in X$, suppose that $a\ D_G\ b$ for some $G \not\ni i$. Then $a\ E_i\ b$ and so, by the construction used in the proof of Lemma 2, $a_i = b_i$. This implies that $a\ \hat{D}_{G'}\ b$ for some $G' \not\ni i$. So, for any $R^N \in \mathcal{R}^N(X, \hat{D}^{2^N})$, it follows that $a\ I_i\ b$. Hence the restrictions for $R^N \in \mathcal{R}^N(X, D^{\mathcal{G}})$ are all satisfied by any $R^N \in \mathcal{R}^N(X, \hat{D}^{2^N})$.

(2) Suppose that $a, b \in X$ is any pair satisfying $a\ D_G\ b$. By Lemma 2, $a_i = b_i$ for all $i \in N \setminus G$. Also $a \neq b$ because D_G is assumed to be irreflexive. Hence there is a non-empty $G' \subset G$ for which $a_i \neq b_i \iff i \in G'$. So $a\ \hat{D}_{G'}\ b$ for this subset G'.

(3) Suppose that C is an SCR that respects $\hat{D}^{2^N}$. Let $a, b \in X$ be any pair of social states and $G \subset N$ any group for which both $a\ D_G\ b$ and $a\ P_G(R^N)\ b$. Then $a\ P_{G'}(R^N)\ b$ for every $G' \subset G$. But by (2) above, there exists $G' \subset G$ for which $a\ \hat{D}_{G'}\ b$. Since C respects $\hat{D}^{2^N}$, it follows that $a\ P^C(R^N)\ b$. This proves that C respects $D^{\mathcal{G}}$.

(4) Suppose that $a, b \in X$ is any pair satisfying $a\ \hat{D}_G\ b$ for the group $G \subset N$. By definition of $\hat{D}_G$, it must be true that $a_i = b_i$ iff $i \in N \setminus G$. Because of Lemma 2, for any POPP R^N in $\mathcal{R}^N(X, \hat{D}^{2^N})$, or in the identical set $\mathcal{R}^N(X, D^{\mathcal{G}})$, it must be true that $a\ I_i\ b$ for all $i \in N \setminus G$. Hence, whenever $a\ P_G(R^N)\ b$ is also true, then $a\ P^*_N(R^N)\ b$, implying

that $a\ P^C(R^N)\ b$ for any strictly Paretian SCR C defined on a domain of POPPs (as in the proof of Theorem 1). Therefore any such C respects the rights profile $\hat{D}^{2^N}$. ∎

3.3. A Counterexample

Under the same assumptions as Theorem 1, and for the special case when there are only individual rights and X is a Cartesian product space with one component for each individual, Coughlin (1986) also proved that any rights-respecting SCR that corresponds to a binary social preference relation must be Paretian. As remarked in Section 3.1, this is the natural converse to Theorem 1. There is no such general result when there are also group rights to respect, however, as can be seen from the following modification of an example considered by Gibbard (1974, p. 398) and Gärdenfors (1981).[4] Suppose that there are three individuals, $N = \{ A, E, J \}$, where A is for Angelina, E is for Edwin, and J is for the (male) judge. Suppose too that $X = \prod_{\emptyset \neq G \subset N} X_G$, where

$$X_G = \begin{cases} \{ 0_G, 1_G \} & \text{if } G \in \{ \{ A, E \}, \{ A, J \} \}; \\ \{\bar{x}_G\} & \text{otherwise.} \end{cases}$$

Here 1_G represents the couple G getting married, while 0_G represents them not doing so. Also, $\bar{x}_G$ denotes a dummy option for groups $G \notin \{ \{ A, E \}, \{ A, J \} \}$, representing the absence of rights for these other groups (and individuals) on their own.

Write the four possible social states in the obvious summary form $0, e, j, b$, where 0 indicates that Angelina marries nobody, e that she marries Edwin, j that she marries the judge, and b that she marries both Edwin and the judge. Though b may well be excluded from the (legally) feasible set, it is in the underlying set because that must always be a Cartesian product. Incidentally, this illustrates how restrictive is Riley's (1989) assumption that the feasible set is always an entire Cartesian product set. Though actually, in the plausible case where the judge is much older than both Angelina and Edwin, b could be interpreted as Angelina marrying the judge first, and then marrying Edwin a few years later when the judge has died!

Assume that each potential couple $\{ A, E \}$, $\{ A, J \}$ has the group right to decide whether or not to get married. Then, to be strongly independent, the two non-trivial

[4] Readers who are not already familiar with the Gilbert and Sullivan operettas may care to see *Trial by Jury* for what seems to be the original story.

rights relations D_{AE} and D_{AJ} must respectively satisfy:

$$0 \; D_{AE} \; e, \quad e \; D_{AE} \; 0, \quad \text{and} \quad j \; D_{AE} \; b, \quad b \; D_{AE} \; j;$$
$$0 \; D_{AJ} \; j, \quad j \; D_{AJ} \; 0, \quad \text{and} \quad b \; D_{AJ} \; e, \quad e \; D_{AJ} \; b.$$

Note that this configuration gives each couple the right to marry, even if Angelina also marries the other man.

Suppose that the three individuals' basic preference orderings R_i ($i \in \{ A, E, J \}$) satisfy $b \; P_A \; j \; P_A \; e \; P_A \; 0$, while $0 \; P_E \; e$ and $j \; P_J \; 0$. Edwin and the judge's preferences can then be extended in a unique (though somewhat perverse) manner to a POPP satisfying $j \; I_E \; 0 \; P_E \; e \; I_E \; b$ and $b \; I_J \; j \; P_J \; 0 \; I_J \; e$, with each man indifferent to the externality that arises when Angelina marries the other.

According to the definition given in Section 2.2, when individuals have this POPP, respecting the joint rights of Angelina and the judge requires that:

(i) $b \; P \; e$ because $b \; P_A \; e$, $b \; P_J \; e$, and $b \; D_{AJ} \; e$;

(ii) $j \; P \; 0$ because $j \; P_A \; 0$, $j \; P_J \; 0$, and $j \; D_{AJ} \; 0$.

On the other hand, because of the conflicting preferences $e \; P_A \; 0$ and $0 \; P_E \; e$, as well as $b \; P_A \; j$ and $j \; P_E \; b$, it follows that any social preferences over the two pairs $\{ 0, e \}$ and $\{ j, b \}$ will respect the joint rights of Angelina and Edwin, as specified by the rights relations $0 \; D_{AE} \; e$, $e \; D_{AE} \; 0$, $j \; D_{AE} \; b$ and $b \; D_{AE} \; j$. Finally, no individual or couple has rights over either of the pairs $\{ 0, b \}$ and $\{ j, e \}$, so that any social preferences over these pairs are consistent with respect for rights. Accordingly, the social preference relation defined by $b \; P \; e \; P \; j \; P \; 0$ respects each couple's rights. Yet the particular social preference $e \; P \; j$ clearly violates even the weak Pareto principle, since all three individuals prefer j to e.

Though this example is somewhat contrived, it does show how Pareto efficiency is not ensured by respecting group rights, even for the case when preferences are privately oriented and rights are strongly independent. The converse to Theorem 1 is therefore not true in general.

4. Power in Game Forms

4.1. Libertarian Game Forms

Suppose that for every feasible set $A \in \mathcal{F}(X)$ there is a corresponding *game form* $\Gamma_A = (S_A^N, g_A)$, with individual *strategy sets* S_{iA} ($i \in N$), and an *outcome function* $g_A(\cdot) : S_A^N \to A$ whose domain is the Cartesian product set $S_A^N := \prod_{i \in N} S_{iA}$. Thus, a unique *outcome* $g_A(s^N) \in A$ is specified for each *strategy profile* $s^N = (s_i)_{i \in N} \in S_A^N$.

In the following, for any group $G \in \mathcal{G}$, let S_A^G denote the Cartesian product set $\prod_{i \in G} S_{iA}$ of strategy profiles for the members of the group G, with typical member s^G, and let $S_A^{N \setminus G}$ denote the set $\prod_{i \in N \setminus G} S_{iA}$ of strategy profiles for the members of the complementary group $N \setminus G$, with typical member $s^{N \setminus G}$.

Given the rights profile $D^{\mathcal{G}}$, say that the game form $\Gamma_A = (S_A^N, g_A)$ is *libertarian* if, whenever $a \; D_G \; b$ for some $G \in \mathcal{G}$ and $a, b \in A$, while $\bar{s}^N \in S_A^N$ and $b = g_A(\bar{s}^N)$, then there exists some $s^G \in S_A^G$ for which $a = g_A(s^G, \bar{s}^{N \setminus G})$. Thus, whenever $G \in \mathcal{G}$ and $a \; D_G \; b$, the group G must have the power to change the outcome from $b = g_A(\bar{s}^G, \bar{s}^{N \setminus G})$ to $a = g_A(s^G, \bar{s}^{N \setminus G})$, no matter what strategies $\bar{s}^{N \setminus G} \in S_A^{N \setminus G}$ may be chosen by individuals outside the group.

Relative to any preference profile R^N, the strategy profile $\bar{s}^N \in S_A^N$ is said to be a $\mathcal{G}$-*strong equilibrium* for the game form (S_A^N, g_A) if there is no group $G \in \mathcal{G}$ with an alternative strategy profile $s^G \in S_A^G$ for which $g_A(s^G, \bar{s}^{N \setminus G}) \; P_G(R^N) \; g_A(\bar{s}^N)$. Thus $\bar{s}^G$ must be an efficient response to $\bar{s}^{N \setminus G}$ for every group $G \in \mathcal{G}$. For each preference profile R^N, denote by $E_{\mathcal{G}}(\Gamma_A, R^N)$ the corresponding set of $\mathcal{G}$-strong equilibria of the game form $\Gamma_A = (S_A^N, g_A)$ — it is a (possibly empty) subset of S_A^N.

Theorem 4. *Let $D^{\mathcal{G}}$ be a given rights profile on X. Suppose that the libertarian game form $\Gamma_A = (S_A^N, g_A)$ has the property that the $\mathcal{G}$-strong equilibrium set $E_{\mathcal{G}}(\Gamma_A, R^N)$ is non-empty for all feasible sets $A \in \mathcal{F}(X)$ and all preference profiles R^N in the restricted domain $\mathcal{P}^N$. Then any SCR satisfying $\emptyset \neq C(A, R^N) \subset g_A(E_{\mathcal{G}}(\Gamma_A, R^N))$ everywhere in the domain $\mathcal{F}(X) \times \mathcal{P}^N$ must respect rights.*

Proof: Suppose that some such SCR $C(\cdot, \cdot)$ did not respect rights. Then there would exist a feasible set $A \in \mathcal{F}(X)$, a profile $R^N \in \mathcal{P}^N$, a group $G \in \mathcal{G}$, and social states $a, b \in A$ such that $a \; D_G \; b$, $a \; P_G(R^N) \; b$ and yet $b \in C(A, R^N) \subset g_A(E_{\mathcal{G}}(\Gamma_A, R^N))$. So there would be a $\mathcal{G}$-strong equilibrium $\bar{s}^N \in E_{\mathcal{G}}(\Gamma_A, R^N) \subset S_A^N$ such that $b = g_A(\bar{s}^N)$.

Because the game form Γ_A is libertarian and $a\ D_G\ b$, there must exist some group strategy profile $s^G \in S_A^G$ for which $a = g_A(s^G, \bar{s}^{N\setminus G})$. Because $a\ P_G(R^N)\ b$, the strategy $\bar{s}^G$ could not then be an efficient response for group G to $\bar{s}^{N\setminus G}$ after all, and so $\bar{s}^N$ could not be a $\mathcal{G}$-strong equilibrium — a contradiction.

It has therefore been proved by contradiction that the SCR $C(\cdot,\cdot)$ must respect the rights profile $D^{\mathcal{G}}$. ∎

Theorem 4 says that rights-respecting SCR's are no less general than libertarian game forms *for the same given configuration of rights*. Note that there was no need even to assume any restrictions such as privately oriented preference profiles or independent rights, though the theorem is in danger of being vacuous without such restrictions.

4.2. An Implementation

The following result shows how, under the rich private domain assumption used in Section 3, it is possible to construct a game form that will implement in strong equilibrium any given strictly Pareto efficient outcome. Moreover, every Nash equilibrium will yield an outcome which every individual finds indifferent to that given outcome. By Theorem 1, the resulting choice of outcome must respect both individual and group rights.

THEOREM 5. *Suppose that the set $\mathcal{R}^N(X, D^{\mathcal{G}})$ of POPPs satisfies the rich private domain assumption so that, by Lemma 2, X is equivalent to a subset of the product space $\prod_{i\in N} X_i$ with the properties that $a\ I_i\ b$ whenever $a_i = b_i$, and also that $a\ D_G\ b$ implies $a_j = b_j$ for all $j \in N \setminus G$. Then, for every feasible set $A \in \mathcal{F}(X)$, every POPP $R^N \in \mathcal{R}^N(X, D^{\mathcal{G}})$, and any $\bar{x} \in A$ which is strictly Pareto efficient given R^N, there exists a game form Γ_A with strategy sets $S_{iA} = X_i$ ($i \in N$) and outcome function*

$$g_A(s^N) := \begin{cases} s^N & \text{if } s^N \in A; \\ \bar{x} & \text{if } s^N \notin A; \end{cases}$$

which has one strong equilibrium with $\bar{s}^N = \bar{x}$, and has all of its Nash equilibria $\hat{s}^N$ satisfying $g_A(\hat{s}^N)\ I_i\ \bar{x}$ (all $i \in N$).

PROOF: First, let $\bar{s}_i = \bar{x}_i$ (all $i \in N$). Now, given any non-empty $G \subset N$, let $s^G \in S_A^G$ and $a \in A$ be such that $a = g_A(s^G, \bar{s}^{N\setminus G}) \neq \bar{x}$. Then it must be true that $a = (s^G, \bar{x}^{N\setminus G}) \in A$. Hence $a\ I_i\ \bar{x}$ for all $i \in N \setminus G$. But then $a\ P_G(R^N)\ \bar{x}$ would imply that $a\ P_N^*(R^N)\ \bar{x}$, contradicting the hypothesis that $\bar{x}$ is strictly Pareto efficient in A. This confirms that $\bar{s}^N$ is a strong equilibrium.

Second, let $\hat{s}^N$ be any other Nash equilibrium. Then, because i could choose $\bar{x}_i \in S_{iA}$ instead, one must have $g_A(\hat{s}^N)\ R_i\ g_A(\bar{x}_i, \hat{s}^{N\setminus\{i\}})$ for all $i \in N$. But by definition of the outcome function $g_A(\cdot)$, it must be true that $g_A(\bar{x}_i, \hat{s}^{N\setminus\{i\}})$ is equal to $\bar{x}$ or to $(\bar{x}_i, \hat{s}^{N\setminus\{i\}})$. In either case $g_A(\bar{x}_i, \hat{s}^{N\setminus\{i\}})\ I_i\ \bar{x}$. Since R_i is transitive, it follows that $g_A(\hat{s}^N)\ R_i\ \bar{x}$ for all $i \in N$. But $\bar{x}$ is strictly Pareto efficient, and so $g_A(\hat{s}^N)\ I_i\ \bar{x}$ for all $i \in N$. ∎

Note that the game form Γ_A need not be libertarian, however. For suppose that $a \in X$ and $G \subset N$ are such that $a\ D_G\ \bar{x}$. But now, if $s^N \notin A$ and so $g_A(s^N) = \bar{x}$, it is generally not true that G can change the outcome from $\bar{x}$ to a by finding an $\tilde{s}^G \in S_A^G$ for which $g_A(\tilde{s}^G, s^{N\setminus G}) = a$. In fact this would require not only that group G choose $\tilde{s}^G = a^G$, but also the coincidence that $s^{N\setminus G} = a^{N\setminus G} = \bar{x}^{N\setminus G}$, even though $g_A(s^N) = \bar{x}$ for every s^N such that $s^N \notin A$. For instance, if $A = \{\, 0, e, j \,\}$ and $\bar{x} = j$ in the example of Section 3.3, there is no way that the game form Γ_A constructed above allows Angelina and Edwin to change the outcome to e while the judge continues to choose the strategy of marrying Angelina.

This absence of libertarianism makes Theorem 5 weaker than the corresponding result in Riley (1989). The difference arises because here the outcome function $g_A(s^N)$ has to be well-defined for all $s^N \in \prod_{i \in N} X_i$ even when A is a proper subset of this product space, and even when A is not itself a product space. In the special case when $A = \prod_{i \in N} A_i$, one could take $\tilde{S}_{iA} = A_i$ (all $i \in N$) and use the alternative outcome function $\tilde{g}_A(s^N) := s^N$ for all $s^N \in S_A^N$. This gives an alternative libertarian game form $\tilde{\Gamma}_A$ with a set of strong equilibria that coincides with the set of all strictly Pareto efficient social states — i.e., $g_A(E_{\mathcal{G}}(\tilde{\Gamma}_A, R^N)) = C^{\text{Par}}(A, R^N)$ for every preference profile R^N.

4.3. Direct Game Forms

Of particular interest in Section 5.2 below will be the special case of *direct game forms*, in which all individuals' strategies coincide with their respective preference orderings. Thus, as in the direct mechanisms which occur in the literature on incentive compatibility, it is as though the game form were being played by having individuals report their preferences directly, after which the outcome function selects the appropriate social state for the reported profile of preferences. Apart from this analogy, direct game forms would also seem appropriate for normative judgements concerning a social system, since they tell us precisely how the social state reflects peoples' preferences.

Of course, restricting oneself to such direct game forms places a potentially serious limitation on what SCR's can be implemented. Indeed, it was by allowing a weak form of implementation through indirect game forms or mechanisms such as those considered by Maskin (1979, 1985) that Riley (1989) was able to demonstrate exact equivalence between formulations A and B — i.e., between the rights respecting social choice approach and his version of the libertarian game form approach. Of course, he also restricted attention to binary SCR's, individual rather than group rights, and feasible sets in the Cartesian product form $A = \prod_{i \in N} A_i$. An indirect game form was also used to prove the closely related Theorem 5 above.

Indirect game forms are crucial here, however. Indeed, given the negative results for fully Pareto efficient dominant strategy mechanisms such as those surveyed by Dasgupta, Hammond and Maskin (1979) or Groves and Ledyard (1987), it is clear that direct game forms cannot implement as many SCR's as indirect game forms do. For suppose it were possible to construct a direct game form for which truthfulness was always a $\mathcal{G}$-strong equilibrium, no matter what the privately oriented preference profile may be. Then, arguing as in Dasgupta, Hammond and Maskin (1979), for each group $G \in \mathcal{G}$, no matter what privately oriented preferences are being reported by the individuals who are not members of G, among the set of all possible reports of privately oriented preferences truthfulness would always be an optimal strategy for each individual in G, as well as an efficient strategy for group G and all its subgroups. In fact, therefore, truthfulness would always be a "$\mathcal{G}$-dominant strategy equilibrium," in an obvious sense, contradicting the negative results cited above.

Yet indirect game forms have their own serious problems. For their equilibrium outcomes are generally sensitive to players' beliefs about each other and about how the game form will be played, as pointed out in the discussions of implementation by means of Bayesian or Nash equilibium in Ledyard (1978, 1986), Dasgupta, Hammond and Maskin (1979), and Hammond (1990, 1993). Indirect game forms typically implement only *extended* social choice rules that can be expressed as $C(A, R^N, \theta^N)$, where $\theta^N = (\theta_i)_{i \in N}$ is a profile of individual types θ_i, each of which is a parameter sufficient to determine i's actions and beliefs in the game form.

5. Rights-Inclusive Social States

5.1. Motivation

As mentioned in the introduction, I now want to call into question the way in which past discussions of rights have usually described the social states themselves. As a reason for doing so, note that it is impossible to tell whether a political system is a meaningful democracy unless one knows not only the social states or outcomes that emerge from the system, but also how well those outcomes reflect both individual preferences and values. This illustrates the rather obvious point, which Sugden (1981, 1986) has also made in a rather different way, that a social choice rule cannot really be judged only on the basis of the social outcomes it generates. It is important to know as well how these outcomes depend on individual preferences. This, of course, takes us to the kind of direct game form introduced in Section 4.3 above. It is true that such game forms can be classified as libertarian or not, according to the definition given in Section 4.1. Yet this treats respect for rights as an absolute standard, to be satisfied entirely. There is no room for compromise, and no way of discussing how serious is the extent of any rights violations. Moreover, we live in a world that confronts us with many unfortunate issues where trade-offs between different kinds of rights for different individuals seem unavoidable. This makes disturbing the lack of any framework whatever for discussing how to make the necessary compromises.

An alternative formulation seems rather obvious, therefore. Following Pattanaik and Suzumura (1992), we should consider rights themselves as part of the social state — in other words, we should have *rights-inclusive social states*. Then, along with social outcomes, rights assignments will become objects of preference according to some higher order individual preference relations. Rights assignments will also become objects of social choice according to some higher order social choice rule. In this new approach, respect for rights and liberty becomes a relative concept. Some social choice rules will unambiguously show more respect for rights than others do because they choose both more extensive rights assignments and social outcomes that heed rights better. But there may be no social choice rule at all which respects rights fully.

This, then, is another way to formulate the issue of how (much) to respect rights. The urgent question to be considered next is how to model an assignment of rights before incorporating it in the social state. Note that Pattanaik and Suzumura (1992) choose to model rights as general game forms. Yet this suffers from the disadvantage that it pays too much attention to the strategies themselves and to the labels they bear, rather than to the outcomes resulting from those strategies. After all, it is not clear why one profile of strategy sets should be preferred to another unless the two profiles are likely to give rise to different social outcomes — or at least to different opportunities for individuals and groups to obtain preferred outcomes by exercising their rights. Accordingly, the rest of this Section will consider two other ways of modelling rights.

5.2. Rights as Direct Game Forms

Recall that direct game forms were defined in Section 4.3 as mappings directly from individual preference profiles to social outcomes. Modelling rights as direct game forms differs from the framework used by Pattanaik and Suzumura (1992), who allow complete general game forms, rather than only direct game forms, to be objects of individual preference and of social choice. Nevertheless, it seems only natural at first that the social choice rule should involve choosing a non-empty set of direct game forms.

For one thing, just as in the theory of mechanism design, given any game form and any set of behaviour rules mapping individual preference profiles into equilibrium strategies, there is an equivalent direct game form mapping individual preference profiles into social outcomes. Direct game forms are also an obvious object of individual preference if we admit that individuals may be unsure of their own preferences, especially as regards future outcomes, and that they may also value flexibility for its own sake. Once again there is an analogy with the literature on incentive compatibility, which teaches us to consider direct mechanisms in their entirety in order to see how well an economic system performs. In both cases, moreover, there is yet a further analogy with the Arrow–Debreu theory of resource allocation under uncertainty, with its suggestion that allocations of all possible state-contingent commodities should be considered. Indeed, if one thinks of individuals' preferences as uncertain or as at least private information, then "state-contingent social outcomes" are the obvious counterparts in social choice theory.

The difficulty with such direct game forms, however, is that they rely excessively on reaching an equilibrium in order to know how the social outcome depends on individual preferences. To see this, consider the "matching shirts" example, originally due to Gibbard (1974), which was briefly mentioned in the introduction. This same example also figures most prominently in the recent interchange between Gaertner, Pattanaik and Suzumura

(1992), Pattanaik and Suzumura (1990), and Sen (1992). Recall that it is really just a version of the well known two-person zero-sum game of "matching pennies," but played with shirts of two different colours — e.g., white and blue. There are two players, one of whom is a "conformist" who wants to match, while the other is a "non-conformist" who wants to be different. It seems natural to give each individual the right to choose what colour shirt to wear. Yet then none of the four different possible allocations of two shirt colours to two individuals respects both individuals' rights. Nor does the game have any Nash equilibrium in pure strategies.

Thus, there is no good way to construct a direct game form that specifies what colour shirt each individual will wear in the case when their preferences take this form. Nevertheless, there is a clear sense in which, no matter what colour shirt each individual chooses to wear, they each exercise their right to wear what they choose. Of course, one player will want to change shirts after observing what colour the other is wearing, and may even claim that his rights have been violated. But this is a spurious claim. It is true that this player has made a choice which is regretted later. This may be because of inappropriate expectations or miscalculation, among many other possible reasons. Yet, as Pattanaik and Suzumura (1990) are right to emphasize, players can make mistakes in playing a game form without necessarily having their rights violated. Indeed, the rights of individuals and groups clearly would be violated if they were to be prevented from ever making any mistake! For this reason, therefore, I have come to understand that rights are not adequately modelled by direct game forms.

5.3. *Rights Relations Induced by Game Forms*

Fortunately, there is another way of inferring what individual and group rights to control the social outcome emerge from the structure of the game form. Indeed, given the feasible set A, let $\Gamma_A = (S_A^N, g_A)$ be a game form, as defined in Section 4.1 above. Then the *individual rights structure* induced by Γ_A consists of the profile $D^N = \langle D_i \rangle_{i \in N}$ of individual rights relations defined so that, for all $i \in N$ and all pairs $a, b \in A$, one has $a \; D_i \; b$ if and only if, whenever there exists $s^N \in S_A^N$ for which $g_A(s^N) = b$, then there also exists some other $\tilde{s}_i \in S_{iA}$ for which $g_A(\tilde{s}_i, s^{N \setminus \{i\}}) = a$. In other words, $a \; D_i \; b$ requires that individual i alone always has the power to change the outcome b into the outcome a, no matter what fixed strategies $s^{N \setminus \{i\}}$ the other individuals choose. Note that, if the individual rights relations really were these D_i $(i \in N)$, then Γ_A would be individually libertarian.

Now suppose that $a\ D_i\ b$, $a\ P_i\ b$, and yet, because of a mistake by i or for some other reason, b is still the social outcome that results from the game form. Even so, it is illegitimate for i to claim any rights violation, because i could have altered the outcome to a instead. Disappointed expectations and mistaken free choices are different from violations of personal rights.

A similar construction is possible for group rights, building recursively from the induced individual rights structures defined above. Indeed, the *group rights structure* induced by $g_A(\cdot)$ consists of the profile $\langle D_G \rangle_{G \subset N}$ of rights relations defined so that, for all non-empty $G \subset N$ and all $a, b \in A$, one has $a\ D_G\ b$ if and only if there is no proper subset H of G for which $a\ D_H\ b$ and also, whenever there exists $s^N \in S_A^N$ for which $g_A(s^N) = b$, then there also exists $\tilde{s}^G \in S_A^G$ for which $g_A(\tilde{s}^G, s^{N \setminus G}) = a$. In other words, $a\ D_G\ b$ requires that: (i) group G always has the power to change the outcome b into the outcome a on its own, no matter what strategies individuals outside the group choose; (ii) no proper subset of G has this same power to change b into a. Note that this definition does *not* presume that individuals and groups always exercise their rights in order to maximize their preferences.

Thus every game form induces a rights structure in a natural way. These rights structures describe what power individuals and groups have to change the social outcome. Each game form is libertarian with respect to its induced rights structure. And it is these rights structures which I believe should be incorporated in the description of each social state.

6. Summary and Concluding Remarks

Section 3 investigated the inevitable limits on the individual and group rights which can be respected, especially if there is to be no conflict with the Pareto criterion. It suggested that it is natural to consider *privately oriented preference profiles*, for which Theorem 1 says that all such conflicts disappear. However, if the domain of such preference profiles is rich enough to allow that, given any pair of different social states, at least one individual can express a strict preference over that pair, then the underlying set of social states can be given a product structure, with individuals (and groups) effectively having independent rights over their own (members') components of the product space.

Thereafter Section 4 set out the relationship between the rights-respecting social choice and libertarian game form formulations of rights. In particular, Theorem 4 tells us that selecting social states from the appropriate strong equilibrium outcomes of a libertarian game form always generates a rights-respecting social choice rule. On the other hand, some rights-respecting social choice rules cannot be implemented by means of a libertarian game form. So, when the two approaches differ, it must always be the rights-respecting or decisive social choice approach that is somewhat more general. It bears repeating, however, that Riley (1989) was able to obtain exact equivalence by restricting attention to feasible sets in the form of Cartesian products.

The characterization results of Sections 3 and 4 are essentially negative, showing how unlikely it is that all individual and group rights can be respected. Generally, therefore, a choice has to be made of which rights to satisfy and which to violate. Accordingly, Section 5 contained three different suggestions for incorporating rights within the description of each social state. It was argued that the rights relations induced by a game form may be the most appropriate way of respecting those rights.

Finally, I should say that I am uncomfortably aware of the ultimate endnote in Sen (1992). This warns us that:

> *While the process through which a state of affairs is reached can be brought into the characterization of that social state (and this adds substantially to the domain of the social-choice formulations of liberty), the implicit nature of this presentation can be sometimes rather unhelpful.*

It refers, however, to a fuller discussion in Sen's 1991 Arrow Lectures on *Freedom and Social Choice* which I have so far not had the opportunity of seeing. Nor was able I to hear the lectures as they were delivered. So presently I am unable to say whether I agree or disagree. In fact, though, the specification of an induced rights structure says virtually nothing about "the process through which" the social state finally emerges, and so I remain unsure whether the point I wish to make is really being addressed at all. I hope to be able to remedy this obvious and glaring defect later on.

Nevertheless, I take heart from some other words of Sen's (1985, pp. 231–2) that I have seen, namely:

> *While there is some obvious advantage in seeing liberty as control, it is a mistake to see it* only *as control. The simpler social choice characterizations catch one aspect of liberty well (to wit: whether people are getting what they* would have chosen *if they had control), but miss another (to wit: who actually controlled the decision). But the view of liberty as control misses the former important aspect altogether even though it catches the latter. A more satisfactory theory of liberty in particular and rights in general would try to capture both aspects ….*

Including in the description of each social state the rights structure induced by a game form does indeed "try to capture both aspects" of liberty. Future work will determine how successful this attempt will be.

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