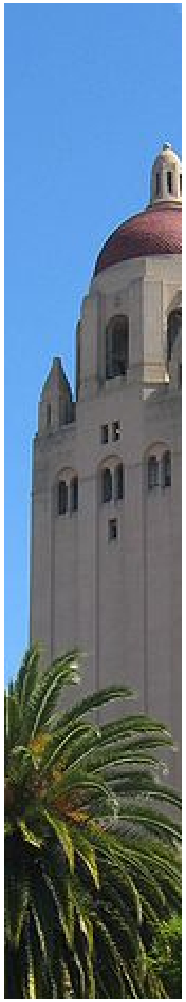
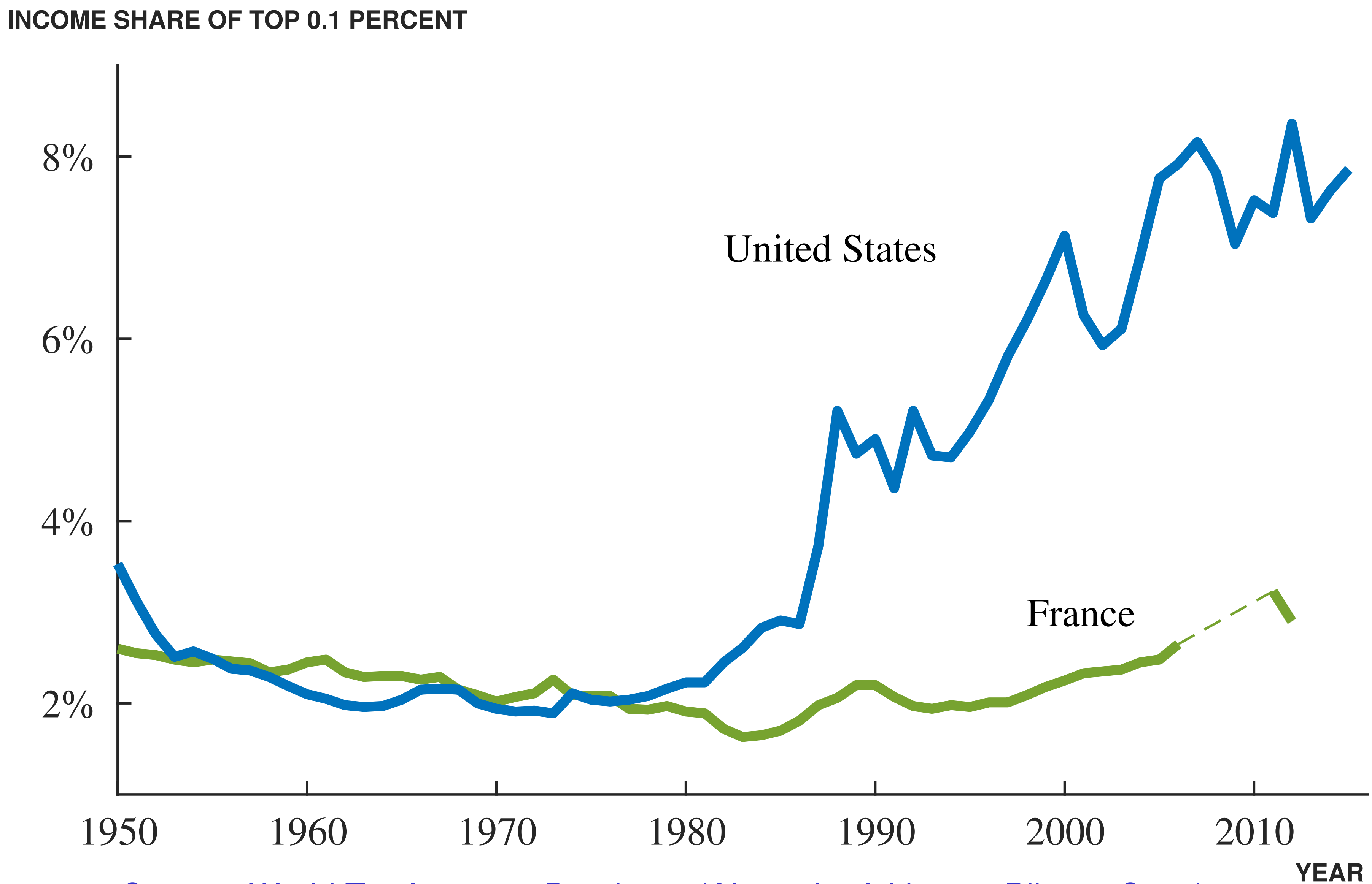


A Schumpeterian Model of Top Income Inequality

Chad Jones and Jihee Kim

Forthcoming, *Journal of Political Economy*

Top Income Inequality in the United States and France



Source: World Top Incomes Database (Alvaredo, Atkinson, Piketty, Saez)

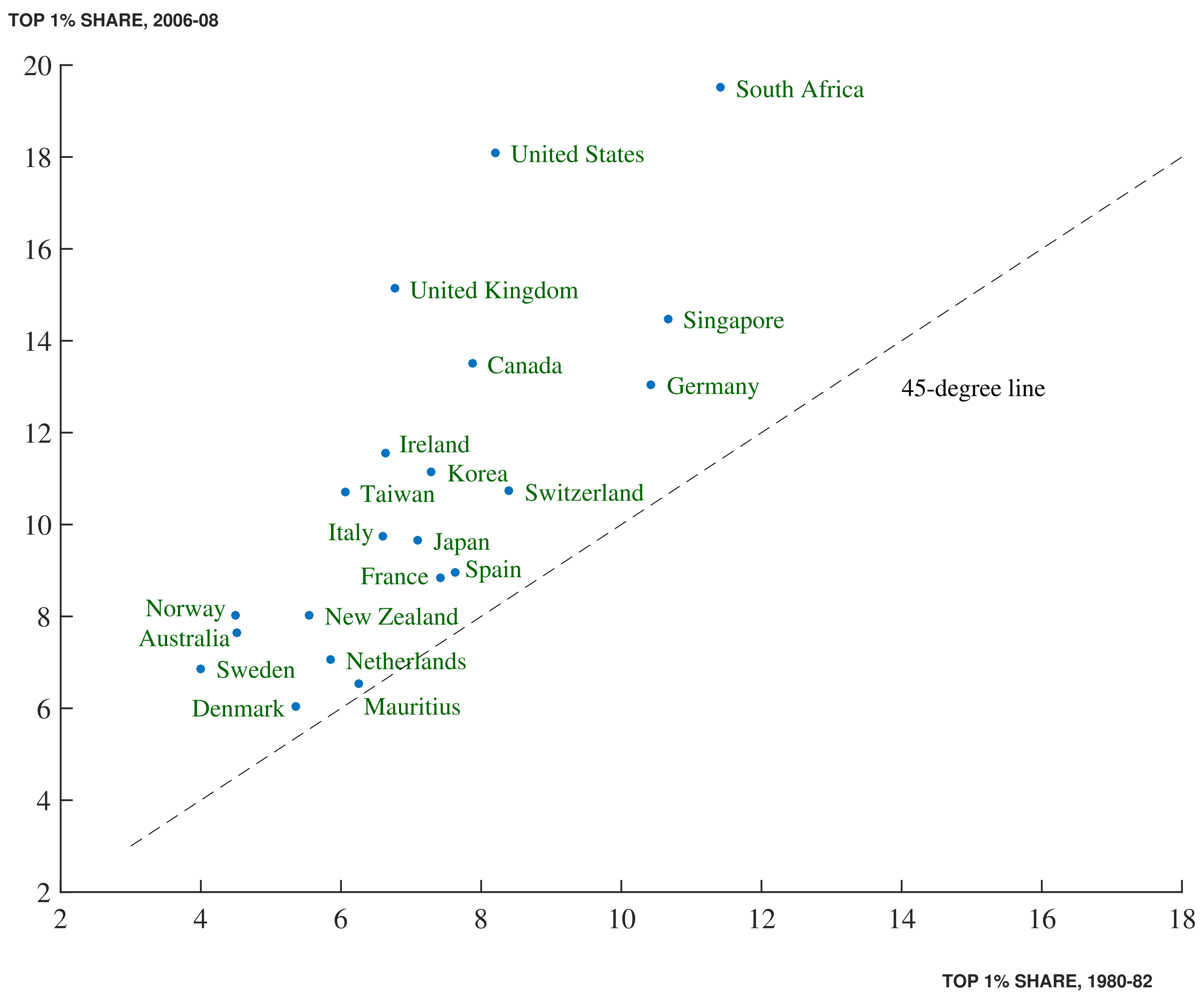
Related literature

- Empirics: Piketty and Saez (2003), Aghion et al (2015), Guvenen-Kaplan-Song (2015) and many more
- Rent Seeking: Piketty, Saez, and Stantcheva (2011) and Rothschild and Scheuer (2011)
- Finance: Philippon-Reshef (2009), Bell-Van Reenen (2010)
- Not just finance: Bakija-Cole-Heim (2010), Kaplan-Rauh
- Pareto-generating mechanisms: Gabaix (1999, 2009), Luttmer (2007, 2010), Reed (2001). GLLM (2015).
- Use Pareto to get growth: Kortum (1997), Lucas and Moll (2013), Perla and Tonetti (2013).
- Pareto wealth distribution: Benhabib-Bisin-Zhu (2011), Nirei (2009), Moll (2012), Piketty-Saez (2012), Piketty-Zucman (2014), Aoki-Nirei (2015)

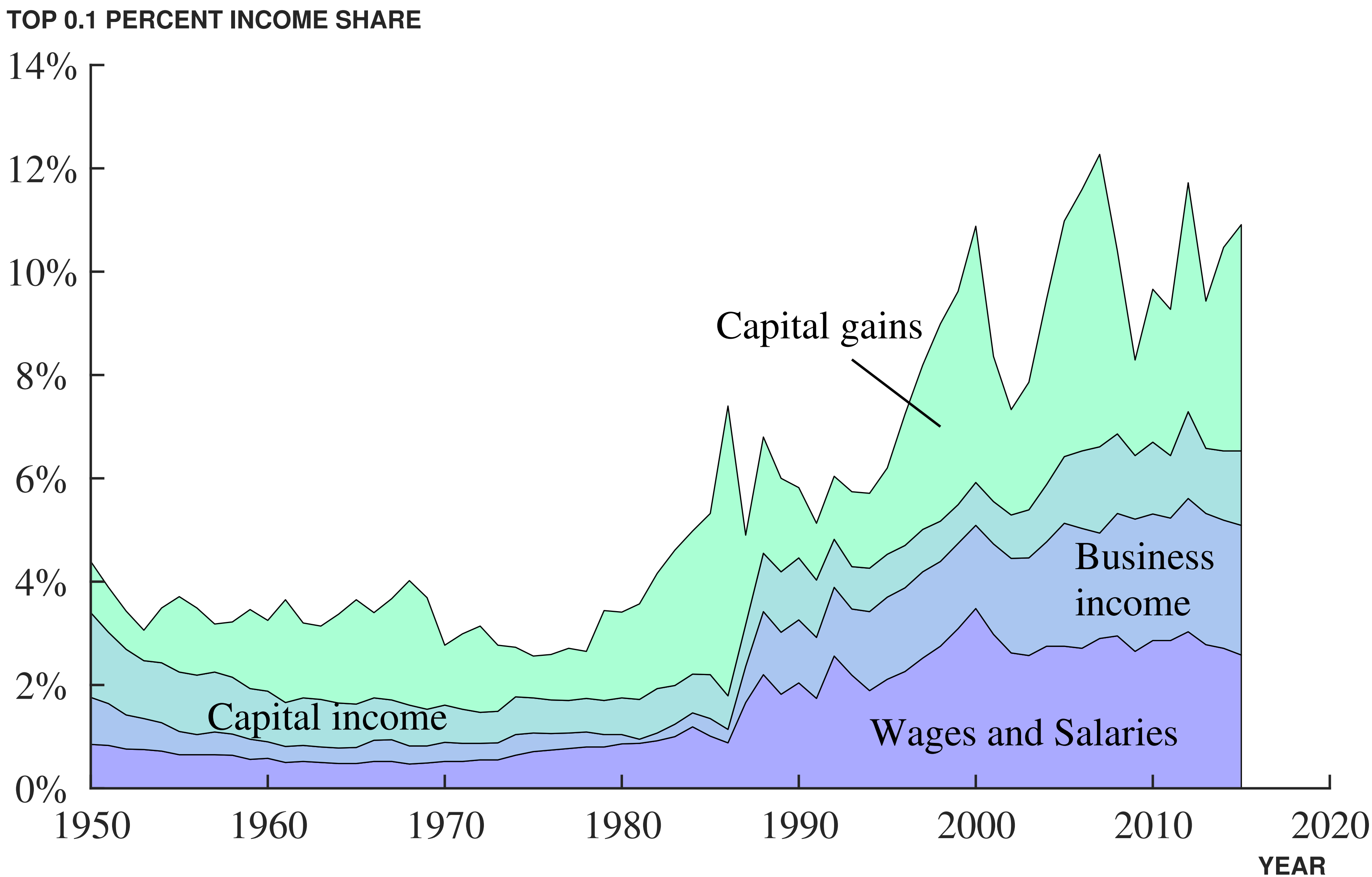
Outline

- Facts from World Top Incomes Database
- Simple model
- Full model
- Empirical work using IRS public use panel tax returns
- Numerical examples

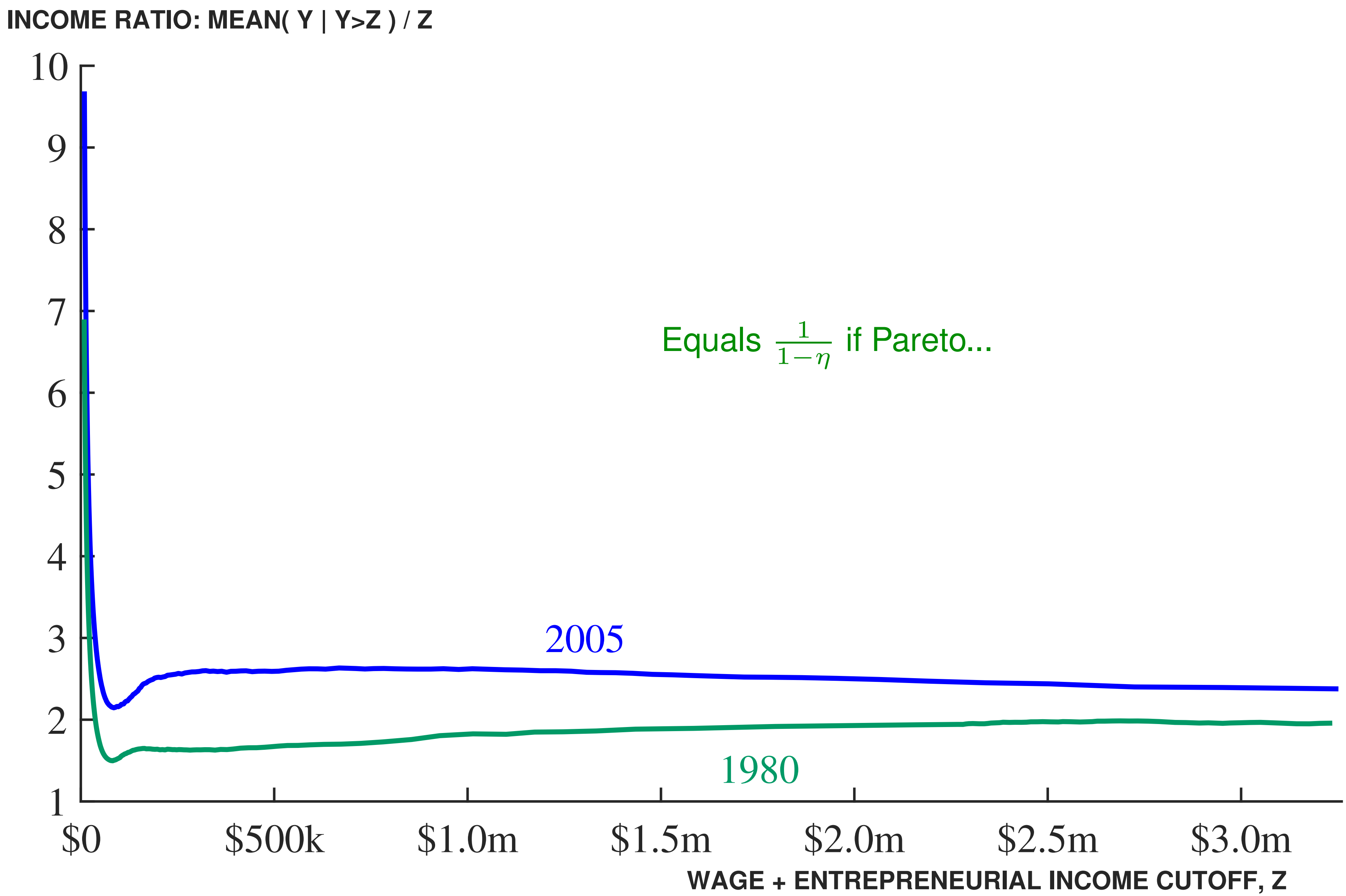
Top Income Inequality around the World



The Composition of the Top 0.1 Percent Income Share



The Pareto Nature of Labor Income



Pareto Distributions

$$\Pr\left[Y > y\right] = \left(\frac{y}{y_0}\right)^{-\xi}$$

- Let $\tilde{S}(p)$ = share of income going to the top p percentiles, and $\eta \equiv 1/\xi$ be a measure of Pareto inequality:

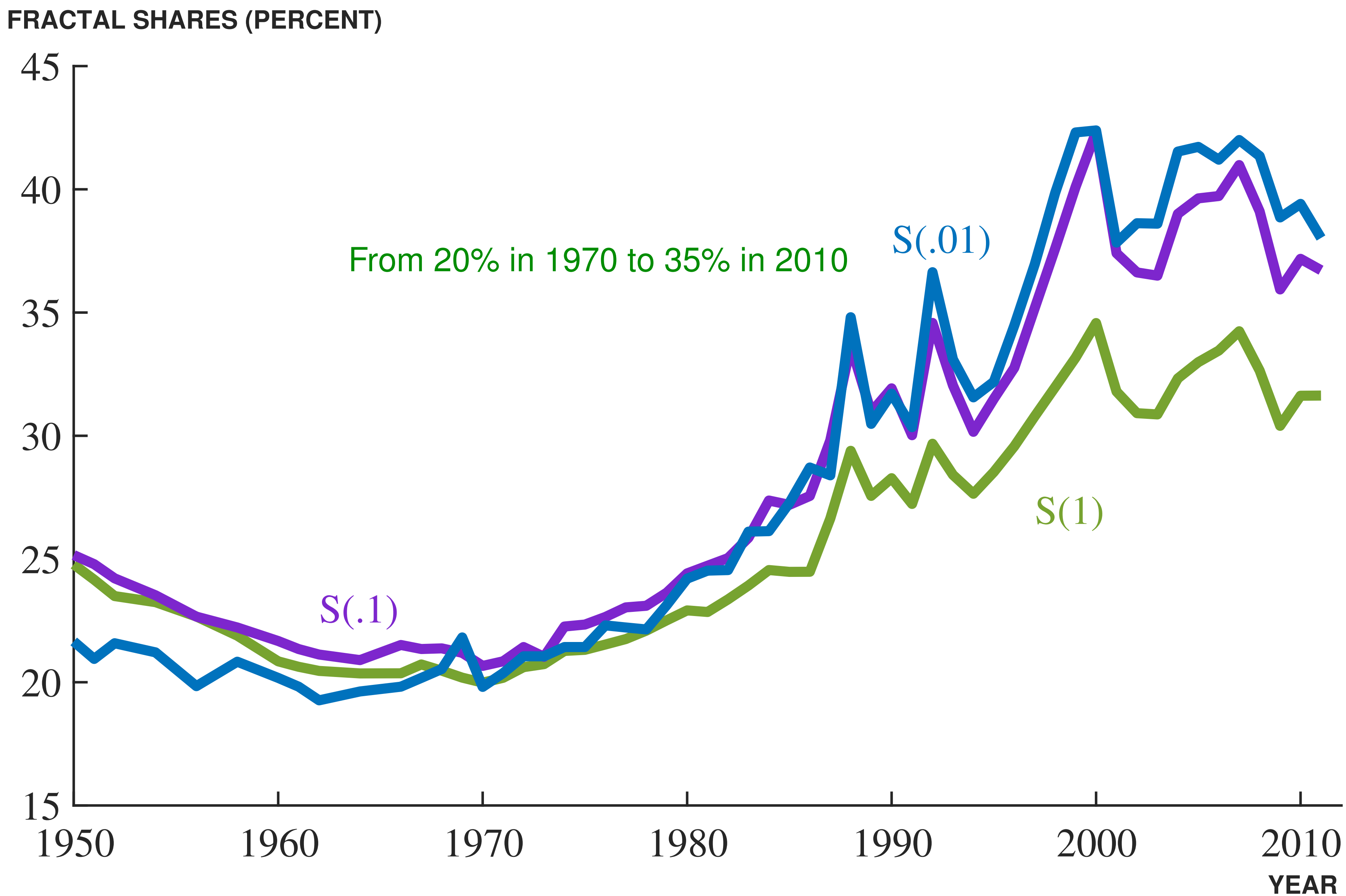
$$\tilde{S}(p) = \left(\frac{100}{p}\right)^{\eta-1}$$

  - If $\eta = 1/2$, then share to Top 1% is $100^{-1/2} \approx .10$
  - If $\eta = 3/4$, then share to Top 1% is $100^{-1/4} \approx .32$

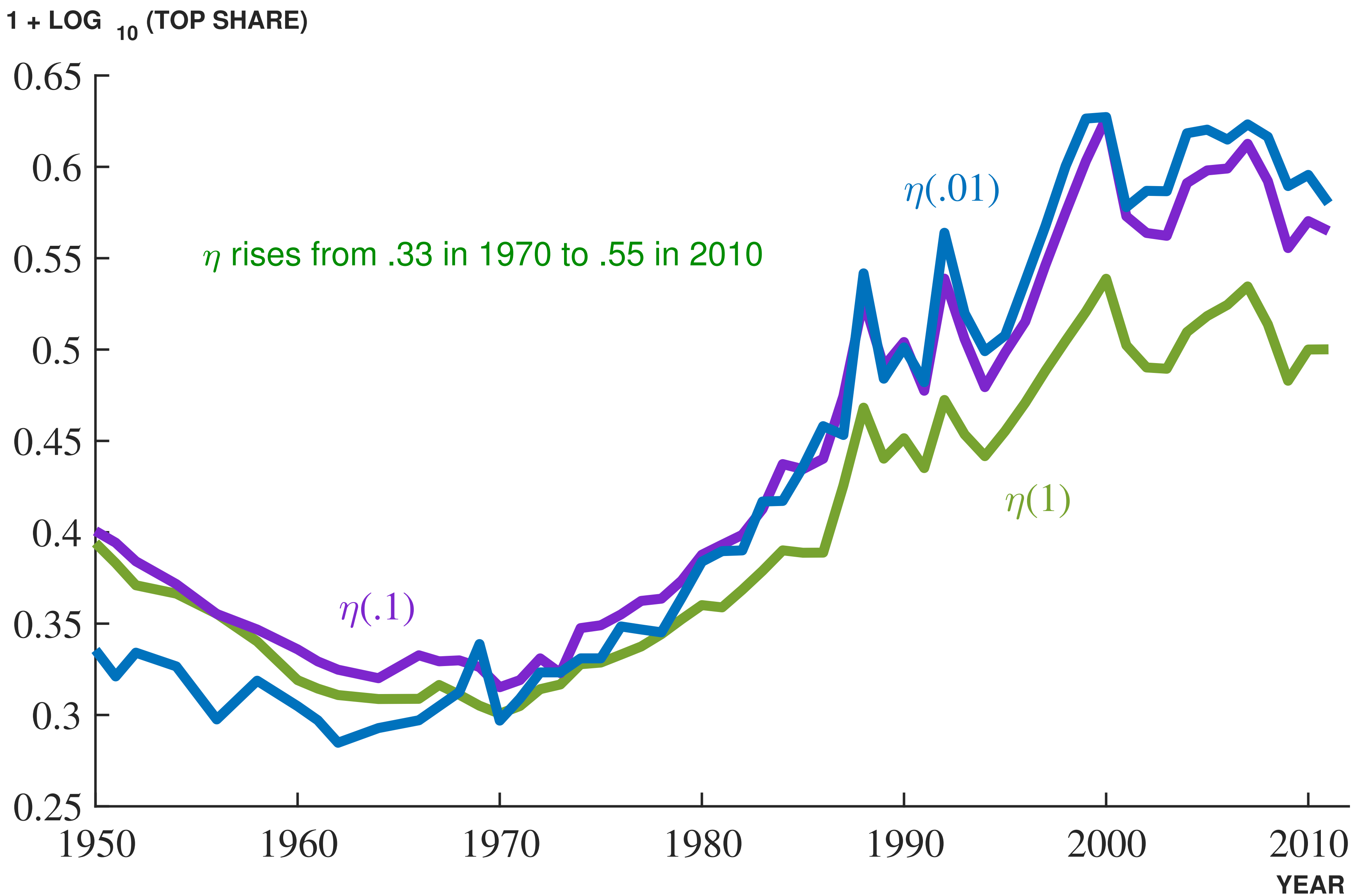
- Fractal: Let $S(a)$ = share of $10a$'s income going to top a:

$$S(a) = 10^{\eta-1}$$

Fractal Inequality Shares in the United States



The Power-Law Inequality Exponent η, United States



Skill-Biased Technical Change?

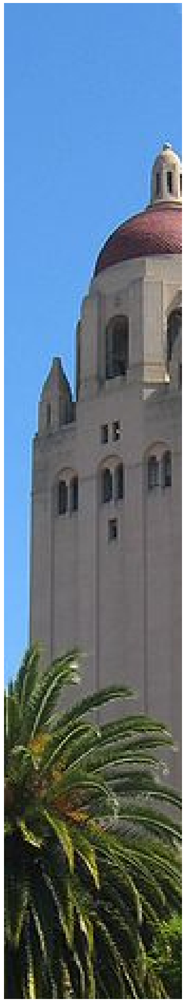
- Let $x_i =$ skill and $\bar{w} =$ wage per unit skill

$$y_i = \bar{w} x_i^{\alpha}$$

- If $\Pr[x_i > x] = x^{-1/\eta_x}$, then

$$\Pr[y_i > y] = \left(\frac{y}{\bar{w}}\right)^{-1/\eta_y} \quad \text{where} \quad \eta_y = \alpha \eta_x$$

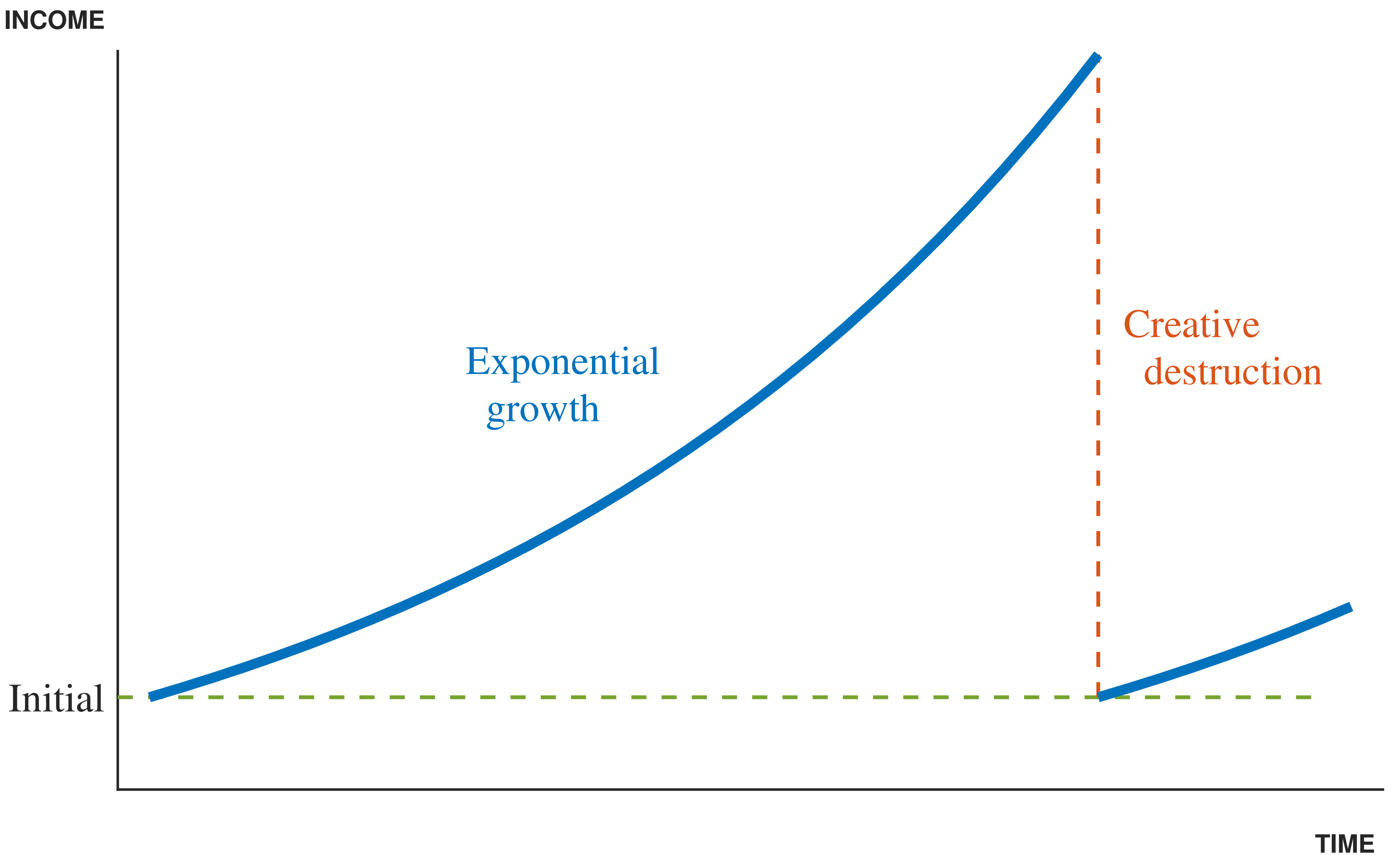
- That is y_i is Pareto with inequality parameter η_y
  - SBTC ($\uparrow \bar{w}$) shifts distribution right but η_y unchanged.
  - $\uparrow \alpha$ would raise Pareto inequality...
  - This paper: why is $x \sim$ Pareto, and why $\uparrow \alpha$



A Simple Model

Cantelli (1921), Steindl (1965), Gabaix (2009)

Key Idea: Exponential growth w/ death ⇒ Pareto



Simple Model for Intuition

- Exponential growth often leads to a Pareto distribution.
- Entrepreneurs
  - New entrepreneur (“top earner”) earns y_0
  - Income after x years of experience:

$$y(x) = y_0 e^{\mu x}$$

- Poisson “replacement” process at rate δ
  - Stationary distribution of experience is exponential

$$\Pr\left[\text{Experience} > x\right] = e^{-\delta x}$$

What fraction of people have income greater than y?

- Equals fraction with at least $x(y)$ years of experience

$$x(y) = \frac{1}{\mu} \log \left(\frac{y}{y_0} \right)$$

- Therefore

$$\begin{aligned} \text{Pr}\left[\text{Income} > y\right] &= \text{Pr}\left[\text{Experience} > x(y)\right] \\ &= e^{-\delta x(y)} \\ &= \left(\frac{y}{y_0} \right)^{-\frac{\delta}{\mu}} \end{aligned}$$

- So power law inequality is given by

$$\eta_y = \frac{\mu}{\delta}$$

Intuition

- Why does the Pareto result emerge?
  - Log of income $\propto$ experience (Exponential growth)
  - Experience $\sim$ exponential (Poisson process)
  - Therefore log income is exponential

$$\Rightarrow \text{Income} \sim \text{Pareto!}$$

- A Pareto distribution emerges from exponential growth experienced for an exponentially distributed amount of time.

Full model: endogenize μ and δ and how they change

Why is experience exponentially distributed?

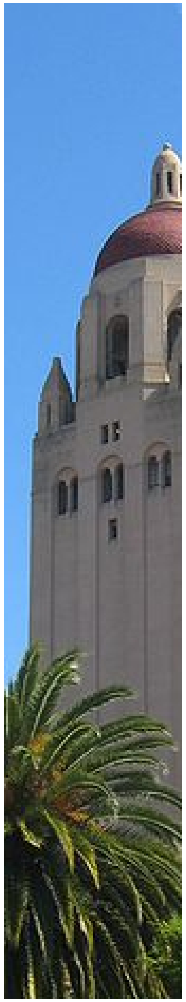
- Let $F(x,t)$ denote the distribution of experience at time t
- How does it evolve over discrete interval Δt?

$$F(x,t+\Delta t) - F(x,t) = \underbrace{\delta \Delta t (1 - F(x,t))}_{\text{inflow from above x}} - \underbrace{[F(x,t) - F(x - \Delta x, t)]}_{\text{outflow as top folks age}}$$

- Dividing both sides by $\Delta t = \Delta x$ and taking the limit

$$\frac{\partial F(x,t)}{\partial t} = \delta(1 - F(x,t)) - \frac{\partial F(x,t)}{\partial x}$$

- Stationary: $F(x)$ such that $\frac{\partial F(x,t)}{\partial t} = 0$. Integrating gives the exponential solution.



The Model

– Pareto distribution in partial eqm
– GE with exogenous research
– Full general equilibrium

Entrepreneur's Problem

Choose $\{e_t\}$ to maximize expected discounted utility:

$$U(c,\ell) = \log c + \beta \log \ell$$

$$c_t = \psi_t x_t$$

$$e_t + \ell_t + \tau = 1$$

$$dx_t = \mu(e_t) x_t dt + \sigma x_t dB_t$$

$$\mu(e) = \phi e$$

x = idiosyncratic productivity of a variety

ψ_t = determined in GE (grows)

δ = endogenous creative destruction

$\bar{\delta}$ = exogenous destruction

Entrepreneur's Problem – HJB Form

- The Bellman equation for the entrepreneur:

$$\rho V(x_t, t) = \max_{e_t} \log \psi_t + \log x_t + \beta \log(\Omega - e_t) + \frac{\mathbb{E}[dV(x_t, t)]}{dt}$$
$$+(\delta + \bar{\delta})(V^w(t) - V(x_t, t))$$

where $\Omega \equiv 1 - \tau$

- Note: the "capital gain" term is

$$\frac{\mathbb{E}[dV(x_t, t)]}{dt} = \mu(e_t) x_t V_x(x_t, t) + \frac{1}{2}\sigma^2 x_t^2 V_{xx}(x_t, t) + V_t(x_t, t)$$

Solution for Entrepreneur's Problem

- Equilibrium effort is constant:

$$e^* = 1 - \tau - \frac{1}{\phi} \cdot \beta(\rho + \delta + \bar{\delta})$$

- Comparative statics:
  - $\uparrow\tau \Rightarrow\downarrow e^*$: higher "taxes"
  - $\uparrow\phi \Rightarrow\uparrow e^*$: better technology for converting effort into x
  - $\uparrow\delta$ or $\bar{\delta} \Rightarrow\downarrow e^*$: more destruction

Stationary Distribution of Entrepreneur's Income

- Unit measure of entrepreneurs / varieties
- Displaced in two ways
  - Exogenous misallocation ($\bar{\delta}$): new entrepreneur $\to x_0$.
  - Endogenous creative destruction (δ): inherit existing productivity x.
- Distribution $f(x,t)$ satisfies Kolmogorov forward equation:

$$\frac{\partial f(x,t)}{\partial t} = -\bar{\delta} f(x,t) - \frac{\partial}{\partial x}\left[\mu(e^*) x f(x,t)\right] + \frac{1}{2} \cdot \frac{\partial^2}{\partial x^2}\left[\sigma^2 x^2 f(x,t)\right]$$

- Stationary distribution $\lim_{t\to\infty} f(x,t) = f(x)$ solves $\frac{\partial f(x,t)}{\partial t} = 0$

- Guess that $f(\cdot)$ takes the Pareto form $f(x) = Cx^{-\xi-1} \Rightarrow$

$$\xi^* = -\frac{\tilde{\mu}^*}{\sigma^2} + \sqrt{\left(\frac{\tilde{\mu}^*}{\sigma^2}\right)^2 + \frac{2\,\bar{\delta}}{\sigma^2}}$$

$$\tilde{\mu}^* \equiv \mu(e^*) - \frac{1}{2}\sigma^2 = \phi(1-\tau) - \beta(\rho + \delta^* + \bar{\delta}) - \frac{1}{2}\sigma^2$$

- Power-law inequality is therefore given by

$$\eta^* = 1/\xi^*$$

Comparative Statics (given δ^*)

$$\eta^* = 1/\xi^*, \quad \xi^* = -\frac{\tilde{\mu}^*}{\sigma^2} + \sqrt{\left(\frac{\tilde{\mu}^*}{\sigma^2}\right)^2 + \frac{2\,\bar{\delta}}{\sigma^2}}$$

$$\tilde{\mu}^* = \phi(1-\tau) - \beta(\rho + \delta^* + \bar{\delta}) - \frac{1}{2}\sigma^2$$

- Power-law inequality η^* increases if
  - $\uparrow \phi$: better technology for converting effort into x
  - $\downarrow \delta$ or $\bar{\delta}$: less destruction
  - $\downarrow \tau$: Lower "taxes"
  - $\downarrow \beta$: Lower utility weight on leisure

Luttmer and GLLM

- Problems with basic random growth model:
  - Luttmer (2011): Cannot produce “rockets” like Google or Uber
  - Gabaix, Lasry, Lions, and Moll (2015): Slow transition dynamics
- Solution from Luttmer/GLLM:
  - Introduce heterogeneous mean growth rates: e.g. “high” versus “low”
  - Here: $\phi_H > \phi_L$ with Poisson rate $\bar{p}$ of transition $(H \to L)$

Pareto Inequality with Heterogeneous Growth Rates

$$\eta^* = 1/\xi_H, \quad \xi_H = -\frac{\tilde{\mu}_H^*}{\sigma^2} + \sqrt{\left(\frac{\tilde{\mu}_H^*}{\sigma^2}\right)^2 + \frac{2\,(\bar{\delta} + \bar{p})}{\sigma^2}}$$

$$\tilde{\mu}_H^* = \phi_H(1-\tau) - \beta(\rho + \delta^* + \bar{\delta}) - \frac{1}{2}\sigma^2$$

- This adopts Gabaix, Lasry, Lions, and Moll (2015)
- Why it helps quantitatively:
  - ϕ_H: Fast growth allows for Google / Uber
  - $\bar{p}$: Rate at which high growth types transit to low growth types raises the speed of convergence $= \bar{\delta} + \bar{p}$.

Growth and Creative Destruction

Final output	$Y = \left(\int_0^1 Y_i^\theta di \right)^{1/\theta}$
Production of variety i	$Y_i = \gamma^{n_t} x_i^\alpha L_i$
Resource constraint	$L_t + R_t + 1 = \bar{N}, \;\; L_t \equiv \int_0^1 L_{it} di$
Flow rate of innovation	$\dot{n}_t = \lambda(1-\bar{z})R_t$
Creative destruction	$\delta_t = \dot{n}_t$

Equilibrium with Monopolistic Competition

- Suppose $R/\bar{L} = \bar{s}$ where $\bar{L} \equiv \bar{N} - 1$.
- Define $X \equiv \int_0^1 x_i di = \frac{x_0}{1-\eta}$. Markup is $1/\theta$.

Aggregate PF	$Y_t = \gamma^{n_t} X^\alpha L$
Wage for L	$w_t = \theta \gamma^{n_t} X^\alpha$
Profits for variety i	$\pi_{it} = (1-\theta)\gamma^{n_t} X^\alpha L \left(\frac{x_i}{X}\right) \propto w_t \left(\frac{x_i}{X}\right)$
Definition of ψ_t	$\psi_t = (1-\theta)\gamma^{n_t} X^{\alpha-1} L$

Note that $\uparrow\eta$ has a level effect on output and wages.

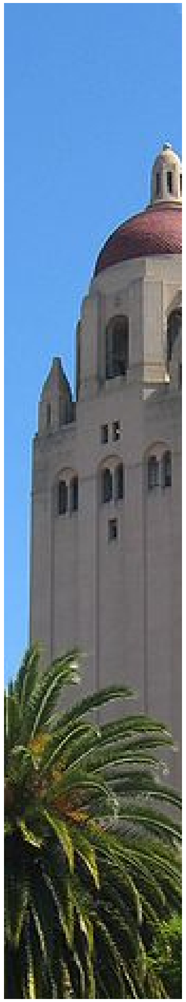
Growth and Inequality in the $\bar{s}$ case

- Creative destruction and growth

$$\delta^* = \lambda R = \lambda(1 - \bar{z})\bar{s}\bar{L}$$

$$g_y^* = \dot{n} \log \gamma = \lambda(1 - \bar{z})\bar{s}\bar{L} \log \gamma$$

- Does rising top inequality always reflect positive changes?
  - No! $\uparrow \bar{s}$ (more research) or $\downarrow \bar{z}$ (less innovation blocking)
  - Raise growth and reduce inequality via $\uparrow$creative destruction.



Endogenizing Research and Growth

Endogenizing $s = R/\bar{L}$

- Worker:

$$\rho V^w(t) = \log w_t + \frac{dV^W(t)}{dt}$$

- Researcher:

$$\rho V^R(t) = \log(\bar{m} w_t) + \frac{dV^R(t)}{dt} + \lambda \left(\mathbb{E}[V(x,t)] - V^R(t) \right) + \bar{\delta}_R \left(V(x_0, t) - V^R(t) \right)$$

- Equilibrium: $V^w(t) = V^R(t)$

Stationary equilibrium solution

Drift of log x

$$\tilde{\mu}_H^* = \phi_H(1-\tau) - \beta(\rho + \delta^* + \bar{\delta}) - \frac{1}{2}\sigma_H^2$$

Pareto inequality

$$\eta^* = 1/\xi^*, \quad \xi^* = -\frac{\tilde{\mu}_H^*}{\sigma_H^2} + \sqrt{\left(\frac{\tilde{\mu}_H^*}{\sigma_H^2}\right)^2 + \frac{2\,(\bar{\delta}+\bar{p})}{\sigma_H^2}}$$

Creative destruction

$$\delta^* = \lambda(1-\bar{z})s^*\bar{L}$$

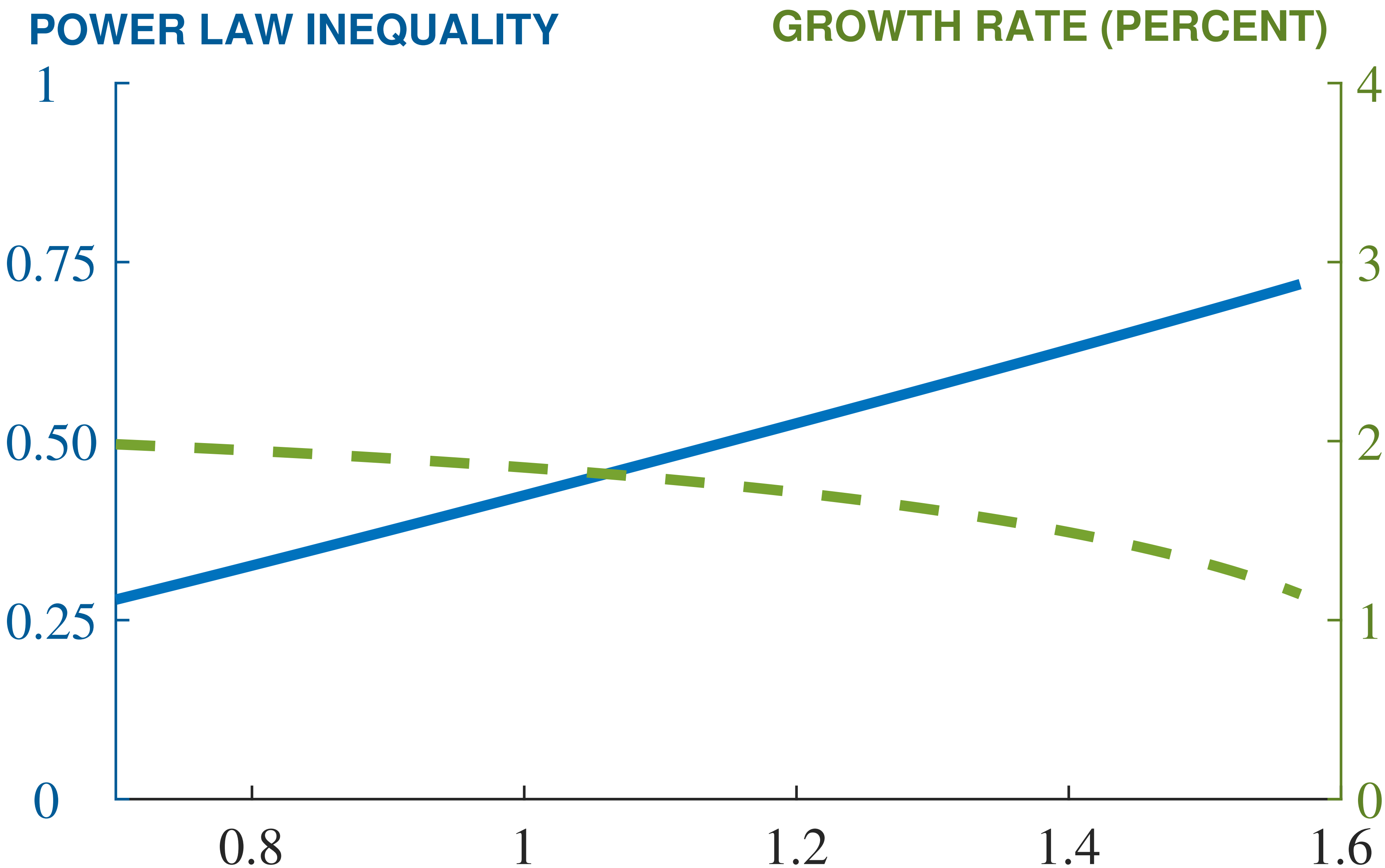
Growth

$$g^* = \delta^* \log\gamma$$

Research allocation

$$V^w(s^*) = V^R(s^*)$$

Varying the x-technology parameter ϕ



Why does $\uparrow\phi$ reduce growth?

- $\uparrow\phi \Rightarrow\uparrow e^* \Rightarrow\uparrow \mu^*$

- Two effects
  - GE effect: technological improvement $\Rightarrow$ economy more productive so higher profits, but also higher wages
  - Allocative effect: raises Pareto inequality (η), so $\frac{x_i}{X}$ is more dispersed $\Rightarrow E \log \pi_i / w$ is lower. Risk averse agents undertake less research.

- Positive level effect raises both profits and wages. Riskier research $\Rightarrow$ lower research and lower long-run growth.

How the model works

- $\uparrow \phi$ raises top inequality, but leaves the growth rate of the economy unchanged.
  - Surprising: a "linear differential equation" for x.
- Key: the distribution of x is stationary!
- Higher ϕ has a positive level effect through higher inequality, raising everyone's wage.
  - But growth comes via research, not through x...

Lucas at "micro" level, Romer/AH at "macro" level

Growth and Inequality

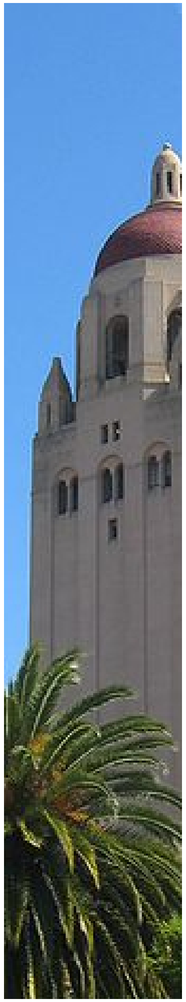
- Growth and inequality tend to move in opposite directions!
- Two reasons
  1. Faster growth $\Rightarrow$ more creative destruction
     - Less time for inequality to grow
     - Entrepreneurs may work less hard to grow market
  2. With greater inequality, research is riskier!
     - Riskier research $\Rightarrow$ less research $\Rightarrow$ lower growth
- Transition dynamics $\Rightarrow$ ambiguous effects on growth in medium run

Possible explanations: Rising U.S. Inequality

- Technology (e.g. WWW)
  - Entrepreneur’s effort is more productive $\Rightarrow \uparrow \eta$
  - Worldwide phenomenon, not just U.S.
  - Ambiguous effects on U.S. growth (research is riskier!)
- Lower taxes on top incomes
  - Increase effort by entrepreneur’s $\Rightarrow \uparrow \eta$

Possible explanations: Inequality in France

- Efficiency-reducing explanations
  - Delayed adoption of good technologies (WWW)
  - Increased misallocation (killing off entrepreneurs more quickly)
- Efficiency-enhancing explanations
  - Increased subsidies to research (more creative destruction)
  - Reduction in blocking of innovations (more creative destruction)

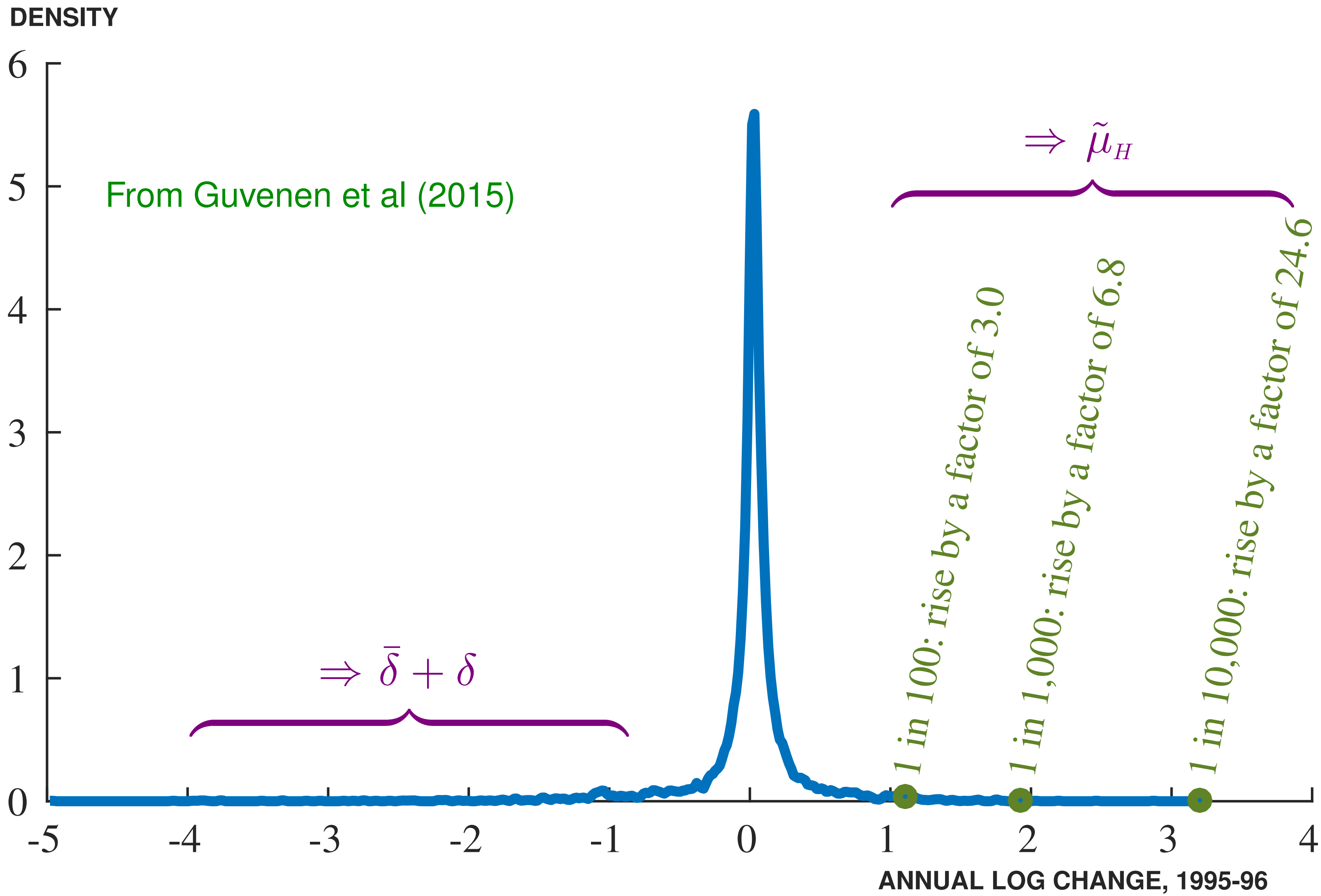


Micro Evidence

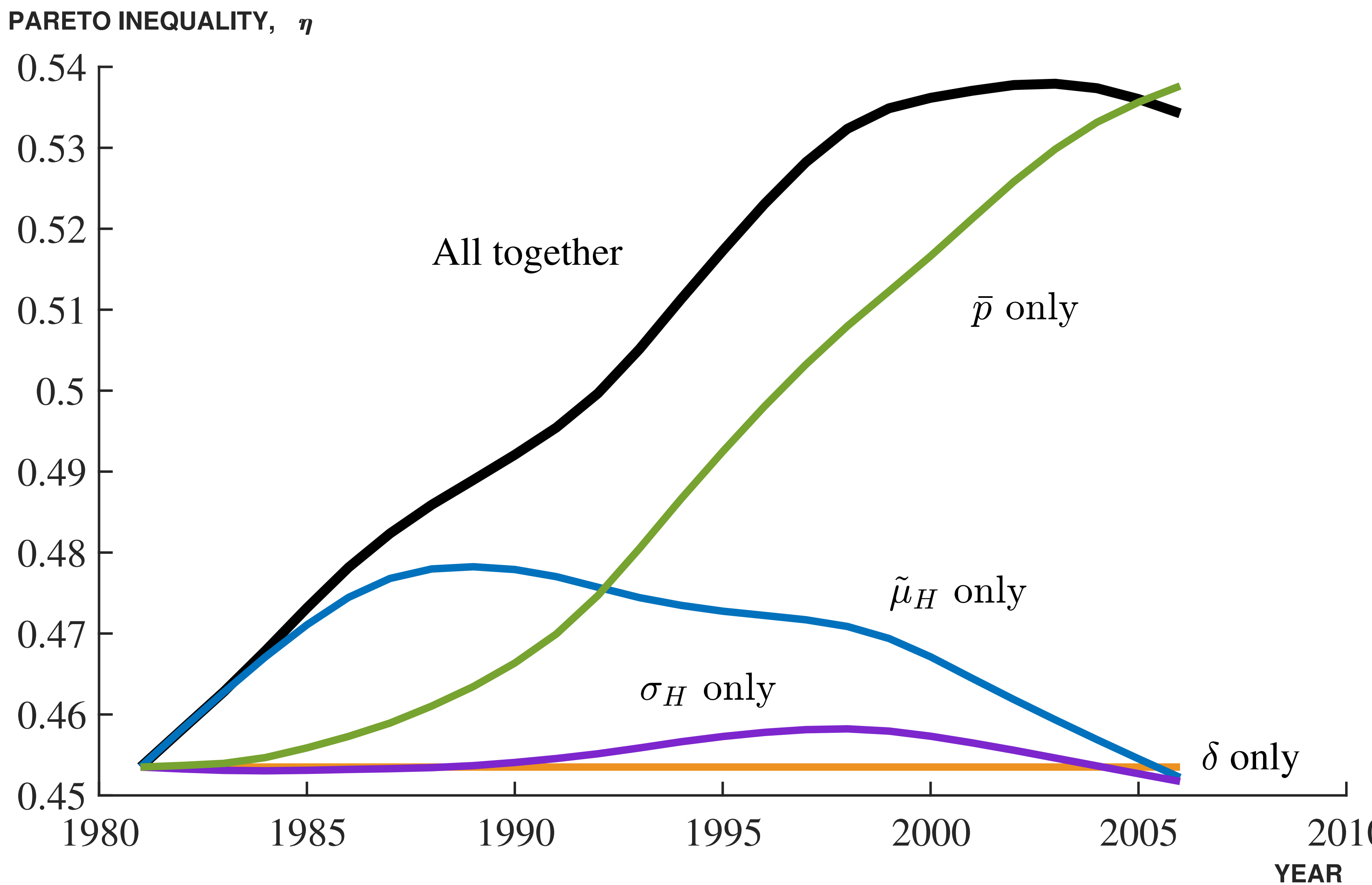
Overview

- Geometric random walk with drift = canonical DGP in the empirical literature on income dynamics.
  - – Survey by Meghir and Pistaferri (2011)
- The distribution of growth rates for the Top 10% earners
  - Guvenen, Karahan, Ozkan, Song (2015) for 1995-96
  - IRS public use panel for 1979–1990 (small sample)

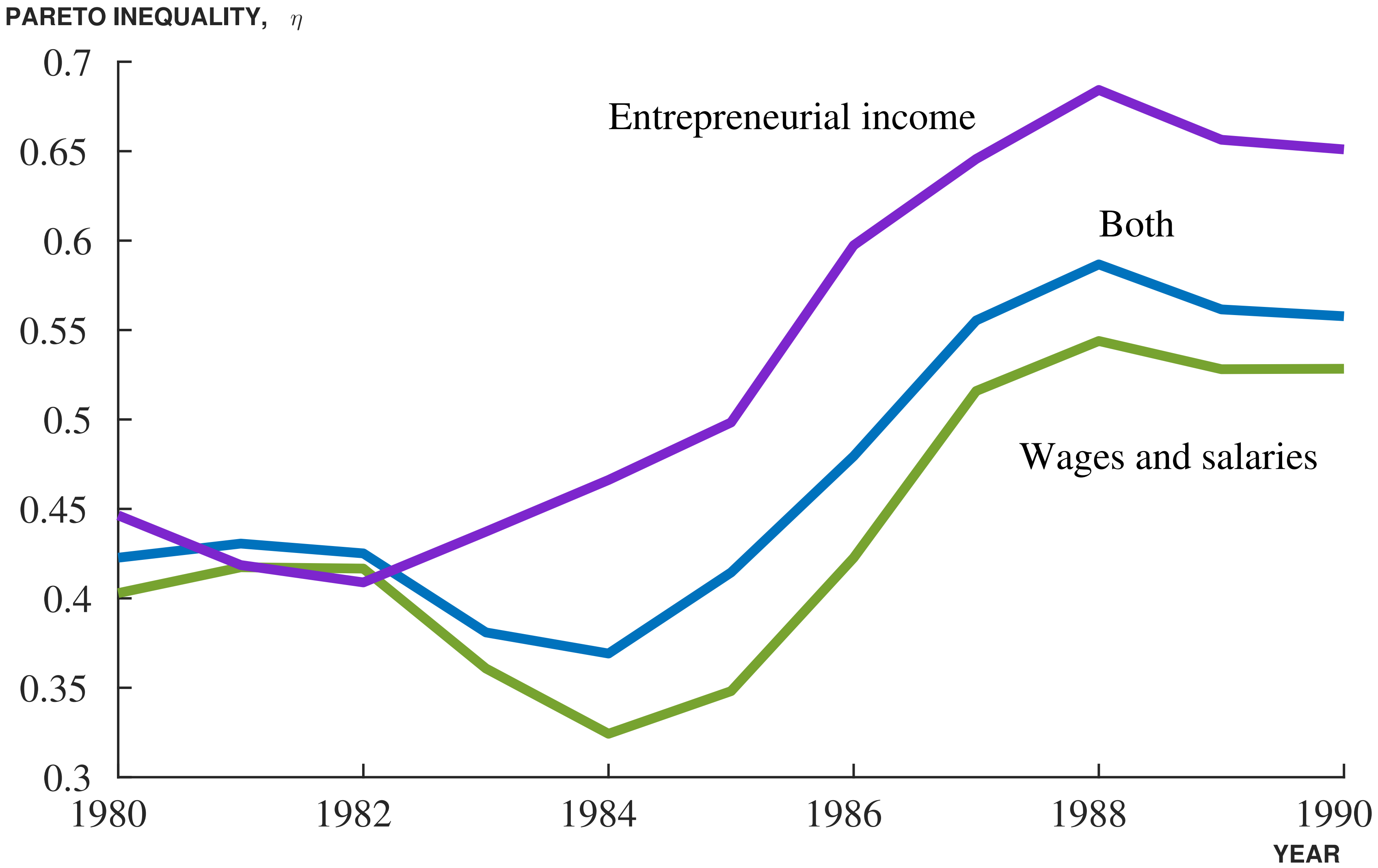
Growth Rates of Top 10% Incomes, 1995–1996



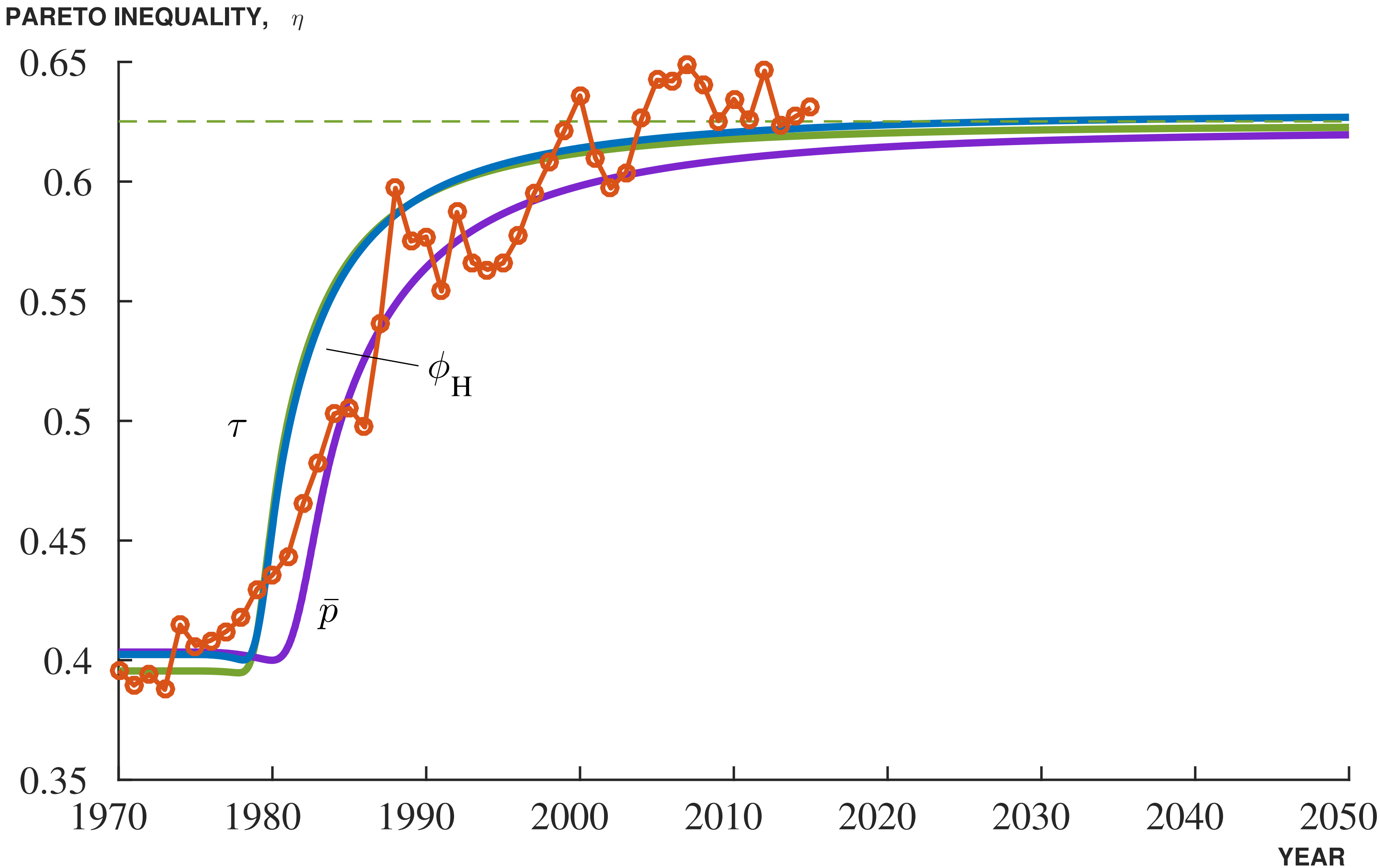
Decomposing Pareto Inequality: Social Security Data



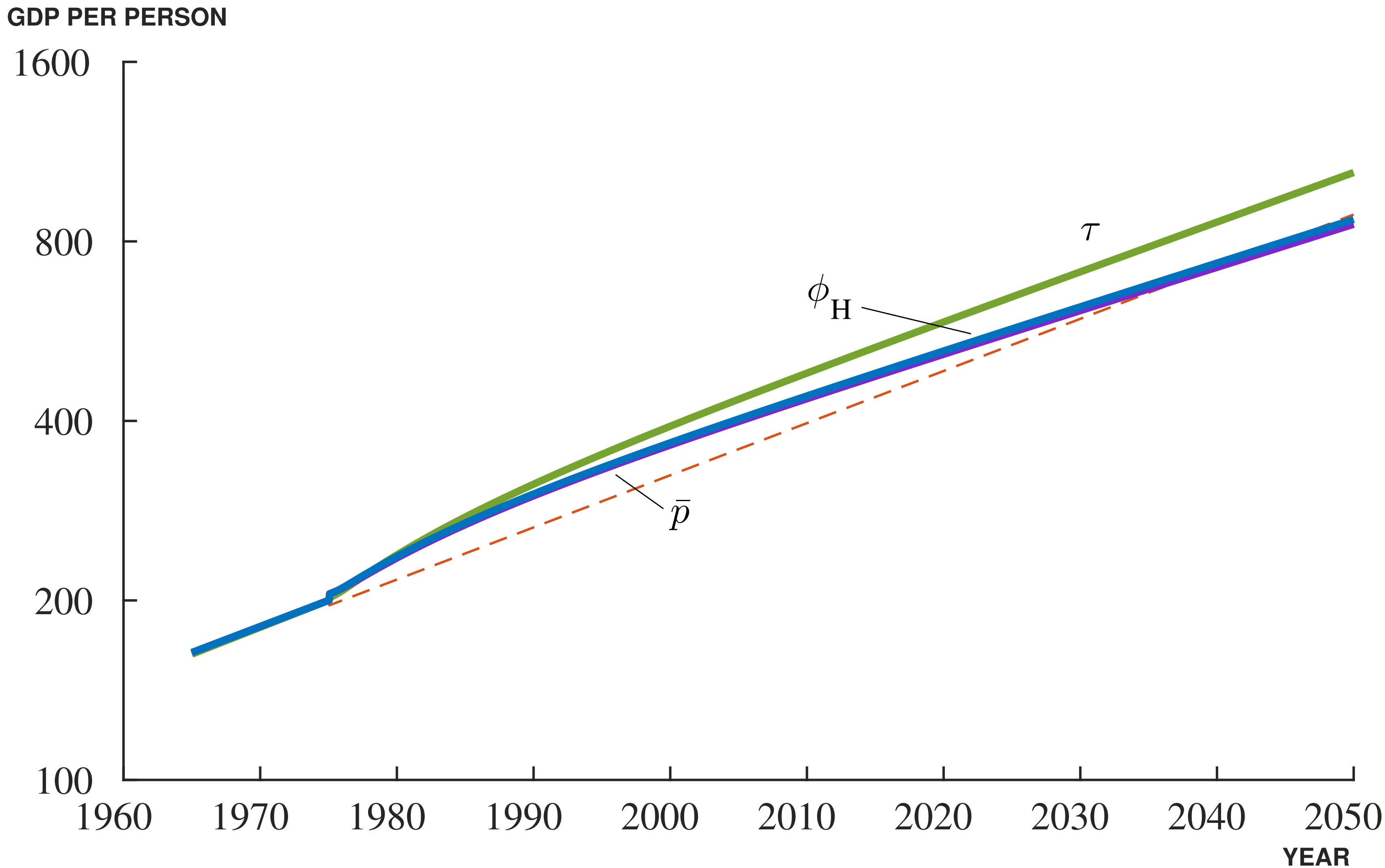
Pareto Inequality: IRS Data



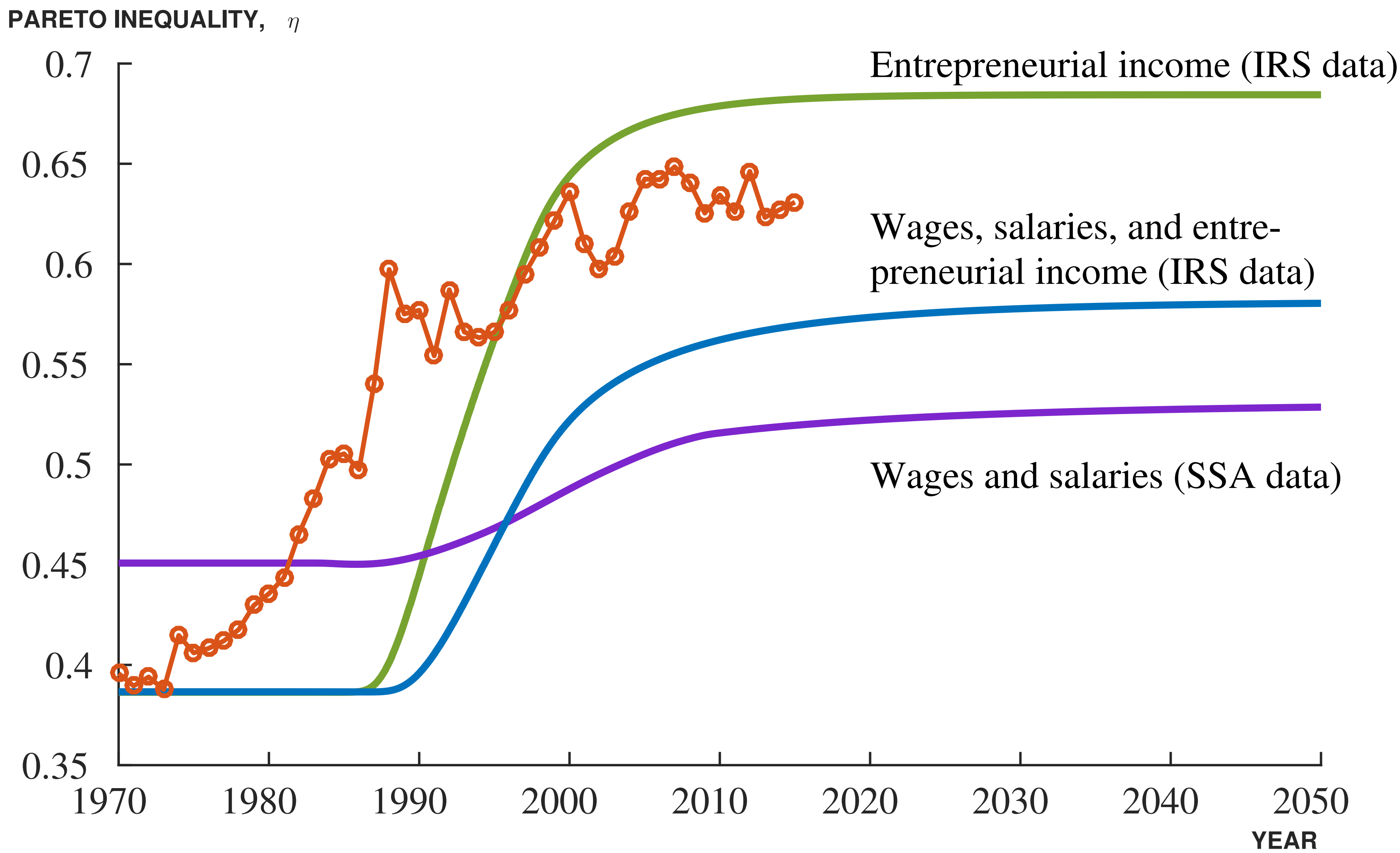
One-Time Shocks to ϕ_H, $\bar{p}$, and τ



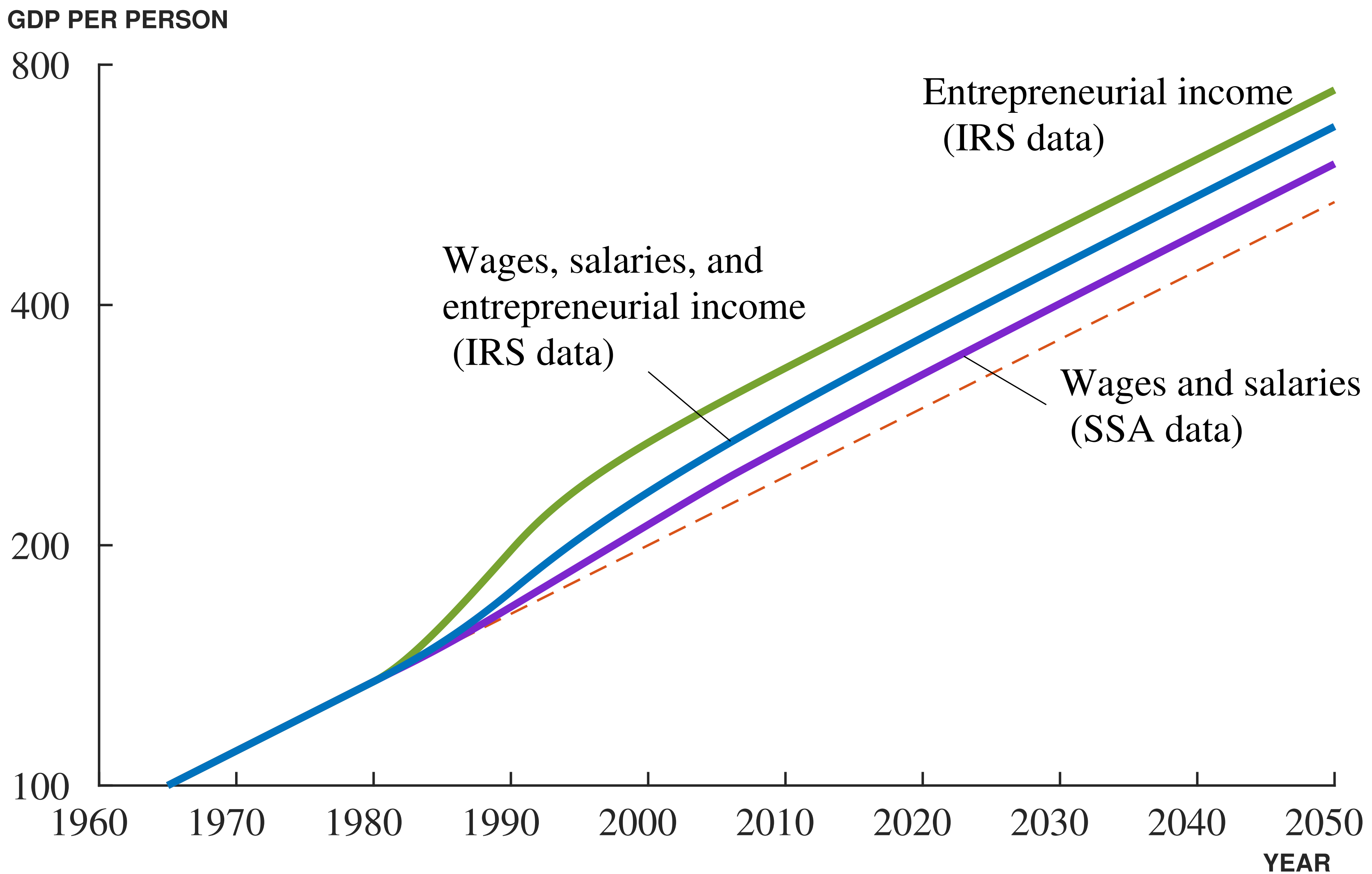
One-Time Shocks to ϕ_H, $\bar{p}$, and τ



The Dynamic Response to IRS/SSA-Inspired Shocks



The Dynamic Response to IRS/SSA-Inspired Shocks



Conclusions: Understanding top income inequality

- Information technology / WWW:
  - Entrepreneurial effort is more productive: $\uparrow \phi \Rightarrow \uparrow \eta$
  - Worldwide phenomenon (?)
- Why else might inequality rise by less in France?
  - Less innovation blocking / more research: raises creative destruction
  - Regulations limiting rapid growth: $\uparrow \bar{p}$ and $\downarrow \phi$

Theory suggests rich connections between:

models of top inequality $\leftrightarrow$ micro data on income dynamics