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Too Much of a Good Thing? The Economics of Investment in R&D

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Research and development is a key determinant of long-run productivity and welfare. A central issue is whether a decentralized economy undertakes too little or too much R&D. We develop an endogenous growth model that incorporates parametrically four important distortions to R&D: the surplus appropriability problem, knowledge spillovers, creative destruction, and duplication externalities. Calibrating the model, we find that the decentralized economy typically underinvests in R&D relative to what is socially optimal. The only exceptions to this conclusion occur when the duplication externality is strong and the equilibrium real interest rate is simultaneously high.

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1. Introduction

Research and development is a key long-run determinant of productivity and consumer welfare. But how much R&D is enough? Much of the theoretical literature has focused on reasons—such as monopoly pricing and the public good nature of knowledge—why there may be too little private R&D. However, there are equally compelling reasons, related to the distorted incentives from patent races and the transfer of rents through creative destruction, to think that there may be too much R&D.[1] In the end, the question as to the sign and magnitude of the deviation of R&D investment from its socially optimal level is an empirical one that can be approached from a number of directions.

Jones and Williams (1998) show how to interpret existing econometric estimates of returns to R&D in the context of endogenous growth models and find that optimal R&D investment is at least four times greater than actual spending. This approach yields a clear answer as to the degree of over- or underinvestment in R&D, but it does not tell us which factors determine the overall result, the knowledge of which may have important policy implications. Furthermore, the answer provided by this method relies on empirical estimates of the return to R&D, which, given the econometric difficulties associated with this undertaking, make an independent check on the results particularly valuable.

In this article, we take a different approach and use a calibrated endogenous growth model to examine the issue of over- and underinvestment in R&D.[2] Existing models in the new growth literature are inadequate for examining this question, either because they focus on a limited set of distortions or because they generate results that are contradicted by empirical evidence. We develop an R&D-based growth model that both incorporates four key distortions to the allocation of resources to R&D and is broadly consistent with empirical evidence on monopoly pricing power and the long-run relationship between R&D and productivity growth.

Two distortions present in the model promote underinvestment in R&D. First and foremost, innovators in a decentralized economy are not able to appropriate the entire "consumer surplus" associated with the good that they create. A markup of price over cost distorts sales downward from the optimum level that would occur if the good were sold at its marginal cost of production. Given the importance of this *surplus appropriability problem*, we employ a generalized specification of production that severs the overly restrictive links between the markup, the capital share, and the consumer surplus otherwise inherent in the standard formulation found in the literature. The second distortion promoting underinvestment is related to the notion that present researchers "stand on the shoulders" of past researchers; that is, part of the output of R&D is knowledge that contributes to the capacity to innovate.

On the other side of the ledger, two distortions in the model promote overinvestment in R&D. One is a negative duplication externality that we refer to as the *stepping on toes effect*. This may arise out of patent races, in which multiple firms run parallel research programs in the hope of being the first to succeed at creating and patenting a new good or process. Holding other factors constant, an increase in R&D effort induces increased duplication that reduces the average productivity of R&D in the economy. Second is the effect of the redistribution of rents from past innovators to current innovators through a process of *creative destruction*. A new good may functionally replace an existing good, causing the current innovator to receive the entire flow of rents while the past innovator gets cut out. On their own, duplication externalities and creative destruction each raise the private return to R&D above the social return.

We calibrate the parameters of the model using both macro and micro data to examine the net effect of the four distortions on investment in R&D. Our central finding, robust to reasonable changes in parameter values, is that in the absence of taxes and subsidies, the decentralized economy underinvests in R&D, with the primary impetus coming from the surplus appropriability problem. The only exceptions to this conclusion occur when there is both a powerful duplication externality and a relatively high equilibrium real interest rate. Overall, there appears to be too little private R&D in our calibrated model. Moreover, because we omit other distortions highlighted by the literature such as imitation and capital market imperfections due to asymmetric information, we believe we have biased the case in favor of overinvestment.[3]

The remainder of the article is organized as follows. We begin in Section 2 by developing a general R&D-based growth model that incorporates a number of distortions to the private incentive to undertake R&D. Section 3 describes the calibration of the model and analyzes the direction and magnitude of the deviation of the allocation of resources from the optimal allocation. Section 4 concludes.

2. The Model

The main building block of the model developed in this section is the variety-based model due to Romer (1990). The basic structure will be familiar from this and later work, so we will pause only to highlight the key extensions in our setup. The main modifications pertain to our incorporation of four determinants of over and underinvestment in R&D: (1) degree of surplus appropriability, (2) creative destruction, (3) knowledge spillovers, and (4) duplication externalities.

2.1. The Final-Goods Sector

A final-goods sector produces a homogenous output good Y_t by using a variety of intermediate capital goods x_{it}, where i indexes the capital good and t denotes time, according to the CES technology

$$Y_t = L_t^{\alpha}\left(\sum_{i=1}^{\infty} x_{it}^{\rho(1-\alpha)}\right)^{\frac{1}{\rho}}. \tag{1}$$

At any given point in time, production and trade in intermediate goods is limited to the set of goods that have been successfully designed in the past, measured by A_t. Given A_t, the production function for final goods exhibits constant returns to scale (to capital and labor inputs). Profit maximization by the final-goods sector then yields a demand curve for intermediate good i defined by

$$q_{it} = (1-\alpha)L_t^{\alpha}\left(\sum_{i=1}^{A_t} x_{it}^{\rho(1-\alpha)}\right)^{\frac{1}{\rho}-1} x_{it}^{\rho(1-\alpha)-1}, \tag{2}$$

where q_{it} is the price of capital good i.[4]

The area under this demand curve, down to the point where the price is equal to the marginal cost of production, is a “consumer” surplus that is available to society for the production of goods when a new design is invested. The inventor of a new design captures only part of this surplus: the private incentive to innovate is, all else equal, less than the gain to society. We refer to this as the *surplus appropriability problem*.

2.2. The Intermediate-Goods Sector and Innovation Clusters

Capital good x_{it} is produced by an intermediate-goods firm that owns an infinitely lived patent for the design of the good. Once the patent is purchased from the R&D sector, we assume that units of raw capital can be costlessly transformed one for one into capital goods. The intermediate goods firm acts as a monopolist and charges a gross markup η_{it} over marginal cost, so that

$$q_{it} = \eta_{it} r_t, \tag{3}$$

where r_t is the real interest rate (cost of capital).

First, consider the case where an intermediate goods firm is unconstrained by competitors offering an equivalent product. In this case, profit-maximizing firms act as monopolists, taking other prices as given. Assuming the number of firms is large, firms choose a markup determined by the elasticity of substitution between intermediate capital goods:

$$\eta_{it}^{U} = \eta^{U} \equiv \frac{1}{\rho(1-\alpha)}. \tag{4}$$

In Romer (1990), ρ is equal to unity so that the monopoly markup is equal to the inverse of the capital share. Empirically, this implies a gross markup (the ratio of price to marginal cost) of approximately 3, sharply exceeding empirical estimates of 1.05 to 1.4.[5] For there to be meaningful discussion of the issue of inefficient allocation of resources to R&D, an R&D-based growth model must conform to empirically supported markups. The more general CES production function above severs the overly restrictive link between the markup and the capital share. In addition, our modeling of creative destruction relaxes the assumption that firms are always unconstrained monopolists in setting prices, as described next. This breaks the link between the equilibrium markup and the "consumer" surplus implied by the creation of a new good.[6]

Now consider the case where firms may face competition from equivalent goods. Here we depart from existing models by introducing creative destruction into a variety model. In doing so we not only introduce an additional incentive for overinvestment in R&D into the model, but we also modify the nature of competition for investment goods. This has the effect of freeing the equilibrium markup from the elasticity of substitution between capital goods.

To introduce creative destruction, we assume that designs or ideas are linked together in *innovation clusters*.[7] An innovation cluster has two key features. First, only a fraction of the cluster, defined as $\frac{1}{1+\psi}$, actually increases the variety of intermediate goods. The remaining fraction $\frac{\psi}{1+\psi}$ consists of "upgrades" that replace existing intermediate goods. Second, all of the goods in the innovation cluster must be used together; the addition to variety cannot be separated from the upgrades.

An analogy may be helpful. Consider the invention of a computer technology that can control all functions of an automobile engine to optimize performance. Use of this innovation cluster increases the output of services from the car for given inputs: it increases productivity. In order for this device to be useful, a subset of engine parts must be redesigned or modified to allow for computer-control. The end function of these modified parts is identical to the existing parts; they differ only in the ability to be computer controlled. In designing an engine system, one has the choice of using the computer controller with computer-controlled parts or not using the computer at all and using existing parts.

The creation of an innovation cluster leads to a choice for final-goods producers. They can either (1) ignore the innovation and continue using the existing technology or (2) take advantage of the productivity improvement associated with the innovation by purchasing quantities of all goods in the new-innovation cluster. We focus on the equilibrium in which the final-goods sector always chooses to adopt the new technology. Competition from producers of existing goods, which can at best price at marginal cost, leads to an upper bound (corresponding to a limit price) on the markup that can be charged for goods in an innovation cluster. Let η_{it}^C be this maximum markup.[8] Then, the *adoption constraint* is

$$\eta_{it} \leq \eta_{it}^C \Longrightarrow \eta_{it} = \min(\eta_{it}^U, \eta_{it}^C). \tag{5}$$

As shown in Appendix A, the constrained markup is given by

$$\eta_{it}^C = \eta^C \equiv \left(\frac{1+\psi}{\psi}\right)^{\frac{1}{\rho(1-\alpha)}-1}. \tag{6}$$

An increase in the size of innovation clusters (larger ψ) reduces the constrained markup, as does a higher elasticity of substitution between capital goods (higher values of $\rho(1-\alpha)$). Whatever the value of the markup, all goods will sell for the same price, and the profit flow to the ith intermediate firm is given by

$$\pi_{it} = \pi \equiv \left(\frac{\eta-1}{\eta}\right)(1-\alpha)\frac{Y_t}{A_t}. \tag{7}$$

2.3. The R&D Sector

R&D constitutes the search for new designs for intermediate goods. A research firm rents labor services (scientists and engineers) and capital inputs (laboratory equipment) to search for new designs. It simplifies the model considerably if we assume that capital goods and labor combine to produce new designs in exactly the same way that they combine to produce output.[9] We also assume the variety of intermediate goods is large so that the summation in the production function can be replaced with an integral. We assume that R&D generates new designs according to

$$(1+\psi)\dot{A}_t = \tilde{\delta}_t R_t = \delta R_t^{\lambda} A_t^{\phi}, \quad \lambda \in (0, 1], \tag{8}$$

where R_t is the amount of output devoted to R&D and $\dot{A}_t$ is the derivative of A_t with respect to t; time is continuous. Researchers take the productivity of R&D, $\tilde{\delta}_t$, as given. From society's standpoint, however, the productivity of R&D may vary with the amount of R&D expenditure and the stock of ideas.

Several aspects of the R&D equation deserve mention. In its simplest form where $\psi = 0$, $\lambda = 1$, and $\phi = 0$, the number of newly discovered designs $\dot{A}_t$ is proportional to the amount of R&D undertaken. We depart from this simple specification in three ways. First, the presence of the ψ term reflects the existence of innovation clusters: for each increase in variety of measure one, existing goods of measure ψ are replaced by "upgrades." From the standpoint of the private inventor, profits are earned on each good in the innovation cluster. However, as we will see, from society's standpoint, only the fraction $\frac{1}{1+\psi}$ that increases variety contributes to welfare.

Second, $\lambda \leq 1$ captures duplication externalities in R&D, which we refer to as the *stepping on toes effect*. At any point in time, an increase in R&D effort induces duplication that reduces the average productivity of R&D. Notice that this duplication may be accidental (unknown to the other, two independent researchers simultaneously discover the same improvement in inkjet printing), or it may be intentional as in a patent race (because of patent protection or first-mover advantages, researchers race to be the first to discover and produce a more powerful computer chip). In either case, in equilibrium private costs are equated to the expected payoff of research that exceeds the marginal social benefits.

Finally, $\phi \neq 0$ allows for "standing on shoulders," an intertemporal knowledge spillover, as well as for diminishing technological opportunities. This effect is assumed to be external to the private agents—that is, firms cannot appropriate the value of the knowledge spillovers from future researchers. At the same time, it is possible that the advance of science and technology is partially stymied by diminishing technological opportunities (Evenson, 1984; Kortun, 1993). It may simply be getting harder to make incremental gains in productivity-enhancing technology. In this case, the "productivity" of R&D is negatively related to the level of technology: ϕ is reduced (and negative if diminishing technological opportunities outweigh knowledge spillovers).

From the point of view of atomistic R&D firms, there are constant returns to R&D, with the firm's perceived marginal product equal to the average product over all R&D firms. Free entry into R&D drives profits to zero. Letting P_{At} denote the market value of a new design, this zero profit condition is

$$P_{At}(1+\psi)\dot{A}_t = R_t \Longrightarrow P_{At} = \frac{s_t Y_t}{g_{At} A_t (1+\psi)}, \tag{9}$$

where g_{At} denotes the growth rate of A_t and $s_t \equiv \frac{R_t}{Y_t}$.

One can now see how creative destruction operates. As in Aghion and Howitt (1992) and Grossman and Helpman (1991), new goods effectively replace old goods, and the profit stream that previously accrued to the inventor of the old good is captured by the inventor of the new good. Creative destruction has two effects on the private incentive to undertake R&D, which we can think of as a carrot and a stick. The carrot is seen in equation (9): firms earn profits even on goods that do not constitute an addition to variety. This "business stealing" effect provides an incentive for overinvestment.

To see the stick associated with creative destruction, notice that the value of a new design P_{At} is equal to the present discounted value of the profits that it commands through the rental of intermediate goods of that type. The arbitrage equation, equating the real flow cost of a design to the sum of the flow of profits, π_t, capital gains due to an increase in the market value of a design, and the expected (negative) capital gain due to creative destruction, is given by

$$r_t P_{At} = \pi_t + \dot{P}_{At} - \psi \frac{\dot{A}_t}{A_t} P_{At}. \tag{10}$$

The third term on the right-hand side of the equation gives the expected capital loss due to creative destruction—the "stick" in our previous analogy, which represents the part of creative destruction that promotes underinvestment. Given the creation of $\dot{A}_t$ new designs, $\psi \dot{A}_t$ existing goods are replaced among the existing set of A_t goods types. The probability of any existing good being replaced at any given point in time is thus $\psi \frac{\dot{A}_t}{A_t}$, and the expected capital loss is $\psi \frac{\dot{A}_t}{A_t} P_{At}$. As shown below, the carrot effect dominates the stick effect, making the net impact of creative destruction one of overinvestment in R&D.

2.4. Steady-State Properties

For the remainder of the article we focus exclusively on the properties of steady state, or balanced-growth path, of the model. We close the model by assuming that the labor supply is exogenous and growing at rate n, the market for labor clears with the wage equal to the marginal product of labor, the demand for capital exhausts the supply, and consumers allocate consumption according to the usual Euler equation (with constant intertemporal elasticity of substitution $\frac{1}{\gamma}$) and dynamic budget constraint. These conditions and the definition of an equilibrium for this economy are discussed further in Appendix B.

As shown in Appendix B, the steady-state growth rate of A_t is constant and given by

$$g_A = \frac{\lambda n}{1 - \phi - \lambda\sigma/\alpha}, \tag{11}$$

where $\sigma \equiv \frac{1}{\rho} - (1 - \alpha)$.[10] The steady-state growth rate of output is given by $g_Y = \frac{\sigma}{\alpha} g_A + n$. The steady-state R&D share for the decentralized economy, s^{DC}, is given by

$$s^{DC} = \frac{\frac{\eta-1}{\eta}(1-\alpha)(1+\psi)g_A}{r - g_Y + (1+\psi)g_A}. \tag{12}$$

This equation relates the steady-state share of investment in R&D to its steady-state rate of return r. Because physical capital and R&D investment both represent foregone consumption, the steady-state rate of return to R&D is equal to that for capital and is given by the Euler equation for consumption: $r = \theta + n + \gamma(g_Y - n)$, where θ is the rate of time preference. Recall that the steady-state values of r, g_Y, and g_A are determined by model parameters outside this equation—that is, by α, ϕ, θ, λ, ρ, and n. Therefore, equation (12) gives the equilibrium R&D share as a function of model parameters. To ease comparison with the socially optimal allocation of resources to R&D, we do not make the additional substitutions.

The intuition underlying equation (12) is simply the zero profit condition for the R&D sector, as can be seen by multiplying both sides of the equation by Y_t. The numerator is the one-period flow of new profits that results from research. The denominator is the relevant discount rate that converts this one-period flow into a present discounted value: private agents discount the future flow at rate $r + \psi g_A$ to reflect the probability that the profit flow will be curtailed by creative destruction, and the profit flow itself grows over time at rate $g_Y - g_A$. The equation then states that the cost of doing R&D, R_t, is equal to the present discounted value of profits that will accrue.

In the decentralized equilibrium steady state, the R&D share is increasing in the markup. A higher equilibrium markup increases the flow of profits to R&D, raising R&D investment. The steady-state share of R&D is also increasing in the steady-state growth rate. Finally, the effect of creative destruction on private R&D investment is apparent from equation (12). The ψ term in the numerator is the carrot, and the ψ term in the denominator is the stick, as discussed earlier. Dividing both the numerator and the denominator by $1 + \psi$, one sees that an increase in the size of innovation clusters raises the R&D share provided that the interest rate is greater than the growth rate of the economy.[11]

2.5. *The Social Planner's Problem and the Social Return to R&D*

How does this decentralized solution compare to the socially optimal allocation of resources? The answer can be obtained by solving the following social planner problem for the economy:

$$\max_{\{C_t, R_t\}} \int_0^\infty u(C_t/L_t)e^{-\theta t}\, dt \tag{13}$$

subject to

$$\begin{aligned} C_t + I_t + R_t = Y_t &= A_t^\sigma K_t^{1-\alpha} L_t^\alpha, \\ \dot{K}_t &= I_t, \quad K_0 > 0 \\ \dot{A}_t &= \frac{\delta}{1+\psi} R_t^\lambda A_t^\phi, \; A_0 > 0 \\ \dot{L}_t/L_t &= n, \quad L_0 > 0. \end{aligned}$$

Applying standard methods yields the optimal allocation of resources to R&D in steady state:

$$s^{SP} = \frac{\lambda \sigma g_A}{r - (g_Y - g_A) - \phi g_A}, \tag{14}$$

where, because the steady-state growth rates are the same in the decentralized and social planner economies, r is also the same. The optimal amount of R&D requires that the marginal cost of doing a unit of R&D equals the marginal benefit, which includes the effects of the duplication externality and knowledge spillovers.

Comparison of equation (14) to equation (12) demonstrates algebraically the four wedges between the equilibrium and optimal allocations of R&D. The surplus appropriability problem is reflected in the numerators of these equations: inventors appropriate the profit flow $\pi_t = \frac{\eta-1}{\eta}(1-\alpha)\frac{Y_t}{A_t}$ while the gain to society is given by $\frac{\partial Y_t}{\partial A_t} = \sigma \frac{Y_t}{A_t}$. The ratio of these two terms is less than one, promoting underinvestment. The *standing on shoulders effect* is associated with $\phi > 0$. The presence of $\lambda < 1$ represents the *stepping on toes effect* associated with duplication externalities. Finally, the *creative destruction effect* associated with ψ promotes overinvestment. Individual agents undertake R&D based on the profits they can earn rather than based on the social surplus created. Note that the social planner's allocation of R&D does not depend on features of the market economy such as the markup or the creative destruction parameter.

3. Results from the Calibrated Model

We now attempt to quantify the degree of under- or overinvestment in R&D by choosing values for model parameters consistent with the micro and macro empirical evidence. This exercise is in the tradition of Stokey (1995). An important difference from that work relates

Table 1. Fundamental parameter values.

	Parameter	Value/Range
Labor share	α	.6400
TFP growth	σg_A	.0125
R&D growth	g_R	.0347
Labor force growth	n	.0144
Interest rate	r	.04–.14
Substitution parameter	ρ	.50–2.77
E (Life of designs)	τ	10 years

Notes: The labor share is taken from Kydland and Prescott (1991). TFP and labor force growth rates are average growth rates in the private business sector over 1948 to 1997 (Bureau of Labor Statistics, U.S. Department of Labor, 1999). R&D growth is the average annual growth rate of real industry-conducted R&D for 1957 to 1997 (National Science Foundation, 1998). See the text for other parameters.

to the "stepping on toes" externality associated with λ, which Stokey leaves as a free parameter in her calibration.[12] Stokey argues, and our results concur, that obtaining estimates of this parameter value is needed to obtain clear results from the calibration. As shown below, empirical evidence on the R&D-productivity relationship provides information regarding plausible values of λ.

3.1. Calibration of the Model

Several parameters of the model have close real-world counterparts so that their calibration is straightforward. Others require a more indirect approach.

The labor share α and the growth rates of total factor productivity (TFP) σg_A, R&D expenditure g_R, and the labor force n are calibrated according to Table 1. We allow the remaining two parameters, the interest rate r and the parameter ρ, to take on a range of values and examine the robustness of the results to this range. We initially consider interest rates ranging from .04 to .14, representing one-half and twice the the average real return on the stock market for the last century of .07 (Mehra and Prescott, 1985). The interest rate here plays two roles. First, it is the relevant summary statistic that captures preferences in the model. Second, it also represents the equilibrium rate of return to R&D for the decentralized economy. For this reason, we were uncomfortable with simply calibrating this interest rate to the risk-free rate on treasury bills—that is, to some number like .01. However, as will be clear later, our results are even stronger in the case of a small r.[13] The highest value of ρ considered is 2.77, which, given $\alpha = .64$, corresponds to an infinite elasticity of substitution between intermediate goods. The smallest value of ρ that we consider is governed by the equilibrium markup that is implied. A value of .5 yields a gross markup of 1.37 over marginal cost for intermediate goods pricing. This value is at the upper end of estimates by Norrbin (1993) and Basu (1996).

The one parameter that requires further discussion is the expected lifetime of a design , τ. The presence of creative destruction means that designs have a finite useful life, at least in expectation. Recall that the probability that a design is destroyed at each instance is ψg_A. Given the assumptions of the model, the expected lifetime of a design is simply the inverse of this probability.

Estimates of the expected life of a design can be obtained in a number of different ways. One is to estimate directly the mean rate of decay of rents from patented goods. The inverse is the mean lifetime. The implied depreciation rate of monopoly rents from patented goods is estimated to be 0.25 by Pakes and Schankerman (1984), implying a mean lifetime of four years. Mansfield, Schwartz and Wagner (1981) find that 60 percent of patented innovations are imitated within four years, which is equivalent to a lifespan of five years. Caballero and Jaffe (1993) estimate a mean rate of creative destruction of 4 percent (lifespan of 25 years) using patent citation data. Their estimated rate for electrical equipment is 13 percent, implying a lifespan of eight years. We assume an intermediate value of 10 years. The results for other parameter values ranging from five years to 20 years are similar, as discussed below.

As noted, a parameter that turns out to be very important in the results that follow is λ. Stokey (1995) argues that the empirical literature does not provide much guidance in choosing a value of λ.[14] In contrast, we find that the empirical evidence on the R&D-productivity relationship does provide information regarding plausible values of λ.

Empirical estimates of returns to R&D often are obtained by regressing total factor productivity growth at the industry level on R&D-output ratios. Such estimates are reviewed in detail by Griliches (1992) and Nadiri (1993), among others. Under an R&D-productivity relationship like that given above in equation (8), Jones and Williams (1998) show that the rate of return parameter that is typically estimated in this productivity literature, denoted $\tilde{r}^{PL}$, is equal to

$$\tilde{r}^{PL} = \frac{\lambda g_{TFP}}{s},$$

where s is the steady-state ratio of R&D spending to output. This implies that

$$\lambda = \frac{\tilde{r}^{PL} s}{g_{TFP}}. \tag{15}$$

While we do not feel confident picking exact values from the literature for these parameters, plausible lower bounds are somewhat easier to choose. For example, taking a lower bound for $\tilde{r}^{PL}$ of 30 percent and a lower bound for s^{DC} of 2.2 percent, together with our estimate of g_{TFP} of 1.25 percent implies a lower bound for λ of about .5.[15] Instead of enforcing this value in our calibrations, we will present results for values of λ between zero and one. This allows our calibration to be separate from our previous results, while allowing for easy evaluation of the results at $\lambda = 0.5$ or higher.

An estimate of ϕ can be derived using the fact that TFP growth has been relatively constant over the postwar period. Log-differentiating the R&D equation in (8) yields

$$\phi = 1 - \lambda\sigma \frac{g_R}{g_{TFP}}. \tag{16}$$

Table 2. A typical calibration result, $r = .07$ and $\rho = 1.8$.

σ	η^U	η^C	η	ψ
0.196	1.543	1.294	1.294	1.648
λ	ϕ	s^{DC}	s^{SP}	s^{SP}/s^{DC}
0.25	0.864	0.066	0.065	0.986
0.50	0.729	0.066	0.111	1.668
0.75	0.593	0.066	0.144	2.169
1.00	0.457	0.066	0.170	2.552

Given values of λ and σ, this equation determines ϕ. Notice that this relationship is valid along any growth path with constant growth rates, not just in steady state. This distinction is somewhat important because the historical growth rate of R&D expenditure has been higher than the growth rate of manufacturing output.[16]

Given values for σ, λ and ϕ, the model implies a steady-state growth rate of A, as in equation (11); this in turn implies a steady-state growth rate of output per worker. The creative destruction parameter ψ is given by $1/(\tau g_A)$. The constrained and unconstrained parameter values for the markup are determined by ψ together with the other fundamental parameter values, and the equilibrium markup is the smaller of the two. This completes the calibration of all the necessary parameter values. The decentralized and optimal R&D shares are then calculated according to equations (12) and (14).

3.2. Equilibrium and Optimal R&D Investment

Table 2 reports the results from a single calibration of the model where $r = .07$ and $\rho = 1.8$. The implied value of σ, the elasticity of output with respect to the level of knowledge, is about .2. The steady-state growth rates of A and output per worker are 6.1 and 1.9 percent, respectively. This is associated with a creative destruction parameter ψ of 1.65, which implies that about 40 percent of each innovation cluster is accounted for by novel goods. Intermediate-goods firms would like to charge a price 54 percent over marginal cost, but the adoption constraint limits them to a markup of about 30 percent over marginal cost. Note that these values, together with the steady-state growth rate of output per hour are independent of the value of λ.

The value of ϕ and the optimal R&D share depend on the particular value of λ. The values of ϕ are relatively robust to the value of λ in this particular example; all are positive and greater than .4, suggesting a relatively large positive knowledge spillover. More generally, for small values of ρ, ϕ can be small and even take negative values. The R&D share chosen by the decentralized economy is invariant to λ, which is external in the decentralized model. For these parameter values, s^{DC} is equal to .066, while s^{SP} is increasing in λ and ranges from .065 to .170. For small values of λ, the decentralized economy slightly overinvests in R&D.

To get an idea of the relative magnitudes of the different effects promoting under- and overinvestment in R&D, we computed the decentralized R&D shares assuming that individual distortions were internalized in the decentralized economy. To be precise, for each case, we altered the equilibrium condition for the R&D share in the decentralized economy so that one distortion was eliminated. For example, in the case of the knowledge spillover, we assumed the research firm was able to internalize the marginal benefits to future R&D productivity resulting from its research investment. For the case of creative destruction, we assumed firms internalized the fact that only a subset of new goods are true innovations and the indirect effect on the expected lifespan of a good but did not eliminate the effect on the equilibrium markup.

In the following, the results correspond to values of λ over the range of .25 to 1. If the knowledge spillover were internalized, the R&D share would rise by 16 to 36 percent (.011 to .024)—with the size of the increase becoming larger as ϕ is larger (or λ is smaller). If the surplus appropriability problem alone were eliminated, the R&D share would increase by 140 percent, regardless of the value of λ. If the creative destruction effect alone were internalized, the R&D share would decrease by 24 percent, again regardless of λ. Finally, if the duplication externality were internalized, the R&D share would fall by $100 * (1 - \lambda)$ percent. Therefore, according to this calibration exercise, the main force promoting underinvestment is the consumer appropriability problem and the main force promoting overinvestment is the "stepping on toes" effect. These results hold more generally for other empirically supported values of the markup and rates of creative destruction discussed below.

3.3. Refining the Results on Over- and Underinvestment

To provide a broader overview of the calibration results, Figure 1 plots the ratio s^{SP}/s^{DC} against λ for three different values of ρ. The title for each subpanel reports the value of ρ together with the implied values of σ, η, and ψ; the superscript on η indicates whether the equilibrium markup η is unconstrained or constrained. The four parameter values move together systematically. Each subpanel contains one of these plots for three different values of the interest rate $r = \{.04, .07, .10\}$.

The decentralized economy only overinvests in R&D when λ is small and the interest rate is relatively high. In comparison, for the case of $\lambda = .5$ and $r = .07$, the optimal R&D share is from 1-1/2 to over three times as large as that in the decentralized economy. Small values of λ promote overinvestment because of the duplication externality. High values of r weaken the intertemporal effects in the model that encourage underinvestment (the knowledge spillover and the "stick" of creative destruction) and therefore indirectly promote overinvestment. Note that the range of η from 1.11 to 1.37 corresponds to the range of estimated markups; thus, attempts to restrict the parameter space on the basis of prior information on η are not particularly helpful here.

The value of ρ has an ambiguous effect on the sign and magnitude of over- or underinvestment in R&D. Increasing ρ reduces σ and η driving down both the social and private rates of return to R&D. On net, the former effect dominates and an increase in ρ reduces the surplus appropriability problem's contribution to s^{SP}/s^{DC}.[17] An increase in ρ also reduces the value of ψ implied by a given value of τ. The decline in ψ lowers the constrained markup and reduces the direct impetus toward overinvestment from creative destruction.

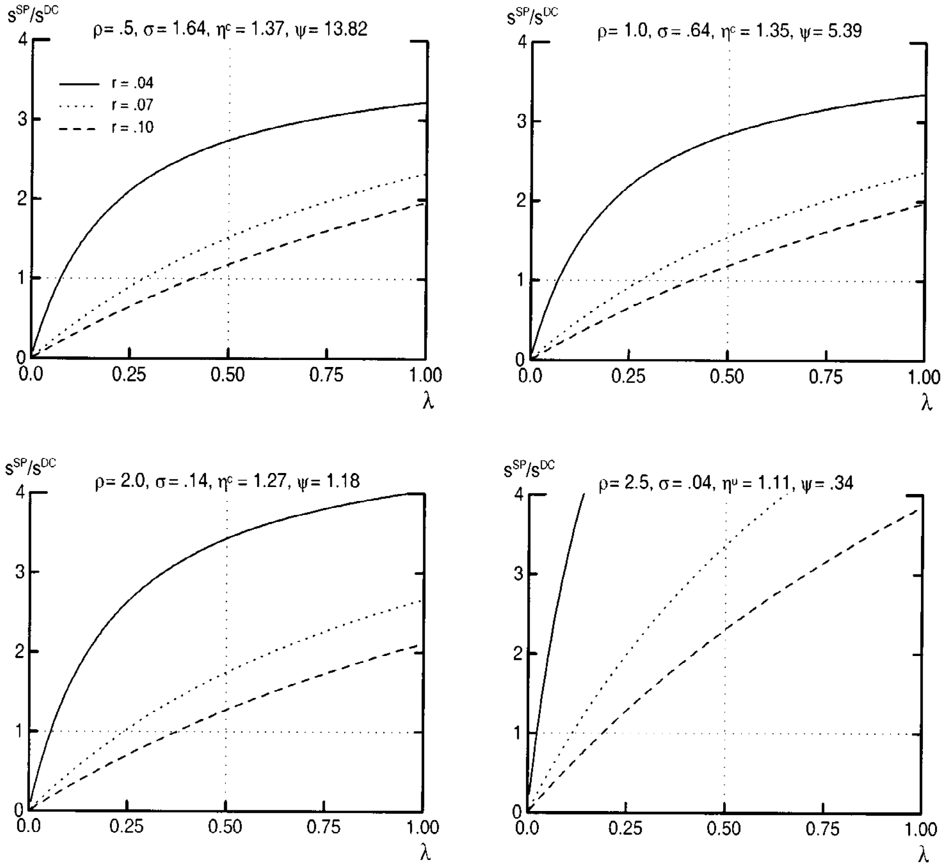

Figure 1. s^{SP}/s^{DC} for different values of ρ.

As shown in the figure, for values of ρ between .5 and 2, the effects on s^{SP}/s^{DC} of changes in ρ roughly cancel; however, for values of ρ above 2, the effect through the reduction in ψ dominates and the region of underinvestment in R&D widens considerably.

Figure 2 summarizes the model's implications for over- versus underinvestment and illustrates the robustness of the basic result that the decentralized economy underinvests in R&D unless λ is small *and* the equilibrium interest rate is high to variations in key model parameters. For three values of ρ, 0.5 (dotted line), 2.0 (dashed line), and 2.5 (solid line), we plot the combinations of the real interest rate r and λ that yield the outcome that the R&D share in the decentralized economy is socially optimal, $s^{DC} = s^{SP}$. The shaded regions indicate the values of λ and r for which $s^{DC} > s^{SP}$ for the case of $\rho = 2.5$. The upper left panel is computed using the baseline values of all fundamental model parameters and provides a benchmark for the other panels. In each of the other panels, one model parameter used in the calibration is perturbed, while the others are left at their baseline values.

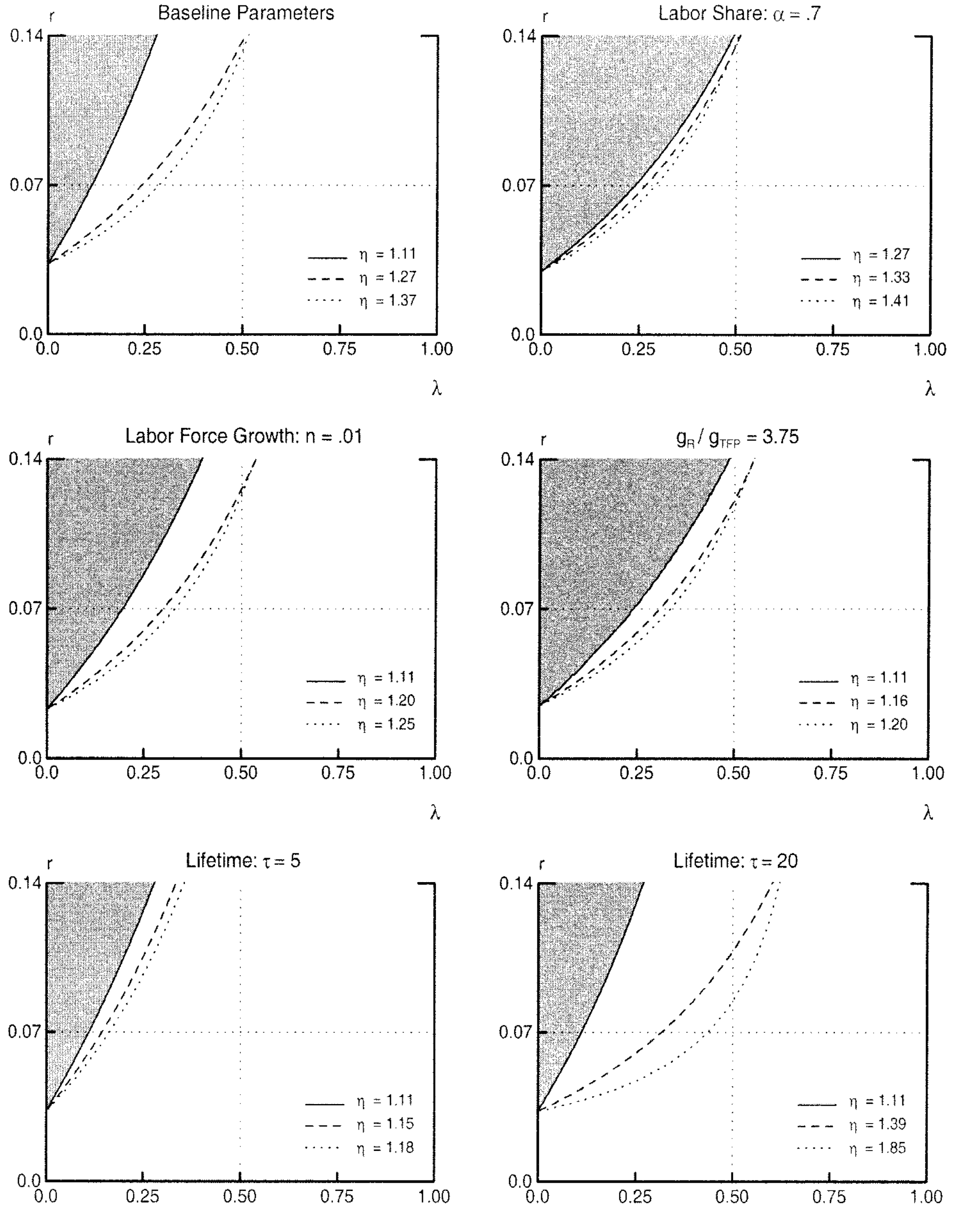

Figure 2. Cutoff values for r at which $s^{DC} = s^{SP}$. Notes: The solid line corresponds to the case of $\rho = 2.5$, the dashed line $\rho = 2.0$, and the dotted line $\rho = 0.5$. The shaded regions correspond to the values of λ and r for which $s^{DC} > s^{SP}$ for $\rho = 2.5$.

The basic results on under- and overinvestment in R&D are insensitive to reasonable changes in the assumed values of the labor share, labor force growth rate, and the ratio of the historical growth rates of R&D spending and total factor productivity. Changes in these parameters generate changes in η, ψ, and other model parameters that partially offset each other in terms of their effect on the difference between the private and social returns to R&D. Note that the value of the equilibrium markup η lies between 1.11 and 1.41—again, within the range of empirical estimates—for these four calibrations of the model.

This insensitivity to the calibration of the model does not, however, extend to the choice of the lifespan of a design, τ, as seen in the lower panels of the figure. Interestingly, lengthening the lifespan of designs—that is, reducing the rate of creative destruction,—actually *expands* the region of overinvestment. This outcome owes to the effect of ψ on the equilibrium markup: the lower ψ is, the larger the constrained markup is, and hence the greater the private return to R&D is (when the constraint is binding, as for the baseline calibration with $\rho > .5$). For example, for $\rho = .5$, increasing τ from 10 to 20 raises the equilibrium markup from 1.37 to 1.85.

An alternative approach to evaluating the robustness of the results to changes in the rate of creative destruction is to fix the equilibrium markup and compare outcomes under different values of ψ. Such an approach can be justified on the grounds that we have a prior that the markup does not exceed 1.4 but do not have strong prior beliefs regarding ψ or τ.

Figure 3 shows the results from such an experiment. For four values of ψ, $\{0, 1, 3, 9\}$, we plot the combinations of the real interest rate r and λ, which yield the outcome that the R&D share in the decentralized economy is socially optimal. Recall that $\frac{1}{1+\psi}$ percent of new designs are novel, so that the four values of ψ correspond to novel design shares of 100, 50, 25, and 10 percent, respectively. For this figure, we set $\rho = 2$ and impose a markup $\eta = 1.39$. This value of the markup equals the equilibrium markup for the case of $\psi = 0$ and lies at the upper end of the range of empirical estimates. Given a fixed markup, an increase in ψ clearly enlarges the region of overinvestment in R&D. Nevertheless, even if only 10 percent of new designs truly add to productive efficiency ($\psi = 9$, $\tau = 1.3$), the decentralized economy underinvests in R&D when $r = .07$ and $\lambda = .5$.

4. Conclusion

To what extent does the economy undertake too much or too little research? Jones and Williams (1998) provide one answer to this question by showing how estimates of returns to R&D from regressions of productivity growth on R&D as a share of output can be interpreted in the context of endogenous-growth models. This approach has the advantage that it incorporates the existing policy environment, including the patent system and subsidies to research. This approach is, however, only as good as the underlying econometric estimates. Endogeneity and measurement problems can cause the estimates to be biased.

In this article, we take a different approach. We develop a growth model that incorporates parametrically four distortions to R&D that have received attention in either the new growth literature or in the micro theory literature on patent races: the surplus appropriability problem and knowledge spillovers (both of which promote underinvestment in R&D) and creative destruction and duplication externalities (both of which promote overinvestment

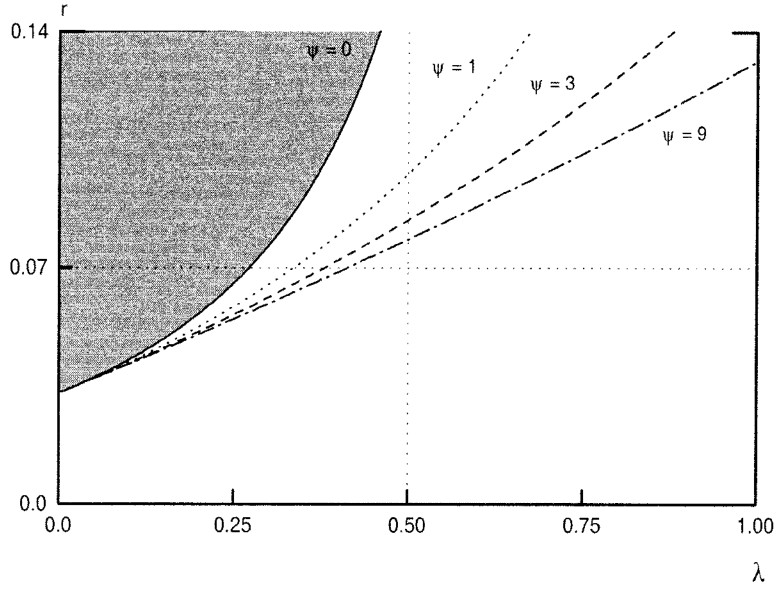

Figure 3. Cutoff values for r at which $s^{DC} = s^{SP}$, imposing $\eta = 1.39$. Note: The shaded region corresponds to the values of λ and r for which $s^{DC} > s^{SP}$ for $\psi = 0$.

in R&D). Moreover, we show that our model is consistent with the available evidence on R&D, growth, and markups.

Calibrating the model to micro and macro data, we find that our decentralized economy typically underinvests in R&D relative to what is socially optimal. This is true unless the equilibrium real interest rate is relatively high and the magnitude of duplication effects is simultaneously large.

We have abstracted from a number of features of the R&D process that could be important and that should be taken into account in future work. Some research effort appears to be intentionally designed to "invent around" existing patents and is fundamentally duplicative. In addition, the research process by its nature involves a great deal of uncertainty. Coupled with asymmetric information, this uncertainty could have a first-order effect on the allocation of resources. Finally, while our model does include some features of quality improvement, it would be interesting to examine the robustness of these results in a "quality ladders" environment, along the lines of Stokey (1995). We suspect that incorporating these features would strengthen the case for underinvestment in research.

Appendix A. Solving for the Adoption Constraint

In solving for the adoption constraint, we look for an equilibrium path along which the final goods producers always choose to adopt the latest innovation cluster. This imposes an upper bound on the markup that can be charged for the goods in the innovation cluster, and this upper bound is essentially the adoption constraint.

To calculate this upper bound, we compare the profits earned by adopting the innovation cluster versus the profits earned by purchasing the old intermediate goods at the limit price of marginal cost. Let A_t be the existing stock of designs, and suppose a new design becomes available. The production function for the final goods producer who adopts the new design is

$$Y_t = L_t^{\alpha}\left(\sum_{i=1}^{A_t+1} \tilde{x}_{it}^{\rho(1-\alpha)}\right)^{\frac{1}{\rho}}.$$

The production function for the final-goods producer who does not adopt the new design is

$$Y_t = L_t^{\alpha}\left(\sum_{i=1}^{A_t-\psi} \hat{x}_{it}^{\rho(1-\alpha)} + \sum_{i=A_t-\psi+1}^{A_t} \bar{x}_{it}^{\rho(1-\alpha)}\right)^{\frac{1}{\rho}},$$

where $\tilde{x}_{it}$ and $\hat{x}_{it}$ are intermediate goods purchased at a markup, and $\bar{x}_{it}$ denotes the ψ intermediate goods that are displaced by the innovation cluster and therefore limit priced at marginal cost.

Let η be the common markup charged on all intermediate goods asymptotically. The relative demands for intermediate goods by the final goods producer in each of these cases are given by

$$\bar{x}_{it} = \eta^{\frac{1}{1-\rho(1-\alpha)}}\hat{x}_{it}$$

and

$$\hat{x}_{it} = \zeta_t^{\frac{1}{\alpha}\frac{1-\rho}{\rho}}\tilde{x}_{it},$$

where

$$\zeta_t = 1 - \frac{1+\psi-\psi\eta^{\frac{\rho(1-\alpha)}{1-\rho(1-\alpha)}}}{A_t+1}.$$

Then, the final-goods producer will adopt the new design if and only if the profits from adopting are higher. That is,

$$L_t^{\alpha}\left[(A_t-\psi)\hat{x}_{it}^{\rho(1-\alpha)} + \psi\bar{x}_{it}^{\rho(1-\alpha)}\right]^{\frac{1}{\rho}} - (A_t-\psi)\eta r_t \hat{x}_{it}$$
$$\leq L_t^{\alpha}(A_t+1)^{\frac{1}{\rho}}\tilde{x}_{it}^{1-\alpha} - (A_t+1)\eta r_t \tilde{x}_{it}.$$

Writing this equation in terms of $\tilde{x}_{it}$ and simplifying with tedious algebra reveals that the final goods producer will adopt if and only if

$$\zeta_t \leq 1,$$

which is satisfied for all A_t if and only if

$$\eta \leq \eta^C \equiv \left(\frac{1+\psi}{\psi}\right)^{\frac{1}{\rho(1-\alpha)}-1},$$

which is the adoption constraint reported in the article.

Appendix B. Deriving the Steady-State Allocation

The additional conditions discussed in the text are given below:

$$L_t = L_0 e^{nt} \tag{17}$$

$$w_t = \alpha \frac{Y_t}{L_t} \tag{18}$$

$$\int_0^{A_t} x_{it} di = K_t \tag{19}$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\gamma}(r_t - n - \theta), \quad c_t \equiv C_t / L_t \tag{20}$$

$$\dot{K}_t = (1 - s_t) Y_t - C_t. \tag{21}$$

We can now solve for the equilibrium in the decentralized model. For our purposes, an equilibrium will be defined as a collection of prices and quantities such that the markets for capital goods, R&D input, capital, and output clear; the agents take all prices $\{P_{At}, w_t, r_t, q_t\}$ as given, except for intermediate-goods firms who set the rental rates for their own capital goods; agents maximize profits in the three sectors; and consumers choose a consumption path to maximize utility. Sequences for the 15 unknowns $\{Y_t, L_t, x_{it}, q_{it}, r_t, \eta_{it}, \eta_{it}^C, \eta_{it}^U, \pi_{it}, A_t, R_t, P_{At}, K_t, w_t, c_t\}$ are determined by equations (1) through (10) in the text and the five equations given above.

Two features of the steady state are of particular interest: the steady-state growth rate of the economy and the steady-state allocation of resources to R&D. To solve for the steady-state growth rate of the economy, rewrite the R&D equation (8) as

$$\frac{\dot{A}_t}{A_t} = \frac{\delta}{1 + \psi} \frac{R_t^{\lambda}}{A_t^{1-\phi}}. \tag{22}$$

By definition, $\dot{A}_t / A_t$ will be constant in steady state, so that the numerator and denominator of the right-hand side of this equation must grow at the same rate. Letting g_x denote the steady-state growth rate of any variable x, this implies

$$g_A = \frac{\lambda}{1 - \phi} g_R. \tag{23}$$

In steady state, the stare of output going to R&D, $s \equiv \frac{R_t}{Y_t}$, will be constant, so that R_t and Y_t will grow at the same rate. To tie down this growth rate, notice that the A_t capital goods will be produced in equal quantities so that the aggregate production function can be rewritten as

$$Y_t = A_t^{\sigma} L_t^{\alpha} K_t^{1-\alpha}, \tag{24}$$

where $\sigma \equiv \frac{1}{\rho} - (1 - \alpha)$. The constancy of the capital-output ratio then implies that

$$g_Y = \frac{\sigma}{\alpha} g_A + n. \tag{25}$$

When combined with the results from the R&D equation, this pins down the steady-state growth rate of knowledge g_A and output g_Y:

$$g_A = \frac{\lambda n}{1 - \phi - \lambda \frac{\sigma}{\alpha}} \tag{26}$$

$$g_Y = \frac{\sigma}{\alpha} g_A + n. \tag{27}$$

These last two equations show that the steady-state growth rate of per capita output is proportional to the population growth rate in this model, a result emphasized in a similar context by Jones (1995). For the assumption of a constant steady-state growth rate to hold on the balanced growth path we need $1 - \phi - \lambda \frac{\sigma}{\alpha} > 0$. If this restriction were violated, a constant R&D output share would imply an increasing growth rate over time as the labor force grows.

To solve for the share of output going to R&D, s, we first solve for the market value of a design, P_{At}, from the arbitrage equation. Differentiating the zero profit condition (9) allows us to solve for $\frac{\dot{P}_{At}}{P_{At}}$ as

$$\frac{\dot{P}_{At}}{P_{At}} = g_Y - g_A \tag{28}$$

and substituting into the arbitrage equation reveals

$$P_{At} = \frac{\pi_t}{r - (g_Y - g_A) + \psi g_A}. \tag{29}$$

The zero-profit condition says that the costs of R&D are equal to the value of R&D, $P_{At}(1+\psi)\dot{A}_t = R_t$, which gives

$$\frac{g_A A_t (1+\psi)\pi_t}{r - (g_Y - g_A) + \psi g_A} = s Y_t. \tag{30}$$

Substituting from the equation for profits and simplifying yields the steady-state R&D share for the decentralized economy, s^{DC}:

$$s^{DC} = \frac{\frac{\eta - 1}{\eta}(1-\alpha)(1+\psi) g_A}{r - g_Y + (1+\psi) g_A}. \tag{31}$$

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Notes

1. Tirole (1988) provides an overview of the relevant literature in the industrial organization literature. Romer (1990), Aghion and Howitt (1992), and Grossman and Helpman (1991) analyze this issue in the context of endogenous growth models.
2. In this, we follow Stokey (1995), who applies the same method using a different endogenous growth model, as discussed in the text below.
3. Stiglitz (1992) and Hall (1992) argue that capital-market imperfections promote underinvestment. Furthermore, in the United States at least, the net effect of the tax system, which is ignored in the present article, likely further reduces the private return to R&D (Williams, 1999).
4. We require $0 < \rho < 1/(1-\alpha)$ below, so that demand curves slope downward and the desired gross markup on intermediate goods is greater than unity.
5. See Norrbin (1993) and Basu (1996) for empirical evidence on markups.
6. The link is inherent in models with CES production and unconstrained monopoly pricing. For example, in Romer (1990), the equilibrium share of the surplus appropriated by the inventor is equal to the *inverse* of the markup. This leads to the somewhat counterintuitive result that higher markups are associated with a lower share of the surplus being captured.
7. While our notion of innovation clusters corresponds to some aspects of real-world innovation, this assumption is largely made for modeling purposes.
8. All goods in a cluster sell for the same markup. This symmetric price rule is an equilibrium. If any firm in the innovation cluster were to raise its price, the final-goods firm would switch to purchasing the old capital goods. Other equilibria for this game may well exist. Our purpose here is to provide a simple mechanism for introducing creative destruction in a variety model and to see how it affects the rate of return calculation in the productivity literature.
9. In the terminology of Rivera-Batiz and Romer (1991), we employ the "lab equipment" version of R&D instead of the "knowledge-driven" specification, in which labor is the only internalized input. The results in this article are very similar if the "knowledge-driven" specification is employed instead.
10. We require $\phi + \lambda\sigma/\alpha < 1$. Otherwise, no balanced-growth path exists, and the growth rate is increasing over time. See Jones (1995).
11. The reason for this is straightforward and has been pointed out by Barro and Sala-i-Martin (1995). In the steady state of these models, the profit earned from innovating grows at the same constant growth rate as the economy. In terms of timing, however, an innovator destroys the incumbent's profits today, while her own profits will not be destroyed until some point in the future. If the interest rate exceeds the growth rate, the profit destruction effect dominates. This condition is required by the transversality condition of the consumer maximization problem.
12. Two other differences are worth noting. First, Stokey's approach is based on a quality ladders model that contains parameters (such as a fixed quality step size) that are empirically difficult to interpret and calibrate. Second, Stokey's model exhibits intertemporal scale effects of the kind criticized by Jones (1995): adding population growth to her model results in accelerating growth rates.
13. An additional restriction is $r > g_Y$. For our parameter values, g_Y is .033.
14. One exception is Kortum (1992), who finds values for a parameter like λ of about 0.2. However, this estimate can be criticized on two grounds. First, the framework is somewhat different from ours, so it is not clear that the estimate is valid. Second, the estimate is obtained using patent data. The puzzling aggregate time series properties for patents (that is, the lack of a clear trend during most of the twentieth century) also suggest problems interpreting the estimates.
15. The number 2.2 percent comes from the average value of the ratio of industry R&D to private business output for 1957 to 1997. The corresponding number for total (that is, including government) R&D is 3.1 percent. It is plausible to think of these numbers as respective lower bounds because a substantial amount of what economists would call R&D is probably not counted as R&D in the collected statistics.
16. Jones (1998) shows that a rising R&D share can be consistent with a constant growth rate of the aggregate variables in the intermediate run.
17. This result can be seen by noting that the partial derivative of the ratio $\frac{\sigma}{(1-\alpha)(\eta-1)/\eta}$ with respect to ρ, holding ψ constant, is negative.

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