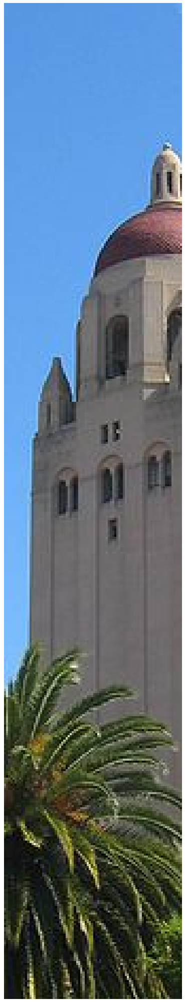

Intermediate Goods and Weak Links

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Introduction

- Huge income differences across countries — why?
- Old ideas: Leontief (1936) and Hirschman (1958)
  - Intermediate goods (linkages)
  - Weak links (complementarity)
- A model to make these ideas precise and quantify their importance

Intermediate Goods and Weak Links

- Intermediate goods
  - Another produced input, like capital $\Rightarrow$ higher multiplier
  - Examples: electricity, materials, financial services
  - electricity $\Rightarrow$ construction, banking $\Rightarrow$ electricity

- Weak links (complementarity, O-rings)
  - Intermediate goods often associated with complementarity (energy)
  - Production requires 10 things to go right
  - In poorest countries, multiple problems...
  - Problems with electricity or infrastructure or financial services can have disproportionately large effects.

Multipliers

- Why are allocations distorted? Political economy (not here)
- Why do distortions lead to large differences? This paper
- Example: Neoclassical growth model
  - Explaining why poor countries have low investment rates $\Rightarrow$ small income differences
  - You need a multiplier...
- Important related work by Ciccone (2002) and Yi (2003)

A Brief History of the Growth/Development Literature

- Capital multiplier: more $K \rightarrow$ more $Y \rightarrow$ more K, etc.
  - Multiplier is $\frac{1}{1-\alpha} = 3/2$ if $\alpha = 1/3$.
  - Mankiw, Romer, and Weil (1992): This is too small...
- Broaden capital: Need $\alpha = 2/3 \Rightarrow$ multiplier = 3.

human capital	Mankiw, Romer, and Weil (1992)
organizational capital	Chari, Kehoe, and McGrattan (1997)
ideas	Howitt (2000), Klenow and Rodriguez-Clare (2005)
human capital	Manuelli/Seshadri (07), Erosa/Koreshkova/Restuccia (09)

A Simple Example

$$Q_t = \bar{A}\left(K_t^{\alpha} L_t^{1-\alpha}\right)^{1-\sigma} X_t^{\sigma}, \quad \sigma = 1/2$$

$$K_{t+1} = \bar{s} Y_t + (1-\delta) K_t$$

$$X_{t+1} = \bar{x} Q_t, \quad Y_t \equiv Q_t (1-\bar{x})$$

- Steady State: Let $\bar{m} \equiv (1-\bar{x})^{1-\sigma} \bar{x}^{\sigma}$:

  $Y = (\bar{A}\bar{m})^{\frac{1}{1-\sigma}} K^{\alpha} L^{1-\alpha}$, and $y \equiv \frac{Y}{L} = \left(\bar{A}\bar{m}\left(\frac{\bar{s}}{\delta}\right)^{\alpha(1-\sigma)}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}}$

- Intermediate goods multiplier (with $\sigma = 1/2$):
  - Share of produced factors is $\alpha(1-\sigma) + \sigma = 2/3$
  - Multiplier is $\frac{1}{1-\sigma} \cdot \frac{1}{1-\alpha} = 2 \cdot 3/2 = 3$

Numbers in the Simple Example

- Suppose neoclassical factors (physical and human capital) contribute a factor of 4 to rich/poor income differences
- Suppose $\bar{A}$ or $\bar{m}$ differs by a factor of 2 (theft? technologies?)
- What is the income ratio y^{rich}/y^{poor}?

Neoclassical model	$\sigma = 0$	$2^{3/2} \times 4 = 11$
Intermediate goods	$\sigma = 1/2$	$2^{3} \times 4 = 32$

Complementarity: Making Socks (Kremer 1993)

- Basic inputs
  - Silk, cotton, polyester.
  - Knitting machines, how to use/repair, spare parts.
  - Competent, motivated, healthy workforce.
  - Factory structure, moving technology, electricity.
- Beyond raw materials
  - Security from expropriation/theft.
  - Matching with high-value buyers (foreign markets?)
  - Means of transport/delivery.
  - Legal requirements.
- Knowledge: How to make / motivate / repair / accounting/etc.

Great idea, not currently emphasized...

The Model

The Economic Environment

Production of Variety i $$Y_i = A_i \left(K_i^{\alpha} H_i^{1-\alpha}\right)^{1-\sigma} X_i^{\sigma}$$

Resource constraint (good i) $$c_i + z_i = Y_i$$

Final uses (substitutes) $$Y = \left(\int_0^1 c_i^{\theta} di\right)^{1/\theta}, \quad 0 < \theta < 1$$

Intermediate uses (complements) $$X = \left(\int_0^1 z_i^{\rho} di\right)^{1/\rho}, \quad \rho < 0$$

Resource constraint (X) $$\int_0^1 X_i \, di \leq X$$

A_i = exogenous productivity, σ = Linkages parameter,
θ = substitutability of final, ρ = complementarity of intermediates

Environment – continued

Resource constraint (K) $\int_0^1 K_i di \leq K$

Resource constraint (H) $\int_0^1 H_i di \leq H \equiv \bar{h}\bar{L}$

Capital accumulation $\dot{K} = I - \delta K$

Resource constraint (GDP) $C + I \leq Y$

Preferences $U = \int_0^\infty e^{-\lambda t} u(C_t) dt$

Allocating Resources

- Two ways:
  1. Symmetric: A “rule of thumb” allocation, like Solow.
  2. Competitive Equilibrium: With micro-level distortions.
- Advantages of starting with symmetric
  - Easy to solve for; delivers some key results.
  - Important benchmark for understanding CE.
- DEFINITION: The *symmetric allocation* has $K_i = K$, $H_i = H$, $X_i = X$, $I = \bar{s}Y$, and $z_i = \bar{z}Y_i$, where $0 < \bar{s}, \bar{z} < 1$.

The Symmetric Allocation

PROPOSITION 1. THE SYMMETRIC ALLOCATION: Given K units of capital, GDP is

$$Y = \phi(\bar{z})(S_\theta^{1-\sigma} S_\rho^\sigma)^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha},$$

where

$$S_\rho \equiv \left(\int_0^1 A_i^\rho di \right)^{\frac{1}{\rho}}$$

and

$$\phi(\bar{z}) \equiv ((1-\bar{z})^{1-\sigma} \bar{z}^\sigma)^{\frac{1}{1-\sigma}}$$

and S_θ is defined in a way analogous to S_ρ.

1. Substitution vs. Complementarity

$$S \equiv S_\theta^{1-\sigma} S_\rho^\sigma, \quad S_\eta \equiv \left(\int_0^1 A_i^\eta di \right)^{\frac{1}{\eta}}$$

- TFP involves both CES combinations of productivities.
  - S_θ is between geometric and arithmetic means
  - S_ρ is between geometric and minimum

  $\Rightarrow$ Weak links crucial; importance of σ.
- Example: $\theta = 1$, $\rho \to -\infty$, $\sigma = 1/2$
  - TFP $= \bar{A} \times \min\{A_i\}$.
  - Aggregate TFP is determined by the weakest link.
- U.S. and Kenya may not be so different on average but several weak links can drag down output.

2. Linkages deliver a multiplier

$$Y = S^{\frac{1}{1-\sigma}} K^{\alpha} H^{1-\alpha}$$

- TFP is the CES average raised to the power $\frac{1}{1-\sigma}$.
- Example: Suppose $Y_t = aX_t^{\sigma}$ and $X_t = sY_{t-1}$.
  - Output depends on intermediate goods
  - Intermediate goods are yesterday's output.

  Solving these two equations in steady state gives

$$Y^* = a^{1/1-\sigma} s^{\sigma/1-\sigma}.$$

- Analogous to the multiplier from capital accumulation.

The Competitive Equilibrium Allocation

- Standard CE with one key difference:
  - Each variety i producer is subject to a variety-specific distortion τ_i
- Motivated by Banerjee-Duflo (2005), CKM (2007), Restuccia-Rogerson (2008), Hsieh-Klenow (2009)
  - Misallocation at micro level $\Rightarrow$ Aggregate TFP.
  - Distortions: Theft, monopoly markups, regulations, preferential credit, taxes
- Firms produce “gross output” not “value-added”
  - It’s not only K and L that can be misallocated, but also intermediate goods.
  - Multiplied because intermediates are a produced input and because of weak links.

CE Optimization Problems

- Final Use Problem

$$\max_{\{c_i\}} \left(\int_0^1 c_i^\theta di \right)^{1/\theta} - \int_0^1 p_i c_i \; di$$

- Intermediate Use Problem

$$\max_{\{z_i\}} q \left(\int_0^1 z_i^\rho di \right)^{1/\rho} - \int_0^1 p_i z_i \; di$$

- Variety i's Problem

$$\max_{\{X_i, K_i, H_i\}} (1-\tau_i) p_i A_i \left(K_i^\alpha H_i^{1-\alpha} \right)^{1-\sigma} X_i^\sigma - (r+\delta) K_i - w H_i - q X_i .$$

Definition of Equilibrium

The *competitive equilibrium with distortions* consists of quantities and prices $\{p_i\}, q, w, r$ such that

1. Firms and households optimize (previous slide).
2. Prices clear markets.
3. Distortion revenue rebated lump sum: $T = \int_0^1 \tau_i p_i Y_i di$
4. Economic environment is respected.

Solving for the CE

PROPOSITION 2. THE COMPETITIVE EQUILIBRIUM, GIVEN CAPITAL: Given K, GDP in the competitive equilibrium is

$$Y = \psi(\tau) \left(B_\theta^{1-\sigma} B_\rho^\sigma\right)^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha},$$

where

$$B_\eta \equiv \left(\int_0^1 (A_i(1-\tau_i))^{\frac{\eta}{1-\eta}} di\right)^{\frac{1-\eta}{\eta}},$$

and

$$\psi(\tau) \equiv \frac{1-\sigma(1-\tau)}{1-\tau} \cdot \sigma^{\frac{\sigma}{1-\sigma}}$$

where $\tau \equiv T/(Y + qX)$ is an average distortion rate.

Three Remarks

1. The intermediate goods multiplier plays its usual role.
2. Wedges work through aggregate TFP.
   - As in CKM, RR, HK
   - Now they get multiplied by IG multiplier as well.
3. Change in curvature parameter in CES... (next slide)

Strengthen weak links, Favor superstars

$$B_\eta \equiv \left(\int_0^1 (A_i(1-\tau_i))^{\frac{\eta}{1-\eta}} di \right)^{\frac{1-\eta}{\eta}}$$

- Curvature parameter is $\frac{\eta}{1-\eta}$ rather than η
  - $\rho \in [0, -\infty)$ implies $\frac{\rho}{1-\rho} \in [0, -1)$
  - $\theta \in [0, 1)$ implies $\frac{\theta}{1-\theta} \in [0, \infty)$
  - A *higher* power mean
    $\Rightarrow$ Strengthen weak links, favor superstars.
- Example: $\theta = 1$, $\rho \to -\infty$, $\sigma = 1/2$, $\tau_i = 0$
  - TFP $= \max\{A_i\} \times \bar{A}$.
  - Aggregate TFP is determined by the superstar.
- Even with Leontief, other margins of substitution:
  - Resources substitute for low A_i.

The Steady State

PROPOSITION 3. THE COMPETITIVE EQUILIBRIUM IN STEADY STATE: Let $y \equiv Y/\bar{L}$. GDP per worker in SS is

$$y^* = \psi(\tau)\left(B_\theta^{1-\sigma} B_\rho^{\sigma}\right)^{\frac{1}{1-\sigma}\frac{1}{1-\alpha}} \left(\frac{\alpha(1-\sigma)}{\lambda+\delta}\right)^{\frac{\alpha}{1-\alpha}} \bar{h}.$$

- The long-run multiplier is $\frac{1}{1-\sigma}\frac{1}{1-\alpha} = \frac{1}{1-\beta}$
- Suppose we compare 2 economies with $Q^{rich} = 2 \times Q^{poor}$
- Income ratios
  - Neoclassical ($\sigma = 0$): $2^{3/2} \approx 2.8$
  - Here ($\sigma = 1/2$): $2^{2\times 3/2} = 2^3 = 8$.

Symmetric Distortions

PROPOSITION 4. SYMMETRIC DISTORTIONS: Suppose $\tau_i = \bar{\tau}$. Let $z^* \equiv \frac{qX}{Y+qX}$ and $m^* \equiv (1-z^*)^{1-\sigma}(z^*)^{\sigma}$. Then $z^* = \sigma(1-\bar{\tau})$, and

$$Y = \left(m^* \tilde{B}_{\theta}^{1-\sigma} \tilde{B}_{\rho}^{\sigma}\right)^{\frac{1}{1-\sigma}} K^{\alpha} H^{1-\alpha},$$

Also, in steady state

$$y^* = (1-\sigma(1-\bar{\tau}))\,(1-\bar{\tau})^{\frac{1}{1-\sigma}\frac{1}{1-\alpha}-1} \left(\tilde{B}_{\theta}^{1-\sigma} \tilde{B}_{\rho}^{\sigma}\right)^{\frac{1}{1-\sigma}\frac{1}{1-\alpha}} \bar{h},$$

- GDP is maximized at $\bar{\tau} = 0$ (i.e. $z^* = \sigma$).
- Why does a symmetric wedge distort? Diamond-Mirrlees/Chamley/Judd.

Another Intuition for the Multiplier

- Diamond-Mirrlees (1971), Judd (1985), Chamley (1986)
  - Taxes on intermediate goods / capital multiply up
  - Monopoly distortions would also be multiplied.
- Example: Theft
  - 1/2 of the steel gets stolen from the steel mill
  - 1/2 of the cars get stolen from the auto plant
  - 1/2 of the pizzas gets stolen from the delivery van

$\Rightarrow$ The steel effectively gets stolen 3 times!

Random Productivity and Distortions

PROPOSITION 5: Let $a_i \equiv log A_i$ and $\omega_i \equiv \log(1-\tau_i)$ be jointly normally distributed so that $a_i \sim N(\mu_a, \nu_a^2)$ and $\omega_i \sim N(\mu_\omega, \nu_\omega^2)$ and $Cov(\omega_i, a_i) = \nu_{a\omega}$. Finally, let $1-\bar{\tau} \equiv e^{\mu_w + \nu_w^2/2}$. Then

$$\log y^* = \underbrace{\log\left(\frac{1-\sigma(1-\tau)}{1-\tau}\right)}_{①} + \underbrace{\frac{1}{1-\sigma}\frac{1}{1-\alpha}\left((1-\sigma)\log B_\theta + \sigma \log B_\rho\right)}_{②} + \zeta_2$$

where

$$① = \log\left(1-\sigma(1-\bar{\tau})\exp[\eta_\rho(\nu_\omega^2 + \nu_{a\omega})]\right) - \left(\log(1-\bar{\tau}) + \eta_\theta(\nu_\omega^2 + \nu_{a\omega})\right)$$

and

$$② = \frac{1}{1-\sigma}\frac{1}{1-\alpha}\left(\mu_a + \log(1-\bar{\tau}) + \frac{1}{2}\tilde{\eta}\nu_a^2 + \tilde{\eta}\nu_{a,\omega} - \frac{1}{2}(1-\tilde{\eta})\nu_\omega^2\right)$$

where $\eta_\rho \equiv \frac{\rho}{1-\rho}$, $\eta_\theta \equiv \frac{\theta}{1-\theta}$, and $\tilde{\eta} \equiv (1-\sigma)\eta_\theta + \sigma\eta_\rho$. Moreover, given capital, $\frac{\partial \log y}{\partial \nu_\omega^2} < 0$.

Corollary

Let $\rho \to 0$ and $\theta \to 0$, and reconsider the result in Proposition 5. In this case, $\tilde{\eta} = \eta_\rho = \eta_\theta = 0$, and we are left with

$$y^* = (1 - \sigma(1 - \bar{\tau}))\,(1 - \bar{\tau})^{\frac{1}{1-\sigma}\frac{1}{1-\alpha}-1} \exp\left(-\frac{1}{2}\left(\frac{1}{1-\sigma}\cdot\frac{1}{1-\alpha}\right)\nu_\omega^2\right)\zeta_3$$

where ζ_3 is a function of terms that do not depend on the distortions.

Quantitative Exercises

Quantitative Exercises

- No good measure of distortions
  - Try many examples and check robustness
- Wealth of empirical evidence supports $\sigma = 1/2$
- Two countries: rich (undistorted) and poor (distorted)
  - Focus on multipliers
  - Even though we do not know the magnitudes of the distortions, whatever they are, they are multiplied by intermediate goods and weak links.

Intermediate Goods Share: σ

- Basu (1995) uses $\sigma = 1/2$ based on Jorgenson, Gollop, and Fraumeni (1987) U.S. average for 1947–1979.
- Chenery, Robinson, and Syrquin (1986) suggest that share rises with development
  - But Korea, Taiwan, and Japan in 1970s are all *higher* than this U.S. number, at 61% to 80%
- OECD I-O database at 1-digit level has

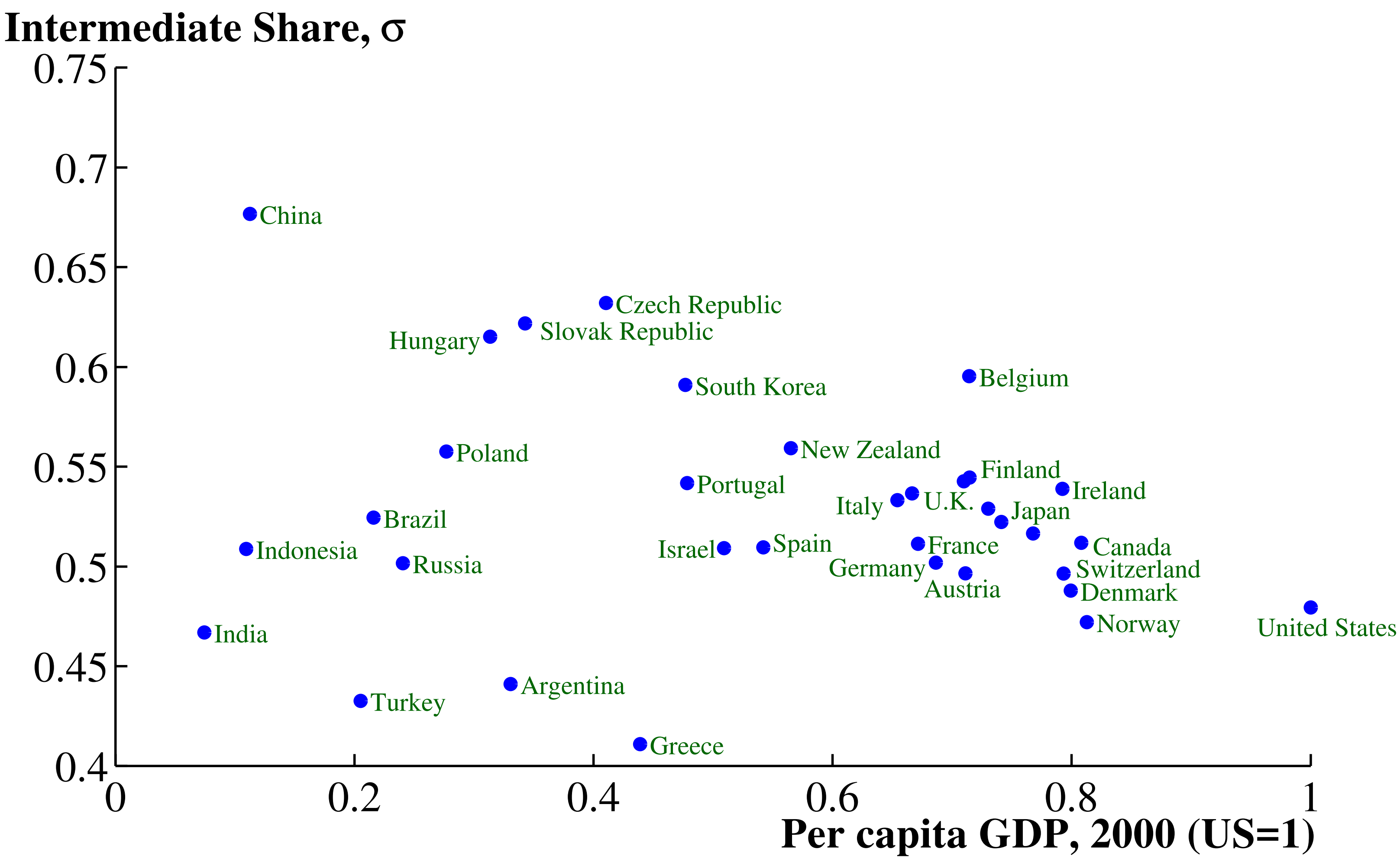
  $\sigma \approx 46\%$ for U.S., Japan, India

  $\sigma = 64\%$ for China

  Across 21 countries: mean = 52.4%, stdev = 6%.

$\Rightarrow \sigma = 1/2$ seems quite reasonable

The Intermediate Goods Share



Parameter Choices

Parameter	Value	Comment
α	1/3	Conventional value for capital share
$\bar{h}^r / \bar{h}^p$	2	Standard contribution from education
θ	2/3	Hsieh and Klenow
ρ	-1	Elasticity of substitution is 1/2
$\bar{\tau}^{poor}$	0.2	Average distortion
$\nu_a^{rich} = 0.84$, $\nu_a^{poor} = 1.23$		HK(US and India)
$\nu_w^{rich} = .45$, $\nu_w^{poor} = .68$		

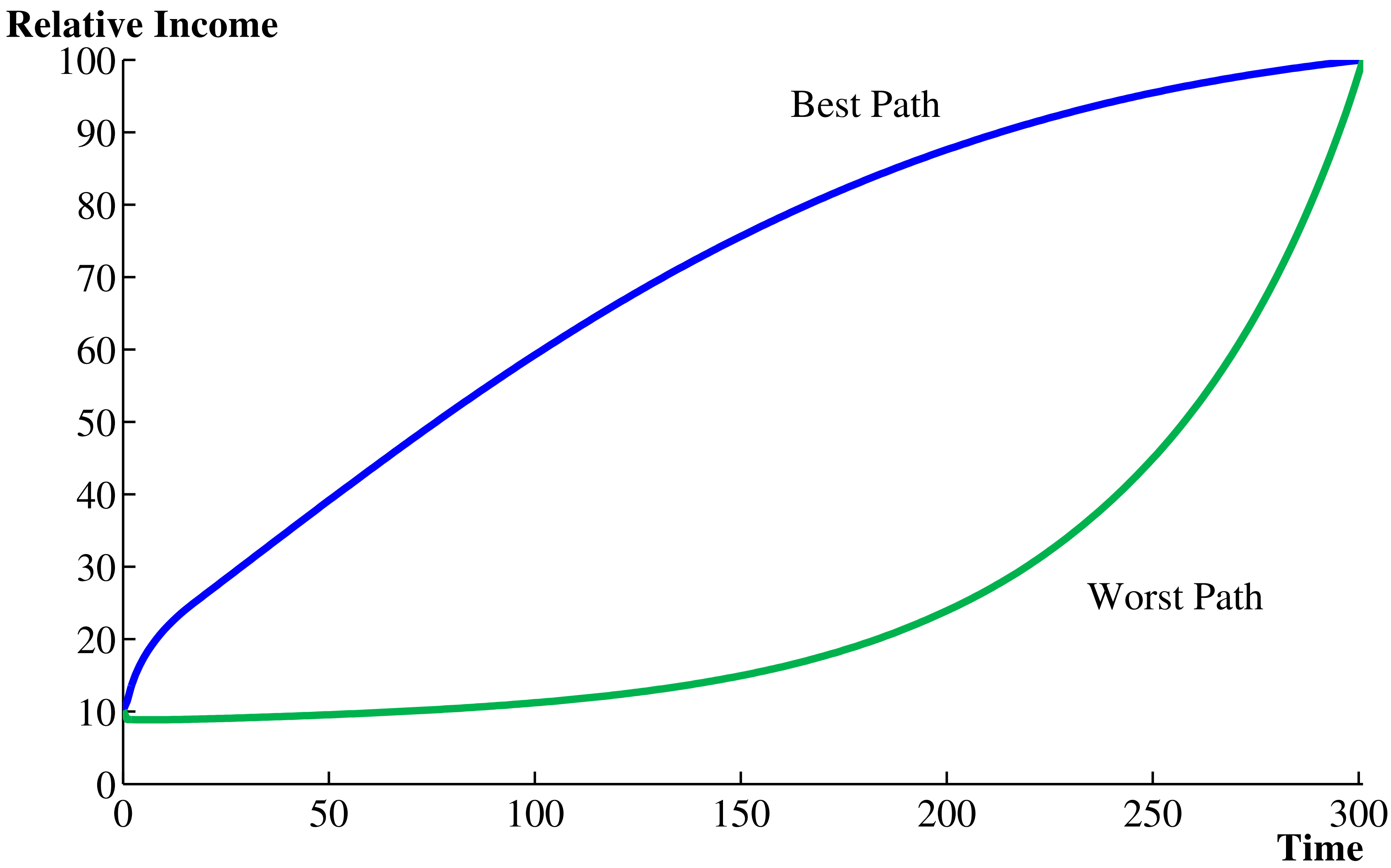
Output per Worker Ratios: "Rich" vs. "Poor"

	Description	"Average" TFP	No Intermediates $\sigma = 0$	Base Case $\sigma = 1/2$	Multiplicative Factor
1.	Baseline	0.604	4.7	29.0	*6.2*
2.	Identical TFPs	1.000	3.4	4.3	*1.3*
3.	$\nu_a^{rich} = \nu_a^{poor} = 0.84$	0.800	4.8	8.4	*1.8*
4.	$\nu_a^{rich} = \nu_a^{poor} = .5$	0.800	4.1	7.7	*1.9*
5.	$\nu_a^{rich} = .5$, $\nu_a^{poor} = .75$	0.654	4.9	16.9	*3.4*
6.	5, but $\nu_{aw} = 0$	0.654	3.5	14.2	*4.0*
7.	6, but $\bar{\tau}^{poor} = 0$	0.654	3.1	10.3	*3.3*

Output per Worker Ratios: Robustness

		— Amplification Factors —		
		Cobb-Doug	Baseline	"Leontief"
Scenario	Description	$\rho = 0$	$\rho = -1$	$\rho = -100$
1.	Baseline	5.5	6.2	6.8
2.	Identical TFPs	1.4	1.3	1.1
3.	$\nu_a^{rich} = \nu_a^{poor} = 0.84$	2.0	1.8	1.5
4.	$\nu_a^{rich} = \nu_a^{poor} = .5$	2.0	1.9	1.8
5.	$\nu_a^{rich} = .5,\ \nu_a^{poor} = .75$	3.4	3.4	3.5
6.	5, but $\nu_{aw} = 0$	3.4	4.0	4.9
7.	6, but $\bar{\tau}^{poor} = 0$	2.9	3.3	3.8

Growth and Reforms?



Conclusions

- Intermediate goods and Complementarity provide multipliers
  - Intermediate goods: large effect, relatively easily calibrated.
  - Complementarity: Great stories. Hard to calibrate, offset by substitution?
- Directions for further research
  - What about a much richer input-output structure?

    "Misallocation, Economic Growth, and Input-Output Economics"

  - Redo Hsieh and Klenow (2009) with intermediate goods
  - Measuring weak links and misallocation

Richer Input-Output Structure

- Long and Plosser (1983) “Real Business Cycles”
  - Multi-sector model
  - Cobb Douglas everywhere $\Rightarrow$ log linear $\Rightarrow$ linear algebra!
- Jones (2013) “Misallocation, Economic Growth, and Input-Output Economics”
- Acemoglu, Carvalho, Ozdaglar, and Tahbaz (2012) “Network Origins of Aggregate Fluctuations”
- Baqaee and Farhi (2017) “The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten’s Theorem”
- Peter and Ruane (2017) “Intermediate Input Elasticities and Industrial Policy”

Economic Environment: N sectors

$$Q_i = A \cdot A_i \left(K_i^{\alpha_i} H_i^{1-\alpha_i}\right)^{1-\sigma_i} \underbrace{m_{i1}^{\sigma_{i1}} m_{i2}^{\sigma_{i2}} \cdot \ldots \cdot m_{iN}^{\sigma_{iN}}}_{\text{intermediates}}$$

Resource constraint (j): $c_j + \sum_{i=1}^N m_{ij} = Q_j$

Aggregation: $Y = c_1^{\beta_1} \cdot \ldots \cdot c_N^{\beta_N}$

Physical capital: $\sum_{i=1}^N K_i = K$ given

Human capital: $\sum_{i=1}^N H_i = H$ given

Equilibrium and the Leontief Inverse

In the competitive equilibrium with misallocation, the solution for total production of the aggregate final good is

$$Y = A^{\tilde{\mu}} K^{\tilde{\alpha}} H^{1-\tilde{\alpha}} \epsilon$$

where

$\mu' \equiv \beta'(I - B)^{-1}$ ($N \times 1$ vector of multipliers)

$\tilde{\mu} \equiv \mu' \mathbf{1}$

$(I - B)^{-1}$ is the “Leontief inverse”, like $1/1 - \sigma$!

$\log \epsilon \equiv \omega + \mu' \tilde{A}$, where $\tilde{A}_i \equiv A_i(1 - \tau_i)$.

Hulten's Theorem (1978)

- Generalize the I-O structure
- How does a change in productivity in one sector (or firm) affect aggregate GDP?
  - Answer: elasticity equals ratio of sector or firm's Sales Revenue to GDP
  - Otherwise independent of I-O structure
  - Basically, the Leontief multiplier
- See Baqaee and Farhi (2017) and Ernest Liu (2017)
  - That's a first-order approximation, but second-order terms can matter
  - Only true in the absence of distortions

Oil Spending Share of World GDP (Baqaee/Farhi)