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A NEW PROOF OF UZAWA'S STEADY-STATE GROWTH THEOREM

Charles I. Jones and Dean Scrimgeour*

Abstract—This note revisits the proof of the steady-state growth theorem, first given by Uzawa in 1961. We provide a clear statement of the theorem, discuss intuition for why it holds, and present a new, elegant proof due to Schlicht (2006).

I. Introduction

THE steady-state growth theorem says that if a neoclassical growth model exhibits steady-state growth, then technical change must be labor augmenting, at least in steady state. It is sometimes added that an alternative is for the production function to be Cobb-Douglas. But this is really subsumed in the original version of the theorem since technical change can always be written in the labor-augmenting form in steady state if the production function is Cobb-Douglas.

It did not escape the attention of economists, either in the 1960s or more recently, that this is a very restrictive theorem. We often want our models to exhibit steady-state growth, but why should technical change be purely labor augmenting? The induced-innovation literature associated with Fellner (1961), Kennedy (1964), Sanmelson (1965), and Drandakis and Phelps (1966) explicitly pondered this question without achieving a clear answer. Recently, Acemoglu (2003) and Jones (2005) have returned to this puzzle.

Perhaps surprisingly, given its importance in the growth literature, we have been unable to find a clear statement and proof of the theorem. In addition, exactly why the result holds is not something that is well understood. What is the intuition for why technical change must be labor augmenting?

Uzawa (1961) is typically credited with the proof of the result,[1] and there is no doubt that he proved the theorem. However, Uzawa is primarily concerned with showing the equivalence of *Harrod-neutral* technical change (that is, technical change that leaves the capital share unchanged if the interest rate is constant) and labor-augmenting technical change, formalizing the graphical analysis of Robinson (1938). It is, of course, only a small and well-known step to show that steady-state growth requires technical change to be Harrod neutral. But the modern reader of Uzawa will be struck by two things. First is the lack of a statement and direct proof of the steady-state growth theorem. Second is the absence of economic intuition, both in the method of proof and more generally in the paper.

Barro and Sala-i-Martin (1995, chapter 2) come close to providing a clear statement and proof of the theorem. However, their statement of the theorem is more restrictive: if technical change is factor augmenting at a constant exponential rate, then steady-state growth requires it to be labor augmenting. This leaves the door open to the possibility that there might be some perverse nonfactor augmenting twist of technical change that could be consistent with steady-state growth. McCallum (1996) also comes close, providing a proof of the

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*U.C. Berkeley and NBER, and U.C. Berkeley, respectively.

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[1] For example, see Barro and Sala-i-Martin (1995) and Solow (1999).

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general theorem very similar to Uzawa's approach: by sticking so closely to Uzawa, however, the intuition for the result remains elusive.

This note fills the gap in the literature. We provide a clear statement and proof of the steady-state growth theorem, together with a concise intuition for why it holds.

The working paper version of this paper (Jones & Scrimgeour, 2005) contained a proof inspired by Uzawa (1961) and focused on developing intuition. Building on our working paper, a number of authors have constructed more straightforward proofs. Russell (2004) provides a quick mathematical proof of the theorem that exploits some methods from the physics literature on a class of partial differential equations called advective equations.

However, the holy grail of an elegant, intuitive proof has been claimed by Schlicht (2006). The Uzawa-style proof now seems tedious by comparison. Therefore, in what follows, we have replaced our previous proof with the new one provided by Schlicht—it is his new proof to which we refer in the title of this paper. The proof is quite straightforward and will surely be taught in first-year graduate macro courses from this point forward. We fix a small technical omission in Schlicht's original proof related to the fact that investment must be positive for the theorem to be valid. We conclude with a paragraph of intuition from our earlier working paper that captures the essence of Schlicht's proof and that explains why the steady-state growth theorem holds.[2]

II. Stating and Proving the Theorem

The steady-state growth theorem applies to the one-sector neoclassical growth model. We begin by defining the model precisely and then defining a balanced growth path. We will follow the usual convention of also referring to a balanced growth path as a steady state. Following the definitions, we state and prove the theorem.

Definition 2.1. *A* neoclassical growth model *is given by the following economic environment:*

$$Y_t = F(K_t, L_t; t), \tag{1}$$

$$C_t + I_t = Y_t, \tag{2}$$

$$\dot{K}_t = I_t - \delta K_t, \quad K_0 > 0, \quad \delta \geq 0, \tag{3}$$

$$L_t = L_0 e^{nt}, \quad L_0 > 0, \quad n \geq 0. \tag{4}$$

The production function F satisfies the standard neoclassical properties: constant returns to scale in K and L, and positive and diminishing marginal products of K and L.

Definition 2.2. *A* balanced growth path *in the neoclassical growth model is a path along which all quantities* $\{Y_t, K_t, L_t, C_t, I_t\}$ *grow at constant exponential rates (possibly zero) for all* $t \geq \tau \geq 0$.

Theorem 2.1 (The Steady-State Growth Theorem, Uzawa, 1961). Suppose the neoclassical growth model exhibits a steady state starting at date τ *where output per worker grows at rate g and* $I_t > 0$ *for* $t \geq \tau$. *Then for all* $t \geq \tau$,

$$Y_t = F(K_t, A_t L_t; \tau), \tag{5}$$

where $\dot{A}_t / A_t = g$. *That is, technical change is labor augmenting in steady state.*

Proof: (*Schlicht*, 2006). From the production function, $Y_\tau = F(K_\tau, L_\tau; \tau)$. Let g_x denote the growth rate of quantity x in steady state. Then $Y_\tau = Y_t e^{-g_Y(t-\tau)}$, for example, so that for all $t \geq \tau$

$$Y_t e^{-g_Y(t-\tau)} = F(K_t e^{-g_K(t-\tau)}, L_t e^{-n(t-\tau)}; \tau).$$

Because F exhibits constant returns in K and L, we can divide through by the exponential to get

$$Y_t = F(K_t e^{(g_Y - g_K)(t-\tau)}, L_t e^{(g_Y - n)(t-\tau)}; \tau). \tag{6}$$

If $g_Y = g_K$, the result is proved, with $A_t \equiv e^{g(t-\tau)}$. But it is well-known that this holds—for example, it is an immediate result in the model with a constant investment rate. In the more general framework here, it follows from some slightly tedious algebra.

In particular, the capital accumulation in equation (3) requires $g_I = g_K$, so if $g_I = g_Y$ we are done. Differentiating $Y_t = C_t + I_t$ with respect to time for $t \geq \tau$ gives

$$g_Y = \frac{C_t}{Y_t} g_C + \frac{I_t}{Y_t} g_I.$$

Differentiating this expression again with respect to time and rearranging gives

$$g_C(g_Y - g_C)C_t = g_I(g_I - g_Y)I_t.$$

If $C_t = 0$, the right side must be 0 so $g_I = g_Y$ and we are done. If $C_t \neq 0$, this expression can hold only if C_t and I_t grow at the same rate, but this too requires $g_I = g_Y$. Therefore, $g_Y = g_K$.

III. Discussion

Notice that our assumption in the statement of the theorem that investment is positive plays its role in the last step of the proof, where we showed that $g_Y = g_K$. In particular, if investment were equal to 0, it is possible to have a steady state with $g_Y > g_K$, but only if technical change is both capital- and labor-augmenting. With zero investment, the capital stock declines exponentially at the rate of depreciation. Since capital is not accumulating with output, the logic of the steady-state growth theorem does not apply. Instead, technical change needs to augment capital: first to offset depreciation, and second to get "effective capital" growing at the same rate as output—look back at equation (6). The theorem and proof as stated in Schlicht (2006) omit the requirement that $I_t > 0$.

In addition to its simplicity, Schlicht's proof has another advantage relative to Uzawa (1961). Uzawa's proof ends with a new production function G such that $F(K_t, L_t; t) = G(K_t, A_t L_t)$. Schlicht shows in equation (5) that technical change is labor augmenting in the *original* production function.

IV. Conclusion

The only asymmetry between capital and labor in the neoclassical growth model is that capital is accumulated as units of the output

[2] After this paper was accepted, Lutz Arnold brought another proof to our (and Ekkehart Schlicht's) attention, found in Wan (1971, p. 59). Wan proves the result in the context of a Solow model—that is, a model with a fixed saving rate—and uses some arguments similar to those rediscovered by Schlicht (2006).

good, while labor is not. This asymmetry must be behind the steady-state growth theorem, and this is confirmed in the proof.

Here is a simple way to connect this intuition with the labor-augmenting result. Divide both sides of the production function by output, yielding the "balance" expression $1 = F(K_t/Y_t, L_t/Y_t;t)$. Capital accumulates and inherits the trend in output, so the capital-output ratio is constant in steady state. Labor does not inherit the trend in output, however, so L_t/Y_t falls in steady state. To satisfy the balance equation, technical change must exactly offset the decline in L_t/Y_t. That is, technical change must be labor augmenting.

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THE YIELD CURVE AS A PREDICTOR OF GROWTH: LONG-RUN EVIDENCE, 1875–1997

Michael D. Bordo and Joseph G. Haubrich*

Abstract—This paper brings historical evidence to bear on the stylized fact that the yield curve predicts future growth. The spread between corporate bonds and commercial paper reliably predicts future growth over the period 1875–1997. This predictability varies over time, however, and has been strongest in the post–World War II period.

I. Introduction

By now, the ability of the yield curve to predict recessions has reached the hallowed status of "stylized fact" among macroeconomists. Inversions (short rates higher than long rates) predict recessions (Estrella & Hardouvelis, 1991) and more generally, a steep yield curve predicts fast growth and a flat curve, slow growth (Harvey, 1988, 1991; Haubrich & Dombrosky, 1996).[1] The late 1990s appeared somewhat anomalous in that a relatively flat yield curve accompanied fast growth; however, an inversion did precede the recession that began in March of 2001.

The evidence for this stylized fact comes primarily from the post–World War II experience of the United States, though an increasing amount of work has looked at other countries (Harvey, 1991; Stock & Watson, 2003; Gonzalez, Spencer, & Walz, 2000). The predictive content of the yield curve for longer historical periods, however, has been curiously neglected.[2] Whether the yield curve's ability to predict emerges as a general property of the American business cycle or depends sensitively on the structure of the economy, financial markets, and monetary policy seems an obvious question. A broader historical perspective may also shed some light on the reasons behind the yield curve's ability to predict future output—for example, one simply cannot ascribe twists in the yield curve during the 1880s to an FOMC ratcheting up short-term rates.

In this paper we look at the relationship between the term spread and movements in real economic activity, focusing on the United States from 1875 to 1997. We examine this relationship using a consistent series for both interest rate spreads and real activity at quarterly frequency. Section II discusses data construction, and section III reports the results of predictive regressions.

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* Rutgers University and NBER, and Federal Reserve Bank of Cleveland, respectively.

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[1] The literature is quite extensive, and Stock and Watson (2003) provide a useful survey, but some other papers we have found useful include Friedman and Kuttner (1998), Dotsey (1998), and Roma and Torous (1997).

[2] Kessel (1965) looks at patterns over a long time period, but does not directly address the predictability issue. Baltzer and Kling (forthcoming) look at the German historical experience.