Online Appendix to "The Allocation of Talent and U.S. Economic Growth"

(Not for publication)

Chang-Tai Hsieh, Erik Hurst, Charles I. Jones, Peter J. Klenow

April 26, 2019

A   Derivations and Proofs

The propositions in the paper summarize the key results from the model. This appendix shows how to derive the results.

Proof of Proposition 1. Occupational Choice

The individual's utility from choosing a particular occupation, $U(\tau_{ig}, w_i, \epsilon_i, \mu_i)$, is proportional to $\mu_i(\bar{\gamma}\widetilde{w}_{ig}\epsilon_i)^{\frac{3\beta}{1-\eta}}$, where $\widetilde{w}_{ig} \equiv w_i s_i^{\phi_i}(1-s_i)^{\frac{1-\eta}{3\beta}} \cdot \frac{\bar{h}_{ig}\tilde{z}_{ig}}{\tau_{ig}}$ and $\bar{\gamma} \equiv 1 + \gamma(2) + \gamma(3)$ is the sum of the experience terms. We first consider the occupation decision for individuals with ability heterogeneity (so no taste heterogeneity or $\mu_i = 1$). For these people, the solution to the individual's problem involves picking the occupation with the largest value of $\widetilde{w}_{ig}\epsilon_i$. To keep the notation simple, we will suppress the g subscript in what follows.

Without loss of generality, consider the probability that the individual chooses occupation 1, and denote this by p_1. Then

$$
\begin{aligned}
p_1 &= \Pr\left[\widetilde{w}_1\epsilon_1 > \widetilde{w}_s\epsilon_s\right] \ \forall s \neq 1 \\
&= \Pr\left[\epsilon_s < \widetilde{w}_1\epsilon_1/\widetilde{w}_s\right] \ \forall s \neq 1 \\
&= \int F_1(\epsilon, \alpha_2\epsilon, \ldots, \alpha_M\epsilon)d\epsilon, \quad\quad (A1)
\end{aligned}
$$

where $F_1(\cdot)$ is the derivative of the cdf with respect to its first argument and $\alpha_i \equiv \widetilde{w}_1/\widetilde{w}_i$.

Recall that

$$
F(\epsilon_1, \ldots, \epsilon_M) = \exp\left[\sum_{s=1}^{M} \epsilon_s^{-\theta}\right].
$$

Taking the derivative with respect to ϵ_1 and evaluating at the appropriate arguments

gives

$$F_1(\epsilon, \alpha_2\epsilon, \ldots, \alpha_M\epsilon) = \theta\epsilon^{-\theta-1} \cdot \exp\left[\bar{\alpha}\epsilon^{-\theta}\right] \tag{A2}$$

where $\bar{\alpha} \equiv \sum_s \alpha_s^{-\theta}$.

Evaluating the integral in (A1) then gives

$$
\begin{aligned}
p_1 &= \int F_1(\epsilon, \alpha_2\epsilon, \ldots, \alpha_M\epsilon)d\epsilon \\
&= \frac{1}{\bar{\alpha}} \int \bar{\alpha}\theta\epsilon^{-\theta-1} \cdot \exp\left[\bar{\alpha}\epsilon^{-\theta}\right] d\epsilon \\
&= \frac{1}{\bar{\alpha}} \cdot \int dF(\epsilon) \\
&= \frac{1}{\bar{\alpha}} \\
&= \frac{1}{\sum_s \alpha_s^{-\theta}} \\
&= \frac{\widetilde{w}_1^{\theta}}{\sum_s \widetilde{w}_s^{\theta}}
\end{aligned}
$$

A similar expression applies for any occupation i, so we have

$$p_i = \frac{\widetilde{w}_i^{\theta}}{\sum_s \widetilde{w}_s^{\theta}} \tag{A3}$$

We now consider individuals with taste heterogeneity (so no ability heterogeneity or $\epsilon_i = 1$). These individuals pick the occupation with the largest value of $\mu_i(\widetilde{w}_{ig})^{\frac{3\beta}{1-\eta}}$. The probability the individual picks occupation 1 is now given by

$$p_1 = \Pr\left[\widetilde{w}_1^{\frac{3\beta}{1-\eta}}\mu_1 > \widetilde{w}_s^{\frac{3\beta}{1-\eta}}\mu_g\right] \ \ \forall s \neq 1$$

The probability of picking occupation 1 is then given by

$$p_1 = \frac{\widetilde{w}_1^{\frac{3\beta}{1-\eta}\omega}}{\sum_s \widetilde{w}_s^{\frac{3\beta}{1-\eta}\omega}}$$

where ω is the shape parameter of the Fréchet distribution for tastes. Using the assumption that $\omega = \frac{\theta(1-\eta)}{3\beta}$, the probability of picking occupation i is still given by equation (A3).

Proof of Proposition 2. Geometric Average of Worker Quality

Efficiency units of labor of an individual of cohort c in occupation i at time t is given by $h_i(c,t) = \bar{h}_i s(c)^{\phi(t)} e_i(c)^\eta$. Using the results from the individual's optimization problem, it is straightforward to show that

$$h_i(c,t)\,\epsilon_i = s_i(c)^{\phi_i(t)}\gamma(t-c)\left(\frac{\eta\,s_i(c)^{\phi_i(c)}w_i(c)(1-\tau_i^w(c))\bar{h}_i\bar{\gamma}}{1+\tau_i^h(c)}\right)^{\frac{\eta}{1-\eta}}\epsilon_i^{\frac{1}{1-\eta}}.$$

For individuals that sort on ability, the geometric average of efficiency units of labor in an occupation is given by

$$e^{\,\mathbb{E}\,log[h_i(c,t)\epsilon_i\,|\,choose\,i]} = s_i(c)^{\phi_i(t)}\gamma(t-c)\left(\frac{\eta s_i(c)^{\phi_i(c)}w_i(c)(1-\tau_i^w(c))\bar{h}_i\bar{\gamma}}{1+\tau_i^h(c)}\right)^{\frac{\eta}{1-\eta}}e^{\,\mathbb{E}\,log\left[\epsilon_i^{\frac{1}{1-\eta}}\,|\,choose\,i\right]}.$$

(A4)

We need to compute $e^{\,\mathbb{E}\,log\left[\epsilon_i^{\frac{1}{1-\eta}}\,|\,choose\,i\right]}$. Let ϵ^* denote ability in the chosen occupation. We need to know the distribution of ϵ^* raised to some power. Let $y_i \equiv \widetilde{w}_i\epsilon_i$ denote the key occupational choice term. Then

$$y^* \equiv \max_i\{y_i\} = \max_i\{\widetilde{w}_i\epsilon_i\} = \widetilde{w}^*\epsilon^*.$$

Since y_i is the thing we are maximizing, it inherits the extreme value distribution:

$$\begin{aligned}
\Pr[y^* < z] &= \Pr[y_i < z]\ \forall i\\
&= \Pr[\epsilon_i < z/\widetilde{w}_i]\ \forall i\\
&= F\left(\frac{z}{\widetilde{w}_1},\ldots,\frac{z}{\widetilde{w}_M}\right)\\
&= \exp\left[-\sum_s \widetilde{w}_s^\theta z^{-\theta}\right]\\
&= \exp\{-mz^{-\theta}\}.
\end{aligned}$$

That is, the extreme value also has a Fréchet distribution, where $m \equiv \sum_s \widetilde{w}_s^\theta$.

Straightforward algebra then reveals that the distribution of ϵ^*, the ability of people in their chosen occupation, is also Fréchet:

$$G(x) \equiv \Pr[\epsilon^* < x] \equiv \exp\left[-m^*x^{-\theta}\right]$$

where $m^* \equiv \sum_{s=1}^{M} (\widetilde{w}_s / \widetilde{w}^*)^\theta = 1/p^*$.

Next, we need an expression for the expected value of the chosen occupation's ability raised to some power. Let λ be some positive exponent. Then,

$$
\begin{aligned}
\mathbb{E}[\epsilon^{*\,\lambda}] &= \int_0^\infty \epsilon^{*\,\lambda} dG(\epsilon^*) \\
&= \int_0^\infty \theta \left(\frac{1}{p^*} \right) \epsilon^{*(-\theta-1+\lambda)} e^{-\left(\frac{1}{p^*}\right)\epsilon^{*\,-\theta}} d\epsilon^*
\end{aligned}
$$

Recall that the "Gamma function" is $\Gamma(\alpha) \equiv \int_0^\infty x^{\alpha-1} e^{-x} dx$. Using the change-of-variable $x \equiv \frac{1}{p^*} \epsilon^{*\,-\theta}$, one can show that

$$
\begin{aligned}
\mathbb{E}[\epsilon^{*\,\lambda}] &= \left(\frac{1}{p^*} \right)^{\lambda/\theta} \int_0^\infty x^{-\frac{\lambda}{\theta}} e^{-x} dx \\
&= \left(\frac{1}{p^*} \right)^{\lambda/\theta} \Gamma\left(1 - \frac{\lambda}{\theta} \right).
\end{aligned}
$$

Applying this result to our model, we have

$$
\mathbb{E}\left[\epsilon_i^{\frac{1}{1-\eta}} \,|\, choose\ i \right] = \left(\frac{1}{p_{ig}} \right)^{\frac{1}{\theta} \cdot \frac{1}{1-\eta}} \Gamma\left(1 - \frac{1}{\theta} \cdot \frac{1}{1-\eta} \right).
$$

Finally note that if $x \sim \text{Frechet}(\theta)$, then $\log x \sim \text{Gumbel}(1/\theta)$, and $\mathbb{E}[\log x] = \frac{\gamma_{em}}{\theta}$, where $\gamma_{em} \approx 0.5772$ is the Euler-Mascheroni constant. Applying this to the expression for $\mathbb{E}\,log\left[\epsilon_i^{\frac{1}{1-\eta}} \,|\, choose\ i \right]$ above, we have

$$
e^{\mathbb{E}\,log\left[\epsilon_i^{\frac{1}{1-\eta}} \,|\, choose\ i \right]} = \left(\frac{1}{p_{ig}} \right)^{\frac{1}{\theta} \cdot \frac{1}{1-\eta}} \widetilde{\Gamma}
$$

where $\widetilde{\Gamma} \equiv e^{\frac{\gamma_{em}}{\theta(1-\eta)}}$. Substituting this expression into equation (A4) yields the geometric mean of ability for individuals who sort on ability:

$$
e^{\mathbb{E}\,log[h_i(c,t)\epsilon_i \,|\, choose\ i]} = \widetilde{\Gamma} s_i(c)^{\phi_i(t)} \gamma(t-c) \left[\left(\frac{\eta s_i(c)^{\phi_i(c)} w_i(c)(1-\tau_i^w(c))\bar{h}_i\bar{\gamma}}{1+\tau_i^h(c)} \right)^\eta \left(\frac{1}{p_{ig}} \right)^{\frac{1}{\theta}} \right]^{\frac{1}{1-\eta}}.
$$
(A5)

The last thing we need is efficiency units of workers that sort on preferences. Remember $\epsilon_i = \Gamma^{1-\eta}$ where $\Gamma \equiv \Gamma\left(1 - \frac{1}{\theta(1-\eta)} \right)$. Therefore quality is the same for all

individuals and given by:

$$[h_i(c,t)\epsilon_i \,|\, choose\, i] = \Gamma s_i(c)^{\phi_i(t)}\gamma(t-c)\left(\frac{\eta s_i(c)^{\phi_i(c)}w_i(c)(1-\tau_i^w(c))\bar{h}_i\bar{\gamma}}{1+\tau_i^h(c)}\right)^{\frac{\eta}{1-\eta}} \quad \text{(A6)}$$

Finally, the geometric average of the expressions in equations (A5) and (A6) where the weights are given by $1-\delta$ (share of workers that sorts on ability) and δ (share of workers that sort on preferences) gives us the geometric average of quality of *all* workers in an occupation in equation (5) where $\bar{\Gamma}$ is defined as:

$$\bar{\Gamma} \equiv \Gamma^\delta \,\widetilde{\Gamma}^{1-\delta} \quad \text{(A7)}$$

Proof of Proposition 3. Occupational Wage Gaps

The proof of this proposition is straightforward after substituting the results from propositions 1 and 2 into the expression for the geometric average of the two groups of workers (workers that sort on ability and those that sort on preferences). The geometric average of the wage of all workers is then the geometric mean of the geometric average wage of the two groups of workers.

Proof of Proposition 4. Relative Propensities

The proof of this proposition is straightforward after substituting the results from propositions 1 and 3 into the expression for relative propensities $\frac{p_{ig}}{p_{i,wm}}$.

Proof of Proposition 5. Relative Labor Force Participation

This proposition is an application of propositions 3 and 4 to the home sector, assuming no distortions for white men in all sectors and no distortions in the home sector for all groups.

B   Determinants of Labor Market and Human Capital Frictions

Following Becker (1957), we assume the owner of the firm in the final goods sector discriminates against workers of certain groups. We model the "taste" for discrimination

as lower utility of the owner when she employs workers from groups she dislikes. Her
utility is given by

$$U_{owner} = Y - \underbrace{\sum_i \sum_g \left(1 - \tau_{ig}^w\right) w_i H_{ig}}_{\text{Profit}} \quad - \quad \underbrace{\sum_i \sum_g d_{ig} H_{ig}}_{\text{Utility loss via discrimination}} \quad . \qquad \text{(B1)}$$

where H_{ig} denotes total efficiency units of workers from group g in occupation i. The
first term denotes profits and the second term captures the extent to which owners
are prejudiced: d_{ig} is the utility loss associated with employing workers from group
g in occupation i. Because all employers are assumed to have these racist and sexist
preferences, perfect competition implies that $\tau_{ig}^w = d_{ig}/w_i$. Intuitively, when the owner
hires a worker from a group she dislikes, she needs to be compensated for her utility
loss via a lower wage for these workers. In equilibrium the utility loss is exactly offset
by the lower wage. Thus the frictions are ultimately pinned down by the discriminatory
tastes of (homogeneous) owners.[38]

A second firm (a "school") sells educational goods e to workers who use it as an
input in their human capital. We assume the school's owner dislikes providing e to
certain groups. The utility of the school's owner is

$$U_{school} = \underbrace{\sum_i \sum_g \left(R_{ig} - \left(1 - \tau_{ig}^h\right)\right) \cdot e_{ig}}_{\text{Profit}} \quad - \quad \underbrace{\sum_i \sum_g d_{ig}^h e_{ig}}_{\text{Utility loss via discrimination}} \qquad \text{(B2)}$$

where e_{ig} denotes educational resources provided to workers from group g in market
sector i, R_{ig} denotes the price of e_{ig}, and d_{ig}^h represents the owner's distaste from pro-
viding educational resources to workers from group g in sector i. We think of this
as a shorthand for complex forces such as discrimination against blacks or women
in admission to universities, or differential allocation of resources to public schools
attended by black vs. white children, or differential parental investments made toward
building up math and science skills in boys relative to girls. Groups that are discrimi-
nated against in the provision of human capital pay a higher price for e, and the higher

---

[38]What is important is not that all firms discriminate but that the marginal firm discriminates (Becker,
1957). We abstract from firm heterogeneity and instead assume all firms within an occupation discrimi-
nate.

price compensates the school owner for her disutility. Perfect competition ensures that $R_{ig} = 1$, and that $\tau_{ig}^h = d_{ig}^h$.

C Equilibrium

A competitive equilibrium in this economy consists of a sequence of individual choices $\{C, e, s\}$, occupational choices in the pre-periods, total efficiency units of labor of each group in each occupation H_{ig}, final output Y, and an efficiency wage w_i in each occupation such that

1. Given an occupational choice, the occupational wage w_i, and idiosyncratic ability ϵ in that occupation, each individual chooses C, e, s to maximize expected lifetime utility given by (1) subject to the constraints given by (2) and $e(c) = \sum_{t=c}^{c+2} e(c,t)$.

2. Each individual chooses the occupation that maximizes expected lifetime utility: $i^* = \arg\max_i U(\tau_{ig}^w, \tau_{ig}^h, \tilde{z}_{ig}, w_i, \epsilon_i, \mu_i)$, taking as given $\{\tau_{ig}^w, \tau_{ig}^h, \tilde{z}_{ig}, w_i, \bar{h}_{ig}, \epsilon_i, \mu_i\}$.

3. A representative firm in the final good sector hires H_{ig} in each occupation to maximize profits net of utility cost of discrimination given by equation (B1).

4. A representative firm in the education sector maximizes profit net of the utility cost of discrimination given by equation (B2).

5. Perfect competition in the final goods and education sectors generates $\tau_{ig}^w = d_{ig}/w_i$ and $\tau_{ig}^h = d_{ig}^h$.

6. $w_i(t)$ clears each occupational labor market.

7. Total output is given by the production function in equation (9).

The equations characterizing the general equilibrium are given in the next result.

Proposition 6 (Solving the General Equilibrium): *The general equilibrium of the model is $H_{ig}^{supply}, H_i^{demand}, w_i$, and market output Y at each point in time such that*

1. $H_{ig}^{supply}(t)$ aggregates the individual choices:

$$H_{ig}^{supply}(t) = \sum_c q_g(c) p_{ig}(c) \, \mathbb{E}\left[h_{ig}(c)\epsilon_{ig}(c) \,|\, \textit{Person chooses } i\right]$$

where $q_g(c)$ denotes the number of workers of group g and cohort c and the average quality of workers is given in equation (5).

2. $H_i(t)^{demand}$ *satisfies firm profit maximization:*

$$H_i^{demand}(t) = \left(\frac{A_i(t)^{\frac{\sigma-1}{\sigma}}}{w_i(t)} \right)^\sigma Y(t)$$

3. $w_i(t)$ *clears each occupational labor market:* $\sum_g H_{ig}^{supply}(t) = H_i^{demand}(t)$.

4. *Total output is given by the production function in equation* (9) *and equals aggregate wages plus total revenues from* τ^w.

D   Identification and Estimation

This section explains how we identify and estimate the frictions and other parameters, carried out in the program `EstimateTauZ.m`.

D.1   Key Equations

To estimate the model, we add one additional feature to the model. In our base case, we assume the return to experience is the same for all occupations, groups, and cohorts. In our robustness checks, however, these parameters may be allowed to vary. We thus index γ (and the sum of the experience terms $\bar{\gamma}$) by group g and occupation i in the equations that follow.

The key equations underlying our estimation are listed below.

- Occupational Choice

$$p_i = \frac{\tilde{w}_i{}^\theta}{\sum_s \tilde{w}_s{}^\theta}$$

$$\text{where } \tilde{w}_{ig} \equiv w_i s_i^{\phi_i} (1 - s_i)^{\frac{1-\eta}{3\beta}} \cdot \frac{\bar{h}_{ig} \tilde{z}_{ig}}{\tau_{ig}}$$

$$\text{and } \tau_{ig} \equiv \frac{(1 + \tau_{ig}^h)^\eta}{1 - \tau_{ig}^w}$$

- Average Quality (geometric mean)

$$e^{\,\mathbb{E}\,log\,[h_{ig}(c,t)\epsilon_{ig}(c)]} = \bar{\Gamma} s_i(c)^{\phi(t)} \gamma_{ig}(t-c) \left[\eta \frac{1 - \tau_{ig}^w(c)}{1 + \tau_{ig}^h(c)} w_i(c) \bar{h}_{ig} \overline{\gamma}_{ig} s_i(c)^{\phi(c)} \right]^{\frac{\eta}{1-\eta}} \left(\frac{1}{p_{ig}(c)} \right)^{\frac{1-\delta}{\theta(1-\eta)}}$$

- Average Wage (geometric mean)

$$\overline{\text{wage}}_{ig}(c,t) \equiv (1 - \tau_{ig}^w(t)) w_i(t) \gamma_{ig}(t-c) e^{\,\mathbb{E}\,log\,[h_{ig}(c,t)\epsilon_{ig}(c)]}$$

$$= \bar{\Gamma} \bar{\eta} \left[p_{ig}(c)^{\delta} m_g(c) \right]^{\frac{1}{\theta} \frac{1}{1-\eta}} z_{ig}(c)^{-\frac{1}{1-\eta}} [1 - s_i(c)]^{-\frac{1}{3\beta}} \frac{(1 - \tau_{ig}^w(t)) w_i(t) \gamma_{ig}(t-c) s_i(c)^{\phi(t)}}{(1 - \tau_{ig}^w(c)) w_i(c) \overline{\gamma}_{ig} s_i(c)^{\phi(c)}}$$

$$\text{where } m_g(c) = \sum_{i=1}^{M} \widetilde{w}_{ig}(c)^{\theta}$$

- Relative Propensity. If $\delta \neq 1$:

$$\frac{p_{ig}(c,c)}{p_{i,wm}(c,c)} = \left(\frac{\bar{h}_{ig}}{\bar{h}_{i,wm}} \right)^{\frac{\theta}{1-\delta}} \left(\frac{\tau_{ig}(c,c)}{\tau_{i,wm}(c,c)} \right)^{-\frac{\theta}{1-\delta}} \left(\frac{\overline{\text{wage}}_{ig}(c,c)}{\overline{\text{wage}}_{i,wm}(c,c)} \right)^{-\frac{\theta(1-\eta)}{1-\delta}} \left(\frac{\overline{\gamma}_{ig}}{\overline{\gamma}_{i,wm}} \right)^{\frac{\theta\eta}{1-\delta}}$$

or when $\delta = 1$, the relative propensity equation simplifies to

$$\frac{\overline{\text{wage}}_{ig}(c,c)}{\overline{\text{wage}}_{i,wm}(c,c)} = \left[\frac{\bar{h}_{ig}/\bar{h}_{i,wm}}{\tau_{ig}(c,c)/\tau_{i,wm}(c,c)} \cdot \left(\frac{\overline{\gamma}_{ig}}{\overline{\gamma}_{i,wm}} \right)^{\eta} \right]^{\frac{1}{1-\eta}}$$

D.2   Estimate ϕ_i, $z_{i,wm}$, and w_i from data of young white men

The following refers to the program `solveWMfor_wZ.m`. This program uses data on wages, years of schooling, and occupational shares of young white men to estimate w_i, $z_{i,wm}$ and ϕ_i.

The s_i and ϕ_i are determined in a straightforward fashion from years of schooling for young white men in each cohort. In particular, we assume that the pre-market period is 25 years long so that

$$s_i = \frac{\text{Years of Education}}{25}$$

Then ϕ_i is determined by the individuals first-order condition for schooling. Rearranging equation (3) gives

$$\phi_i = \frac{1 - \eta}{3\beta} \cdot \frac{s_i}{1 - s_i}.$$

Next, we have 67 values of $\tilde{z}_{i,wm}$ as well as m_{wm} to recover, for a total of 68 parameters. However, we only observe wages in 66 occupations for young white men (there is no wage data for the home occupation), so we need two further assumptions to pin down these parameters. One assumption is that $\tilde{z}_{home,wm} = 1$. The other is that average earnings per person in the home sector for young white men are equal to their average earnings in some other occupation. We choose "Secretaries" for this other occupation, but the results are robust to choosing another occupation (such as Sales).

Then, we use the equation for the average wage to back out m_{wm} from the home occupation, since $\tilde{z}_{home} = 1$. After omitting the indices for cohort and time, the specific equation is:

$$m_{wm} = \left[\frac{\overline{wage}_{i,wm}\, \tilde{z}_{i,wm}\, (1 - s_i)^{\frac{1}{3\beta}}}{\bar{\Gamma}\bar{\eta}} \cdot \frac{\overline{\gamma}_{i,wm}}{\gamma_{i,wm}} \right]^{\theta(1-\eta)} \cdot p_{ig}(c)^{-\delta}$$

where $i = home$. Furthermore, we need to make an initial guess about the return to experience term γ (We describe later how we do this).

Third, we estimate $\tilde{z}_{i,wm}$ for the other occupations from the equation we use above to back out m_{wm} from data on wages. In this case, we use data on the average wage in the occupation, and the estimate for m_{wm} we obtained from step 2 to back out the $\tilde{z}_{i,wm}$ that fits the wage equation.

Fourth, we estimate w_i from the observed occupational shares. After some algebra, the occupational share equation can be expressed as:

$$w_i = \frac{[p_{i,wm} \cdot m_{wm}]^{\frac{1}{\theta}}}{\overline{\gamma}_{i,wm} \cdot s_i^{\phi_i} [(1 - s_i)z_{i,wm}]^{\frac{1-\eta}{3\beta}}}$$

Again, $\tau = 1$ for white men so these two terms do not show up.

Fifth, we estimate γ and $\bar{\gamma}$ (remember we assumed a value for the experience terms for the previous steps) from the change in the average wage of a given cohort and occupation over time. Specifically, the ratio of the average wage in an occupation at time t to that at time c is:

$$\frac{\overline{wage}_{i,wm}(c,t)}{\overline{wage}_{i,wm}(c,c)} = \frac{w_i(t)(\gamma_{ig}(t-c))s_i^{\phi(t)}}{w_i(c)s_i^{\phi(c)}}$$

We estimate $\gamma_{i,wm}(t-c)$ from the change in the average wage in an occupation, after controlling for the change in w_i and the returns to schooling. In our base case, we assume $\gamma_{i,wm}(t-c)$ is the same across all occupations and cohorts so simply take the average across all occupations and cohorts.

D.3 Estimating τ

The next part of the estimation obtains the composite of the distortions $\tau_{ig} \equiv \frac{(1+\tau^h)^\eta}{1-\tau^w}$. Remember we assume $\tau_{i,wm}^w = \tau_{i,wm}^h = 0$ and $\bar{h}_{ig} = \bar{h}_{i,wm}$. These two normalizations imply that we can express relative propensities as:

$$\tau_{ig} = \widehat{p}_{ig}^{-\frac{1-\delta}{\theta}} \cdot \widehat{\mathbf{wage}}_{ig}^{-(1-\eta)} \cdot \widehat{\gamma}_{ig}^{\eta}$$

where a "hat" denotes the value of the variable relative to white men. In this equation, $\widehat{wage}_i$ and $\widehat{p}_{ig}$ are data and $\widehat{\gamma}_{ig}$ and $\widehat{\overline{\gamma}}_{ig}$ are estimated from the previous step.

D.4 Estimating τ^w, τ^h, and z

The next step is to estimate z and the components of τ (i.e. τ^w and τ^h) for the other groups (non-white men). This is done in the program estimatetauz.m. We define α as the Cobb-Douglas split of τ that recovers $1 - \tau^w$. Specifically,

$$\tau^\alpha = \frac{1}{1-\tau^w} \text{ and } \tau^{1-\alpha} = (1+\tau^h)^\eta$$

This implies the following definitions of τ^w and τ^h as a function of τ and α:

$$\tau^w = 1 - \tau^{-\alpha}$$
$$\tau^h = \left(\tau^{1-\alpha}\right)^{\frac{1}{\eta}} - 1$$

Our estimation of τ^w and τ^h is expressed in terms of α.

First, the home sector is assumed to be undistorted, so τ^w and τ^h for that sector are set to zero. We then use the "relative propensity" key equation for $\hat{p}_{ig}$ at the start of this section, together with the wage in the home sector for white men, to recover the wage at home for the other groups.

Second, we normalize $z = 1$ for the home sector and back out m_g for the group

based on data on the average wage in the home sector. Specifically, after some manip-
ulation, the average wage equation for the sector can be expressed as:

$$m_g(c) = \left[\frac{\overline{wage}_{home,g}(c,c)(1 - s_{home}(c))^{\frac{1}{3\beta}}}{\bar{\Gamma}\,\bar{\eta}} \cdot \frac{\overline{\gamma}_{home,g}}{\gamma_{home,g}} \right]^{\theta(1-\eta)} \cdot p_{home,g}(c)^{-\delta}$$

For the other sectors, we use the same wage equation to back out z. Specifically, the
wage equation can be expressed as:

$$z_{ig} = \frac{1}{1 - s_i} \cdot \left[\bar{\Gamma}\,\bar{\eta}\,(p_{ig}^{\delta}m_g)^{\frac{1}{\theta(1-\eta)}} \frac{\gamma_{ig}}{\overline{\gamma}_{ig}} \frac{1}{\overline{wage}_{ig}} \right]^{3\beta}$$

We now have z for all cohorts and τ^w and τ^h for the young cohort in 1960. What is
left is to pin down τ^w and τ^h for the years after 1960. From the "Average Wage" equation
in our list of key equations, we can express wage growth in a given group-occupation
as

$$\frac{\overline{wage}_{ig}(c, t+1)}{\overline{wage}_{ig}(c, t)} = \frac{1 - \tau_{ig}^w(t+1)}{1 - \tau_{ig}^w(t)} \cdot \frac{w_i(t+1)}{w_i(t)} \cdot \frac{\gamma_{ig}(t+1-c)}{\gamma_{ig}(t-c)} \cdot \frac{s_i(c)^{\phi_i(t+1)}}{s_i(c)^{\phi_i(t)}} \qquad \text{(D1)}$$

We use solve this equation for $\tau_{ig}^w(c, t+1)$ and this becomes our estimate since every-
thing else in the equation is now observed. Then, $\tau_{ig}^h(t+1)$ is obtained from $\tau_{ig}(t+1)$.
In other words, τ^w is the time effect in wage growth, while τ^h is the cohort effect.

There are two small modifications we make to this in practice. First, we set the
minimum value of τ^h to –0.80, though we relax this constraint in the robustness checks
(without this constraint, the revenue required to subsidize women secretaries with τ^h
gets implausibly large).

Second, in our model, occupations are chosen when young, so all groups have the
same labor-force participation when middle-aged and old. In the data, this is clearly
not the case. Therefore, we strip out from wage growth for a given group-occupation
using our model's estimate of the selection effect from differential participation. Based
on the "Relative Propensity" equation in our "Key Equation" list, this effect has an
elasticity of $\theta(1 - \eta)/(1 - \delta)$. Absent data on labor-force participation by group, we
use a common adjustment across all occupations to obtain the wage growth estimate

used in equation (D1):

$$\left(\frac{\text{wagegrowth}_{ig}}{\text{wagegrowth}_{i,wm}}\right)^{\text{for estimation}} = \left(\frac{\text{wagegrowth}_{ig}}{\text{wagegrowth}_{i,wm}}\right)^{\text{data}} \left(\frac{\text{LFPgrowth}_{ig}}{\text{LFPgrowth}_{i,wm}}\right)^{\frac{1-\delta}{\theta(1-\eta)}}.$$
(D2)

We also report results without making this adjustment in our robustness checks.

D.5   Geometric and Arithmetic Averages

To get a closed-form solution, our model relies on geometric averaging, both with the taste and ability types and in particular across the two types. In the micro data, however, we cannot distinguish the two types and take a simple arithmetic average. This section describes how we go from the arithmetic average in the data to the geometric average (of the wage or the quality of workers) in the model. In particular, the formulas below apply specifically to the average wage, but a similar argument applies to average quality.

To see how these are related, let $x \sim Frechet(\alpha)$ with $\mu_x \equiv \mathbb{E}x = S\,\Gamma(1 - 1/\alpha)$ where $\Gamma(\,)$ is the gamma function. It is straightforward to show that $\log x \sim Gumbel(1/\alpha)$ with mean $\mathbb{E}[\log x] = \log S + \gamma_{em}/\theta$, where $\gamma_{em} \approx 0.5772$ is the Euler-Macheroni constant. Finally, if $g_x \equiv e^{\mathbb{E}\log x}$ denotes the geometric mean of x, then the arithmetic mean and the geometric mean are related by a constant factor of proportionality:

$$\mu_x = g_x \cdot \frac{\Gamma(1 - 1/\alpha)}{e^{\gamma_{em}/\alpha}}.$$
(D3)

Now let G denote a geometric average (e.g. of the wage) and let A denote an arithmetic average. In the model, we have

$$G = G_a^{1-\delta} G_t^{\delta}$$

where G_a is the geometric mean within the ability types and G_t is the geometric mean within the taste types. As part of the proof of Proposition 3, we showed that

$$G_a = \tilde{\Gamma} S$$

$$G_t = \Gamma S p_{ig}^{\frac{1}{\theta(1-\eta)}}$$

where S denotes most of the "stuff" in the model. This means that

$$G = G_a^{1-\delta} G_t^\delta = \bar{\Gamma} S p_{ig}^{\frac{\delta}{\theta(1-\eta)}}. \tag{D4}$$

Because heterogeneity in the ability types is Frechet, by equation (D3) above, the arithmetic means are given by

$$A_a = G_a \frac{\Gamma}{\tilde{\Gamma}}$$

and, because there is no heterogeneity among the taste types

$$A_t = G_t.$$

Then the arithmetic mean in the data is relaed to the geometric means by

$$
\begin{aligned}
A &= (1-\delta)A_a + \delta A_t \\
&= (1-\delta)G_a \frac{\Gamma}{\tilde{\Gamma}} + \delta Q_t \\
&= (1-\delta)\Gamma S + \delta \Gamma S p_{ig}^{\frac{1}{\theta(1-\eta)}} \\
&= \Gamma S \left(1 - \delta + \delta p_{ig}^{\frac{1}{\theta(1-\eta)}}\right)
\end{aligned}
$$

Combining this last result with (D4) gives the key relationship between the arithmetic mean of the wage or quality and the geometric mean:

$$G = A \cdot \frac{\bar{\Gamma}}{\Gamma} \cdot \frac{p_{ig}^{\frac{\delta}{\theta(1-\eta)}}}{1 - \delta + \delta p_{ig}^{\frac{1}{\theta(1-\eta)}}} \tag{D5}$$

Given the arithmetic mean from the data, this is how we construct the geometric mean for the wage or quality that is used in the estimation of the model.

E   Numerically Solving for an Equilibrium

The numerical solution of the equilibrium of the model begins by guessing values for $Y(t)$ and $m_g(c)$, where $c = 7 - t$ is the cohort born at date t. Given these values, we compute the equilibrium solution for year t. The main part of this solution is solving

for the wages per unit of quality $w_i(t)$ in each occupation. These are chosen to clear the labor market in each occupation, as in Proposition 6.

The only subtlety in this process is that Proposition 2 characterizes the geometric average of quality in each occupation, while the equilibrium depends on the arithmetic average instead.[39]

Perhaps not surprisingly, it is straightforward to show that the arithmetic mean of quality in each occupation, corresponding to Proposition 2, is

$$\Gamma\left(1 - \frac{1}{\theta(1-\eta)}\right) S \left[(1-\delta)\left(\frac{1}{p_{ig}}\right)^{\frac{1}{\theta(1-\eta)}} + \delta\right]$$

where $S \equiv s_i(c)^{\phi_i(t)}\gamma(t-c)\left(\frac{\eta s_i(c)^{\phi_i(c)}w_i(c)(1-\tau_i^w(c))\bar{h}_i\bar{\gamma}}{1+\tau_i^h(c)}\right)^{\frac{\eta}{1-\eta}}$. This is the expression we use in determining the supply of talent in each occupation when solving for the equilibrium.

F Additional Data Details

Table F1 provides sample sizes used in our analysis for each census year. The table also shows the share of the sample in each census year that pertains to white and black men and women by age. Table F2 lists the 67 occupational groupings we use for our main analysis (including the home sector).

G Additional Robustness and Results

Table G3 explores an additional set of robustness exercises for our $\delta = 0$ results (where all sorting is based on occupational productivities). For comparison, the first row repeats our benchmark results in the main paper for the share of market GDP per person growth explained by the changing τ's. The next two rows show that the productivity

---

[39]To see how these are related, let $x \sim Frechet(\theta)$ with $\mu_x \equiv \mathbb{E}x = S\,\Gamma(1-1/\theta)$ where $\Gamma(\ )$ is the gamma function. It is straightforward to show that $\log x \sim Gumbel(1/\theta)$ with mean $\mathbb{E}[\log x] = \log S + \gamma_{em}/\theta$, where $\gamma_{em} \approx 0.5772$ is the Euler-Macheroni constant. Finally, if $g_x \equiv e^{\mathbb{E}\log x}$ denotes the geometric mean of x, then the arithmetic mean and the geometric mean are related by a constant factor of proportionality:

$$\mu_x = g_x \cdot \frac{\Gamma(1-1/\theta)}{e^{\gamma_{em}/\theta}}.$$

Table F1: Sample Statistics by Census Year

	1960	1970	1980	1990	2000	2010
Sample Size	624,579	674,059	3,943,034	4,607,829	5,084,891	2,889,513
Share of Sample:						
White Men, Age 25-34	0.141	0.148	0.185	0.172	0.130	0.133
White Men, Age 35-44	0.155	0.140	0.131	0.156	0.162	0.136
White Men, Age 45-55	0.136	0.145	0.118	0.108	0.139	0.157
White Women, Age 25-34	0.158	0.160	0.194	0.175	0.130	0.134
White Women, Age 35-44	0.171	0.151	0.138	0.159	0.163	0.137
White Women, Age 45-55	0.145	0.157	0.127	0.113	0.143	0.162
Black Men, Age 25-34	0.015	0.016	0.021	0.023	0.021	0.021
Black Men, Age 35-44	0.015	0.015	0.014	0.019	0.024	0.022
Black Men, Age 45-55	0.013	0.014	0.012	0.012	0.017	0.023
Black Women, Age 25-34	0.019	0.020	0.026	0.027	0.024	0.024
Black Women, Age 35-44	0.018	0.019	0.018	0.022	0.027	0.025
Black Women, Age 45-55	0.015	0.016	0.015	0.015	0.020	0.026

Note: Data comes from the 1960-2000 U.S. Censuses and the pooled 2010-2012 American Community Survey (designated as 2010). Samples restricted to black and white, men and women between the ages of 25 and 54 (inclusive). Those in the military are excluded. Also, excluded are those not working but actively searching for a job. Sample shares are weighted using Census and ACS provided sample weights.

Table F2: Occupation Categories for our Base Occupational Specifications

0.	Home Sector (0)		34.	Police (12)
1.	Executives, Administrative, and Managerial (1)		35.	Guards (12)
2.	Management Related (2)		36.	Food Preparation and Service (13)
3.	Architects (3)		37.	Health Service (6)
4.	Engineers (3)		38.	Cleaning and Building Service (13)
5.	Math and Computer Science (3)		39.	Personal Service (13)
6.	Natural Science (4)		40.	Farm Managers (14)
7.	Health Diagnosing (5)		41.	Farm Non-Managers (14)
8.	Health Assessment (6)		42.	Related Agriculture (14)
9.	Therapists (6)		43.	Forest, Logging, Fishers, & Hunters (14)
10.	Teachers, Postsecondary (7)		44.	Vehicle Mechanic (15)
11.	Teachers, Non-Postsecondary (8)		45.	Electronic Repairer (15)
12.	Librarians and Curators (8)		46.	Misc. Repairer (15)
13.	Social Scientists and Urban Planners (4)		47.	Construction Trade (15)
14.	Social, Recreation, and Religious Workers (4)		48.	Executive (14)
15.	Lawyers and Judges (5)		49.	Precision Production, Supervisor (16)
16.	Arts and Athletes (4)		50.	Precision Metal (16)
17.	Health Technicians (9)		51.	Precision Wood (16)
18.	Engineering Technicians (9)		52.	Precision Textile (16)
19.	Science Technicians (9)		53.	Precision Other (16)
20.	Technicians, Other (9)		54.	Precision Food (16)
21.	Sales, All (10)		55.	Plant and System Operator (17)
22.	Secretaries (11)		56.	Metal and Plastic Machine Operator (17)
23.	Information Clerks (11)		57.	Metal & Plastic Processing Operator (17)
24.	Records Processing, Non-Financial (11)		58.	Woodworking Machine Operator (17)
25.	Records Processing, Financial (11)		59.	Textile Machine Operator (17)
26.	Office Machine Operator (11)		60.	Printing Machine Operator (17)
27.	Computer & Communication Equip. Operator (11)		61.	Machine Operator, Other (19)
28.	Mail Distribution (11)		62.	Fabricators (18)
29.	Scheduling and Distributing Clerks (11)		63.	Production Inspectors (18)
30.	Adjusters and Investigators (11)		64.	Motor Vehicle Operator (19)
31.	Misc. Administrative Support (11)		65.	Non Motor Vehicle Operator (19)
32.	Private Household Occupations (13)		66.	Freight, Stock, & Material Handlers (18)
33.	Firefighting (12)			

Notes: Our 66 market occupations are based on the 1990 Census Occupational Classification System. We use the 66 sub-headings (shown in the table) to form our occupational classification. See http://www.bls.gov/nls/quex/r1/y97r1cbka1.pdf for the sub-heading as well as detailed occupations that correspond to each sub-heading. We also include the home sector as an additional occupation. When computing racial barriers at the state level for an appendix exercise, we use only twenty broader occupations. The number in parentheses refers to how we group these 67 occupations into the twenty broader occupations for the cross-state analysis. For example, all occupations with a 11 in parentheses refers to the fact that these occupations were combined to make the 11th occupation in our broader occupation classification.

Table G3: Additional Robustness

| | Market GDP per person growth accounted for by | | |
	τ^h and τ^w	τ^h only	τ^w only
Benchmark	41.5%	36.0%	7.7%
Wage gaps halved	37.5%	31.1%	9.6%
Zero wage gaps	33.5%	25.5%	11.9%
Half the return to experience	42.1%	37.6%	6.4%
2/3, 1/3 split of $\tau_{i,g}$ in 1960	39.2%	34.5%	5.3%
1/3, 2/3 split of $\tau_{i,g}$ in 1960	41.7%	33.3%	12.8%
No constraint on τ^h	46.1%	42.5%	4.3%

Note: See notes to Table 5 in the main text. The baseline splits τ in 1960 evenly into τ^h and τ^w in 1960, but not in future years. The baseline also constrains τ^h to be at most –0.8. The robustness of 2/3, 1/3 split of $\tau_{i,g}$ means that 2/3 of the initial $\tau_{i,g}$ is assigned to $\tau_{i,g}^w$ in 1960. Conversely, the robustness of 1/3, 2/3 split of $\tau_{i,g}$ means that 1/3 of the initial $\tau_{i,g}$ is assigned to $\tau_{i,g}^w$ in 1960.

gains we estimate are *not* proportional to the gender and race wage gaps we fed into the model. We can halve the wage gaps in all years, or even eliminate them in all years, and the implied τ's still explain 37.5% or 33.5% of growth in market GDP per person, vs. 41.5% in the baseline. One reason is that misallocation of talent by race and gender can occur even if average wages across groups are similar. The misallocation of talent is tied to the *dispersion* in the τ's, whereas the wage gaps are related to both the mean and variance of the τ's. Another reason is that the wage gap for white women would have widened in the absence of the changing τ's due to changes in the A's and ϕ's. A key take away from this exercise is that productivity gains from changing labor market discrimination and barriers to human capital accumulation cannot be gleaned from the changing wage gaps alone.

Another assumption we make in our base specification is that the returns to experience are constant across groups and occupations over time. We want to stress that allowing for general returns to experience is not adding much to our inference. The fourth row in Table G3 illustrates this point. Specifically, in this robustness exercise, we cut productivity growth over the life cycle (old/middle and middle/young) in half for

each group. Such a change barely alters our baseline results.

The final three rows of Table G3 considers additional robustness checks. In our benchmark we split the composite τ_{ig} in 1960 evenly into τ^w and τ^h. Our procedure estimates changes in τ's over time but we need to make an assumption on the initial split between τ^h and τ^w given that we have nothing to discipline this initial split in the data. If we put more weight (2/3) on τ^w we account for 39.2% of growth in market GDP per person, versus 41.5% in the baseline. If we put less weight (1/3) on τ^w we account for 41.7% of growth. Finally, our benchmark case constrains the values of τ^h to be no smaller than –0.8. If we put no constraint on how negative τ^h can get, we explain 46.1% of growth vs. 41.5% in the baseline. Absent this constraint, the estimation implies human capital subsidies that exceed GDP because of large subsidies for white women (e.g. as secretaries).

The baseline model also implies that the "revenue" from τ^w and τ^h is changing over time. Figure G1 shows how such revenue in our baseline model evolves from 1960 to 2010. The figure displays that the revenue from both τ^w and τ^h combined shrinks from around 4% of GDP in 1960 to -4% of GDP in 2010. This decline results in the market earnings growth of workers being slightly larger than market GDP growth over the sample.

H   Further Model Implications

While our model is stylized in many respects, it is able to match at least two other important facts that were not targeted in the estimation: trends in female labor supply elasticities and cross-state variation in survey measures of racial discrimination. In this section of the Appendix, we discuss these results.

H.1   Trends in Female Labor Supply Elasticities

Using data from the Current Population Survey, Blau and Kahn (2007) estimate that there was a dramatic decline in female labor supply elasticity during the 1980–2000 period. Helpful for comparing with the predictions of our model, they report female labor supply elasticities specifically for 25-34 year olds. We compare the model's implied labor supply elasticities — equal to $\theta(1 - LPF_g)$ — for young white women to the es-

Figure G1: Revenue from τ as share of GDP in the Model
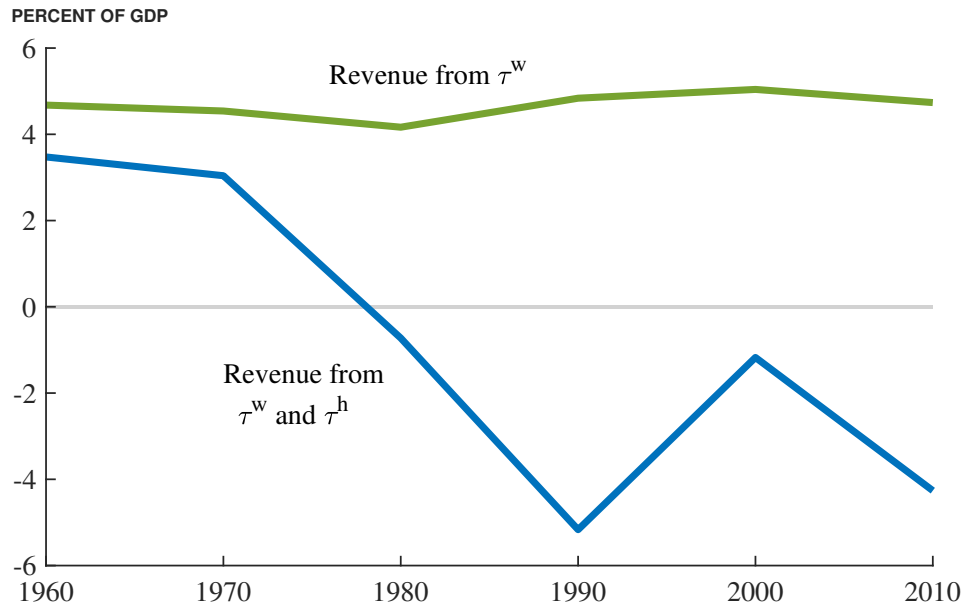
PERCENT OF GDP

Note: The graph shows the employer revenue from discrimination in the labor and human capital markets as a percent of GDP.

timated labor supply elasticities reported in Blau and Kahn (2007). Using our baseline θ, the model matches both the level and the trend female labor supply elasticities well. Blau and Kahn (2007) report labor supply elasticities for women aged 25-34 of 0.75, 0.60 and 0.35, respectively in 1980, 1990, and 2000 — a change of 0.40 over the time period. Our comparable model estimates for young women are 0.90, 0.70, and 0.65 for the three years - a change of 0.25 over the time period. Our estimates are only slightly higher in levels than the Blau and Khan estimates over the three years with a roughly similar trend.

Nothing in our model is calibrated to match either the level or the trend in labor supply elasticities for women. As discussed earlier, we estimated θ to match the labor supply elasticity of men in 1980. With that parameter pinned down, our model implies that women's labor supply elasticity is only a function of female labor force participation. The fact that we can roughly match the level of the labor supply elasticity for young women in three different time periods suggests that our model is broadly consistent with empirical moments outside the ones we used to calibrate the model.
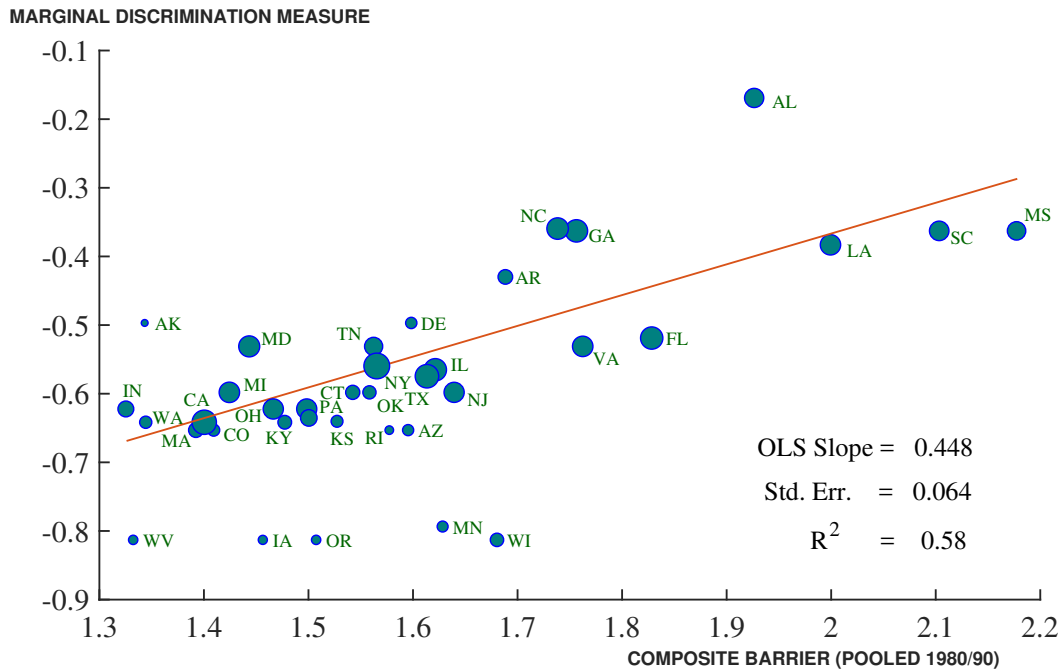
H.2   Cross State Measures of Discrimination

There are very few micro-based measures of discrimination to which we can compare
our estimated τ's. One such exception is the recent work by Charles and Guryan (2008).
Charles and Guryan (CG) used data from the General Social Survey (GSS) to construct
a measure of the taste for discrimination against blacks for every state. The GSS asks a
large nationally representative sample of individuals about their views on a variety of
issues. A series of questions have been asked over the years assessing the respondents
attitudes towards race. For example, questions were asked about individuals' views on
cross-race marriage, school segregation, and the ability for homeowners to discrimi-
nate with respect to home sales. Pooling together survey questions from the mid 1970s
through the early 1990s and focusing only a sample of white respondents, Charles and
Guryan make indices of the extent of racial discrimination in each state.[40] Higher
values of the CG discrimination measure imply more discrimination. They compute
their measure for 44 states.

Figure H2 shows a simple scatter plot between the CG measure of discrimination
and our measure τ_{bm} at the state level.[41] Each observation in the scatter plot is a U.S.
state where the size of the circle represents the number of black men within our Census
sample. We also show the weighted OLS regression line on the figure. As seen from
the figure, there is a very strong relationship between our measures of τ_{bm} and the CG
discrimination index. The adjusted R-squared of the simple scatter plot is 0.6 and the
slope of the regression line is 0.45 with a standard error of 0.06. Places we identify as
having a high τ_{bm} are the same places Charles and Guryan find as being highly discrimi-
natory based on survey data from the GSS. The findings in Figure H2 provide additional

---

[40]We focus on their marginal discrimination measure. The concept of the marginal discriminator comes
from Beckers theory of discrimination. If there are 10 percent of blacks in the state labor market, it is only
the discrimination preferences of the white person at the 10th percentile of the white distribution that
matters for outcomes (with the first percentile being the least discriminatory).

[41]From our earlier estimates, we compute a composite τ measure for black men relative to white men
in each U.S. state. To ensure we have enough observations in each state, we make a few simplifying
assumptions. First, we assume that there are no cohort effects in our composite measure of τ. This allows
us to pool together all cohorts within a year when computing our measure of τ. Next, we collapse our
67 occupations to 20 occupations; see Appendix Table E2. Also, we pool together data from 1980 and
1990; we do this because the CG discrimination measure is based on data pooled from the GSS between
1977 and 1993. We then aggregate $\tau_{i,bm}$ from our 20 different occupations to one measure of τ_{bm} for each
state by taking a weighted average of the occupation level τs where the weights are based on share of the
occupations income out of total income across all occupations (both for the country as a whole). Finally,
we exclude states with an insufficient number of black households to compute our measure of τ_{bm}. Given
the CG restrictions from the GSS and our restrictions from the Census data, we are left with 37 states.

Figure H2: Model τ's for Black Men vs. Survey Measures of Discrimination, by U.S. State

Note: Figure plots measures of our model's implied composite τ's for black men for each state using pooled data from the 1980 and 1990 census (x-axis) against survey-based measures of discrimination against blacks for each state as reported in Charles and Guryan (2008). The Charles and Guryan data are complied using data from the General Social Survey between 1977 and 1993. We use their marginal discrimination measure for this figure.

external validity that our procedure is measuring salient features of the U.S. economy over the last five decades.