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Finding Ultimate Limits of Performance for Hybrid Electric Vehicles

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ABSTRACT

Hybrid electric vehicles are seen as a solution to improving fuel economy and reducing pollution emissions from automobiles. By recovering kinetic energy during braking and optimizing the engine operation to reduce fuel consumption and emissions, a hybrid vehicle can outperform a traditional vehicle. In designing a hybrid vehicle, the task of finding optimal component sizes and an appropriate control strategy is key to achieving maximum fuel economy.

In this paper we introduce the application of convex optimization to hybrid vehicle optimization. This technique allows analysis of the propulsion system's capabilities independent of any specific control law. To illustrate this, we pose the problem of finding optimal engine operation in a pure series hybrid vehicle over a fixed drive cycle subject to a number of practical constraints including:

- nonlinear fuel/power maps
- min and max battery charge
- battery efficiency
- nonlinear vehicle dynamics and losses
- drive train efficiency
- engine slew rate limits

We formulate the problem of optimizing fuel efficiency as a nonlinear convex optimization problem. This convex problem is then accurately approximated as a large linear program. As a result, we compute the globally minimum fuel consumption over the given drive cycle. This optimal solution is the lower limit of fuel consumption that any control law can achieve for the given drive cycle and vehicle. In fact, this result provides a means to evaluate a realizable control law's performance.

We carry out a practical example using a spark ignition engine with lead acid (PbA) batteries. We close by discussing a number of extensions that can be done to improve the accuracy and versatility of these methods. Among these extensions are improvements in accuracy, optimization of emissions and extensions to other hybrid vehicle architectures.

INTRODUCTION

Two areas of significant importance in automotive engineering are improvement in fuel economy and reduction of emissions. Hybrid electric vehicles are seen as a means to accomplish these goals.

The majority of vehicles in production today consist of an engine coupled to the road through a torque converter and a transmission with several fixed gear ratios. The transmission is controlled to select an optimal gear for the given load conditions. During braking, velocity is reduced by converting kinetic energy into heat.

For the purposes of this introduction, it is convenient to consider two propulsion architectures: pure parallel and pure series hybrid vehicles.

A parallel hybrid vehicle couples an engine to the road through a transmission. However, there is an electric motor that can be used to change the RPM and/or torque seen by the engine. In addition to modifying the RPM and/or torque, this motor can recover kinetic energy during braking and store it in a battery. By changing engine operating points and recovering kinetic energy, fuel economy and emissions can be improved.

A series hybrid vehicle electrically couples the engine to the road. The propulsion system consists of an engine, a battery and an electric motor. The engine is a power source that is used to provide electrical power. The electrical power is used to recharge a battery or drive a motor. The motor propels the vehicle. This motor can also be used to recover kinetic energy during braking.

For a given type of hybrid vehicle, there are three questions of central importance:

- What are the important engine, battery and motor requirements?
- When integrated into a vehicle, what is the best performance that can be achieved?
- How closely does a control law approach this best performance?

Answers to these questions can be found by solving three separate problems:

- Solving for the maximum fuel economy that can be obtained for a fixed vehicle configuration on a fixed drive cycle independent of a control law.
- Given a method to find maximum fuel economy, vary the vehicle component characteristics to find the optimal fuel economy.
- Apply the selected control law to the system and determine the fuel consumption. Calculate the ratio between this control law's fuel consumption and the optimal value to give a metric for how close the control law comes to operating the vehicle at its maximum performance.

There are many hybrid vehicle architectures[1]. For the sake of simplicity, a pure series hybrid was chosen for this study. However, the methods used for series hybrid vehicles can be extended to apply to other hybrid vehicle architectures. This study was restricted to minimizing fuel economy. This method can be extended to include emissions.

DISCUSSION: FINDING THE MAXIMUM FUEL ECONOMY FOR A GIVEN VEHICLE

There are many approaches that can be used to determine the maximum fuel economy that can be obtained by a particular vehicle over a particular drive cycle. One common approach is to select a control law and then optimize that control law for the system. Other techniques search through control law architectures and control parameters simultaneously. Since these techniques select a control law before beginning the optimization, the minimum fuel economy found is always a function of the control law. This leaves open the question of whether selection of a better control law could have resulted in better fuel economy.

The approach presented here finds the minimal fuel consumption of the vehicle independent of any control law. Because a control law is not part of the optimization, the fuel economy found is the best possible. It is noncausal in that it finds the minimum fuel consumption using knowledge of future power demands and past power demands. Therefore it represents a limit of performance of a causal control law. Furthermore, since the problem is formulated as a convex problem and then a linear program, the minimum fuel consumption calculated is guaranteed to be the global minimum solution. The discussion that follows details:

1. The formulation of the fuel economy minimization problem as a convex problem.
2. The reduction of this convex problem to a linear program.
3. Solution of the linear program to find the minimum fuel consumption.

DESCRIBING THE PROBLEM

To solve for maximum fuel economy, a model of the series hybrid vehicle is used. To simplify the model, the following assumptions are made:

- The voltage on the electrical bus is constant. Voltage droop and ripple can be ignored.
- The relationship between power output from the engine and fuel consumption can be assumed to be a fixed relationship that is not affected by transients.
- The battery's storage efficiency is constant. It does not change with state of charge or power levels.

These simplifications are used to reduce the complexity of the resulting linear program and to maintain a problem description which is convex. These simplifications illustrate one of the challenges that arises in the application of convex analysis to engineering problems – finding a description of the problem which is convex.

<u>The System Model</u>

Using these simplifications, Figure 1 provides a signal flow diagram of the model.

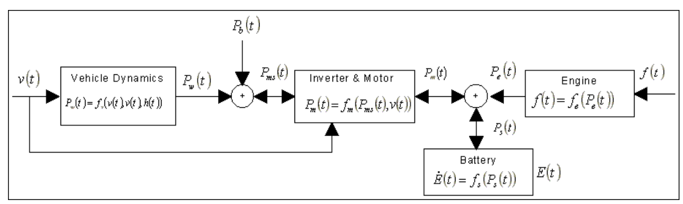

Figure 1 - Series Hybrid Vehicle Model

From this model, the equations that describe the behavior follow.

The fuel consumption at time t will be denoted $f(t)$, and is assumed to be related to the engine electrical power output, denoted $P_e(t)$ by a nonlinear, memoryless function f_e.

$$f(t) = f_e(P_e(t)) \quad (1)$$

We assume that f_e is increasing (since more power requires more fuel) and also convex, which is accurate for most engines. This function is formed by considering the engine, generator and inverter as a single component. This component has fuel as input and electrical power as output. It is assumed that this component is optimized to produce electrical power for minimum fuel consumption under steady state conditions. A possible fuel curve for such a component is illustrated in Figure 2.

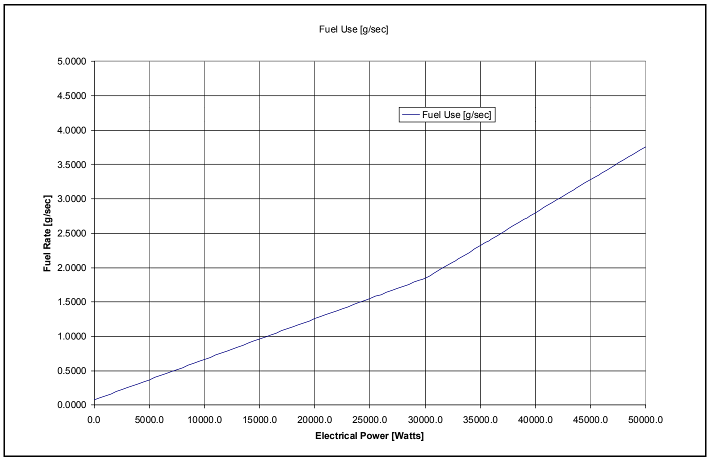

Figure 2 - Illustration of a fuel map

The energy stored in the battery at time t is denoted $E(t)$ and evolves according to the differential equation

$$\dot{E}(t) = f_s(P_s(t)) \qquad (2)$$

where $P_s(t)$ denotes the electrical power flowing into the battery (or out if $P_s(t) < 0$) and f_s is a nonlinear memoryless function that relates the energy in the battery to the charging power. The subscript s is used to denote storage. For example, a lossless battery would have $\dot{E}(t) = P_s(t)$. To model a battery with a 10% loss during charging, we would use

$$f_s(P) = \begin{cases} P, \text{if } P < 0 \\ 0.9 \cdot P, \text{if } P >= 0 \end{cases}$$

More sophisticated models are possible. These models are illustrated in Figure 3. Any of these models can be used. For the purposes of this paper, the simple fixed losses model is used.

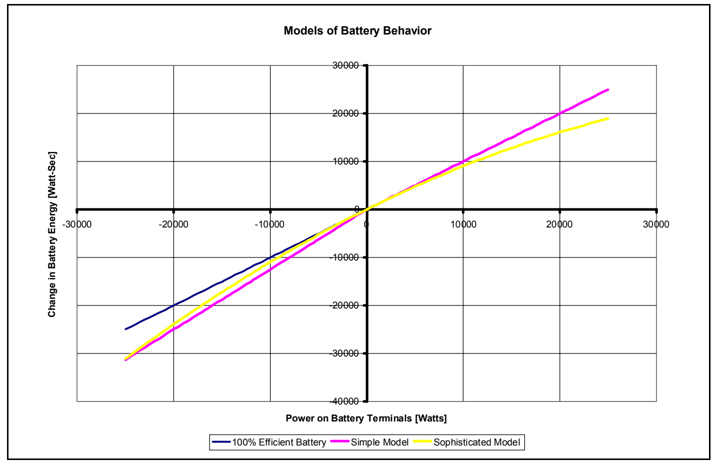

Figure 3 - Illustration of Battery Losses

The balance of electrical power between the battery ($P_s(t)$), the engine($P_e(t)$) and the electrical side of the motors ($P_m(t)$) gives

$$P_e(t) = P_s(t) + P_m(t) \qquad (3)$$

The losses incurred in converting electrical power into mechanical power via the inverters and motors gives

$$P_{ms}(t) = f_m(P_m(t), v(t), \dot{v}(t)) \qquad (4)$$

Where f_m relates the motor's electrical power, P_m, to the shaft power $P_{ms}(t)$ at a wheel speed and acceleration as indicated by $v(t)$ and $\dot{v}(t)$. This function includes effects such as inverter efficiency, motor efficiency, transmission and motor inertia. Additionally, the accessory power loads on the vehicle are accounted for in this function. These accessory loads include blowers, radio, instrument panel, onboard controllers, etc.

The balance of mechanical power at the motor shaft, brakes and wheels gives the equation

$$P_w(t) + P_b(t) + P_{ms}(t) = 0 \qquad (5)$$

Finally, the power at the wheels of the vehicle at time t is denoted $P_w(t)$. We assume that the power is related to vehicle velocity, acceleration and road slope as

$$P_w(t) = f_v(v(t), \dot{v}(t), h(t)) \qquad (6)$$

The function $f_v(\cdot,\cdot,\cdot)$ includes aerodynamic losses, rolling losses, acceleration power and the power related to changing the vehicle's altitude. This relationship is typically expressed as

$$f_v(v,\dot{v},h) = \left(0.5 \cdot \rho \cdot (v)^2 \cdot Cd \cdot A + m \cdot g \cdot (Crr + \dot{v}/g + h/100)\right) \cdot v \quad (7)$$

where

ρ = density of air
Cd = Coefficient of drag of the vehicle
Crr = Coefficient of rolling resistance
A = frontal area of the vehicle
m = vehicle mass
g = acceleration due to gravity
h = road slope (0 for level terrain)
v = velocity of the vehicle.
$\dot{v}$ = acceleration of the vehicle

Constraints and Objective

In the previous section, we identified a set of functions that describe the vehicle and its pure series hybrid power system. In this section, we describe a set of constraints that are imposed on these variables, either by underlying physics or by engineering design.

The first constraint is on engine power levels. The engine can only produce power. This is expressed as

$$P_e \geq 0 \quad (8)$$

When producing power, the engine is limited to a maximum output power. This is expressed as

$$P_e \leq Max\_Engine\_Power \quad (9)$$

The engine output power can only change at finite rates. This rate is limited by inertia and the desire to eliminate misfueling due to load transients. The rate of change is limited differently for increasing and decreasing power changes through

$$\dot{P}_e \leq Max\_Engine\_Slew\_Up \quad (10)$$

$$\dot{P}_e \geq Max\_Engine\_Slew\_Down \quad (11)$$

The brakes are constrained to only absorb power. When absorbing power, the brakes are constrained to absorb a limited amount of power. To simplify this study, the maximum power absorbed by the brakes is assumed to be a constant. A more sophisticated model would compute the maximum power that can be absorbed at each instant in the drive cycle and have a time varying limit on braking power. The limits on braking power are represented by

$$P_b \geq 0 \quad (12)$$

$$P_b \leq Max\_Braking\_Power \quad (13)$$

The battery is limited to a maximum energy set by the storage capacity of the battery. The minimum energy represents the reserve energy that is required by some battery systems. The limits on battery energy are

$$E \geq Min\_Battery\_Energy \quad (14)$$

$$E \leq Max\_Battery\_Energy \quad (15)$$

The charge and discharge rates of the battery are constrained by

$$P_s \geq Max\_Discharge\_Rate \quad (16)$$

$$P_s \leq Max\_Charge\_Rate \quad (17)$$

To act as a charge sustaining hybrid, the battery is constrained to have the same amount of energy at the start of the test and at the end of the test by

$$E(t_0) = E(t_f) \quad (18)$$

The total fuel used is

$$F = \int_{t_0}^{t_f} f(t) \cdot dt$$

Now, we can describe the optimization problem we consider. We make the following assumptions:

- The trajectories $v(t), \dot{v}(t), h(t)$ are known. For many automotive applications, this trajectory would be the FTP, US06 or similar drive schedule.
- The conditions of the test are known and constant. Therefore ρ and g are constant
- The functions f_m, f_e, f_v and f_s appearing in the system model are known. These functions are defined by the vehicle and powertrain characteristics.
- The vehicle characteristics and parameters Cd, A, m and Crr are known.
- The fuel use function f_e is convex.
- The battery charge/discharge function f_s is convex.

The variables in this problem are the trajectories of the engine power (P_e), the battery power (P_s) and the brake power (P_b) over the time t_0 to t_f. The constraints are given by equations 1 through 6 and 8 through 18. We will use the minimization of total fuel use as the objective in our optimization problem. This problem is summarized in Figure 4.

$$\min_{P_e(t)} \left(\int_{t=t_0}^{t=t_f} \dot{f}(t) \cdot dt \right)$$

(19)

subject to equations (1) through (6) and (7) through (18)

Figure 4 - The minimization problem

Note carefully, the interpretation of this optimal control problem: we are asking for the minimum fuel trajectory, given complete, perfect information about the trajectory ahead of time. In contrast, a real power control law must be causal, that is, it must base its engine power at time t on the information available at time t, not on the future trajectory.

SETTING UP THE PROBLEM AS A LINEAR PROGRAM

Posing the Problem as a Convex Optimization Problem

The problem described above in equation (19) is a complex optimal control problem involving a number of trajectory (function) variables, all coupled together via a variety of equality and inequality constraints. In this section, we show how the problem can be approximated accurately by a large, but finite dimensional convex optimization problem. This is done by first simplifying the model. Next, the trajectories are discretized. Then the nonlinear functions are approximated using piecewise-linear approximations.

Simplifying the Model

By modifying the model, the nonlinearities introduced by $f_m(\cdot,\cdot,\cdot), f_v(\cdot,\cdot,\cdot)$ can be moved outside of the optimization problem. Figure 5 illustrates the changes to the model. For the purposes of minimizing fuel consumption, these two models are equivalent. The difference is that the braking power ($P_b(t)$), which originally indicated heat power at the brakes, now shows up as electrical power dissipation. This is not how the braking behaves, however for the purposes of determining minimum fuel consumption, this is an accurate simplification.

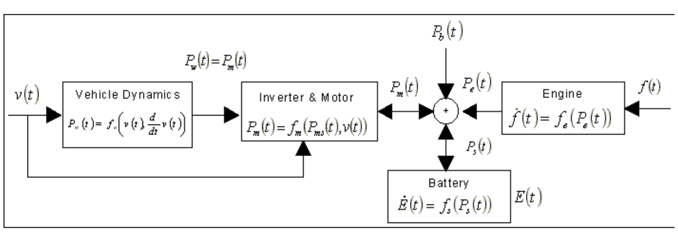

Figure 5 - Modified Series Hybrid Vehicle Model

Given this simplified model, $P_m(t)$ is completely determined by $v(t)$. So, given $v(t)$, $P_m(t)$ is now known. If $P_m(t)$ is now considered the input to the optimization problem, the model can be further reduced as illustrated in Figure 6.

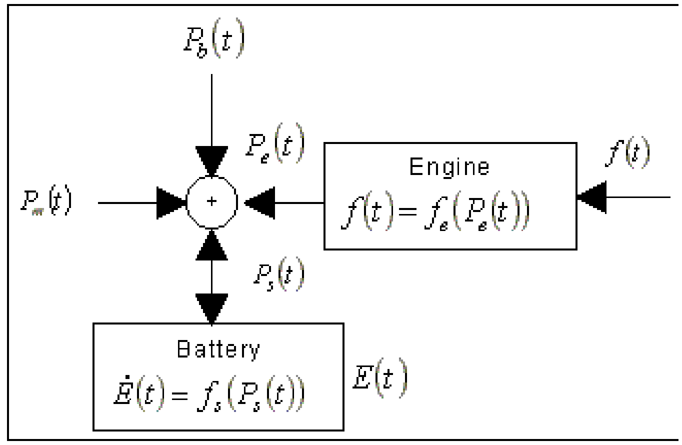

Figure 6 - Simplified Model

This simplified model yields a new set of equations to describe the behavior of the vehicle. These equations follow.

$$f(t) = f_e(P_e(t)) \quad (20)$$

$$\dot{E}(t) = f_s(P_s(t)) \quad (21)$$

$$P_e(t) = P_s(t) + P_m(t) + P_b(t) \quad (22)$$

The constraints in equations (8) through (18) are not affected by these model changes. So now, equations (20) through (22) and (8) through (18) form the constraints on the minimization problem.

Posing the Problem as a Linear Program

Solving this simplified problem in continuous time is possible. However, by converting the problem into discrete time, the problem can be solved as a Linear Program (LP). The rest of this section will illustrate the steps used in converting the problem statement in equations (8) through (22) into a LP.

The first step is to convert the problem statement into discrete time. Since the problem statements contain time derivatives, equation (23) will be used to approximate derivatives.

$$\frac{d}{dt}x(t) = \dot{x}(t) \approx \frac{x(k \cdot T) - x((k+1) \cdot T)}{T} \quad (23)$$

Integrals will be approximated as shown in equation (22).

$$\int_{t=t_i}^{t_f} x(t) \cdot dt \approx T \cdot \sum_{j=k_i}^{k_f} x(j \cdot T) \quad (24)$$

More sophisticated approximations can be used. However for the purposes of this study, these methods were found to provide adequate accuracy. Additionally, T is assumed to be 1 second for the discrete time model.

Given these approximations, the optimization problem statement is expressed in equations (25) through (39)

$$\min \sum_{j=k_0}^{k_f} (f(k)) \quad (25)$$

subject to

$$f(k) = f_e(P_e(k)) \quad (26)$$
$$E(k) - E(k+1) = f_b(P_s(k)) \quad (27)$$
$$P_e(k) = P_s(k) + P_m(k) + P_b(k) \quad (28)$$
$$P_e(k+1) - P_e(k) \le Max\_Engine\_Slew\_Up \quad (29)$$
$$P_e(k+1) - P_e(k) \ge Max\_Engine\_Slew\_Down \quad (30)$$
$$P_e(k) \ge 0 \quad (31)$$
$$P_e(k) \le Max\_Engine\_Power \quad (32)$$
$$P_b(k) \ge 0 \quad (33)$$
$$P_b(k) \le Max\_Braking\_Power \quad (34)$$
$$P_s(k) \ge Max\_Disch\arg e\_Rate \quad (35)$$
$$P_s(k) \le Max\_Ch\arg e\_Rate \quad (36)$$
$$E(k) \ge Min\_Battery\_Energy \quad (37)$$
$$E(k) \le Max\_Battery\_Energy \quad (38)$$
$$E(k_0) = E(k_f) \quad (39)$$

For review, the problem variables and constants are shown in Table 1.

Table 1 - Constants and Variables

Variables	Constants	Constant Functions
$f(k)$ $E(k)$ $P_e(k)$ $P_b(k)$ $P_s(k)$	Max_Engine_Power Max_Discharge_Rate Max_Charge_Rate Min_Battery_Energy Max_Battery_Energy Max_Engine_Slew_Down Max_Engine_Slew_Up $P_m(k)$	$f_e(\cdot)$ $f_b(\cdot)$

The problem is now a finite dimensional, but large, nonlinear optimization problem.

For the problem to be an LP it must be cast in the standard LP form shown in Figure 7. The advantage of posing the LP in standard form is that there is readily available software to efficiently solve the problem. This software includes PCx [3] and Matlab's optimization toolbox.

$$\min c^T \cdot x$$

subject to

$$A \cdot x = b$$
$$x \ge 0$$

Where

c	- is a $n \times 1$ vector
x	- is a $n \times 1$ vector
A	- is an $m \times n$ array
b	- is an $m \times 1$ vector

Figure 7 - Standard Form of LP

To pose our problem as a LP, we will use piecewise-linear approximations of the functions f_s and f_e, then apply a number of transformations to the problem, finally arriving at the standard LP form.

Finding an equivalent statement for Equation (26)

To change equation (26) into a statement that can be used in an LP, the following technique is used. Consider using an LP to find the minimum value of a continuous convex function. The problem can be stated as: $\min f_c(x)$ with no constraints on x. This problem can not be reduced to an LP in this form. If $f_c(x)$ is approximated using a piecewise linear function $f_d(x)$, with N pieces, where $f_d(x) = \max_i \{a_i \cdot x + b_i\}$, then the problem can be restated as $\min f_d(x)$, with no constraints on x. This is still not an LP, however, introducing a new variable and restating this problem yet again yields:

$$\min y$$

subject to

$$y = \max_i \{a_i \cdot x + b_i\}, \{i = 1..N\}$$

Finally, problem statement can be converted into an LP by restating as

$$\min y$$

subject to

$$y \ge a_i \cdot x + b_i, \{i = 1..N\}$$

Since the LP tries to minimize y, the optimal solution set is constrained to the curve described by $y = \max_i \{a_i \cdot x + b_i\}, \{i = 1..N\}$. Since it is assumed that $f_e(x)$ is convex, this same approximation can be used. Applying this concept to equation (26) requires first finding the approximation

$$f_e(x) = \max_i \{a_i \cdot x + b_i\}, \{i = 1..N\}.$$

Given this approximation, the equation is expanded into a set of inequalities such that

$$f(k) \geq a_i \cdot P_e(k) + b_i, \{i = 1..N\}. \quad (40)$$

This set of inequalities can be directly used in an LP in place of equation (26). For the studies that were conducted as part of this research, N was 2.

Finding an equivalent statement for Equation (27)

Equation (27) describes the change in battery energy as a result of power at the battery terminals. The problem with this equation is to approximate a continuous function in a way that results in constraints that can be used in an LP. Since we choose to assume that efficiency is constant, a very simple approximation is used:

$$f_b(x) = \begin{cases} g_1 \cdot x, x \geq 0 \\ g_2 \cdot x, x < 0 \end{cases} \quad (41)$$

By selecting the coefficients of equation (41) properly, this equation describes a battery with a fixed energy storage efficiency.

Equation (41) can be implemented in the LP because variables that are positive and negative must be separated into a negative portion and a positive portion in the standard LP form as shown in Figure 7. A standard notation for this separation is to refer to positive portion of x as x^+ and the negative portion as x^-. Because of the nature of the solutions, only one of the variables x^+ or x^- will be nonzero at any time. Therefore, equation (41) can be restated as

$$f_b(x) = g_1 \cdot x^+ - g_2 \cdot x^-, x = x^+ - x^- \quad (42)$$

Describing $P_s(k)$ using Nonnegative Variables

As described previously, a variable that has positive and negative values can be described using two nonnegative variables. Equation (43) shows how this can be done for $P_s(k)$.

$$P_s(k) = P_s^+(k) - P_s^-(k) \quad (43)$$

Then using this substitution, equations (27),(28),(35), and (36) can be rewritten as follows.

$$E(k) - E(k+1) = g_1 \cdot P_s^+(k) - g_2 \cdot P_s^-(k) \quad (44)$$

$$P_e(k) + P_s^-(k) = P_s^+(k) + P_m(k) + P_b(k) \quad (45)$$

$$P_s^-(k) \leq Max\_Discharge\_Rate \quad (46)$$

$$P_s^-(k) \geq 0 \quad (47)$$

$$P_s^+(k) \leq Max\_Charge\_Rate \quad (48)$$

$$P_s^+(k) \geq 0 \quad (49)$$

Creating the LP

Combing these results yields a equations which can be formed into an LP that solves for minimum fuel consumption. The resulting equations are illustrated in Figure 5

Optimization Goal:

$$\min \sum_{j=k_0}^{k_f} (f(k))$$

Constraints: *Vehicle Dynamics*

$$f(k) \geq a_i \cdot P_e(k) + b_i, \{i = 1..N\}$$

$$E(k) - E(k+1) = g_1 \cdot P_s^+(k) - g_2 \cdot P_s^-(k)$$

$$P_e(k) + P_s^-(k) = P_s^+(k) + P_m(k) + P_b(k)$$

Constraints: *Operating Constraints*

$$P_b(k) \geq 0$$

$$P_b(k) \leq Max\_Braking\_Power$$

$$P_e(k) \geq 0$$

$$P_e(k) \leq Max\_Engine\_Power$$

$$P_e(k+1) - P_e(k) \leq Max\_Engine\_Slew\_Up$$

$$P_e(k+1) - P_e(k) \geq Max\_Engine\_Slew\_Down$$

$$P_s^+(k) \leq Max\_Charge\_Rate$$

$$P_s^+(k) \geq 0$$

$$P_s^-(k) \leq Max\_Discharge\_Rate$$

$$P_s^-(k) \geq 0$$

$$E(k) \geq Min\_Battery\_Energy$$

$$E(k) \leq Max\_Battery\_Energy$$

$$E(k_0) = E(k_f)$$

Figure 8 – Equations for LP to Solve for Minimum Fuel Use

The final step, that must be done to solve this as an LP, is to create the matrices to get the problem into standard form. There are many good texts which explain these steps. For further explanation see Bertsimas & Tsitiklis,97 [2].

NUMERICAL EXAMPLE

To illustrate the results obtained using this method, a numerical example was performed. Figure 9 through Figure 14 illustrate a sample run for a passenger car roughly based on a mid-sized sedan. The resulting LP has 26068 variables and 19210 constraints. The matrices in the LP were very sparse with a total of 69971 nonzero coefficients. The problem was solved in less than 2 minutes on a Pentium Pro running at 200 MHz using PCx [3].

The drive cycle is the first 1371 seconds of the FTP. The vehicle mass is 1072 kg with a CD of 0.3 and a frontal area of 1.96 meters^2. The coefficient of rolling resistance (Crr) is 0.015. Air density is 1.22 kg/m^3. The vehicle is modeled with a 2 kW accessory power load.

Performance and mass scaling for the components are based on PNGV [4] recommendations.

The engine has 50 kW maximum power with a maximum slew up rate of 10 kW/sec and a slew down rate of 20 kW/sec. The fuel consumption curve for the engine is approximated by

$$f(t) \geq P_e(t) * 0.000059 + 0.075000$$
$$f(t) \geq P_e(t) * 0.000096 - 1.041667$$

where $f(t)$ is in units of grams per second and P_e are in units of kW.

The battery is lead acid (PbA) with 0.60 kW-hr maximum energy capacity and a reserve of 0.12 kW-hrs. The maximum charge and discharge rates are 9.54 kW. The battery has a charge efficiency of 80% and a discharge efficiency of 100%.

The resulting global optimal fuel economy is 44.44 mpg (calculated as total mileage/total fuel) for the first 1371 seconds of the FTP. Without the battery, this vehicle configuration achieves 41.55 mpg. It is important to note that, because of the problem description, this optimization result does not shut the engine off at any time. Additional methods are required to incorporate this into the optimization problem.

The detailed results of the optimization are illustrated in Figure 9 through Figure 14. Figure 9 shows the electrical power required to meet the drive schedule and the electrical power available from 100% regeneration at the terminals of the inverter. This is a plot of $P_m(t)$.

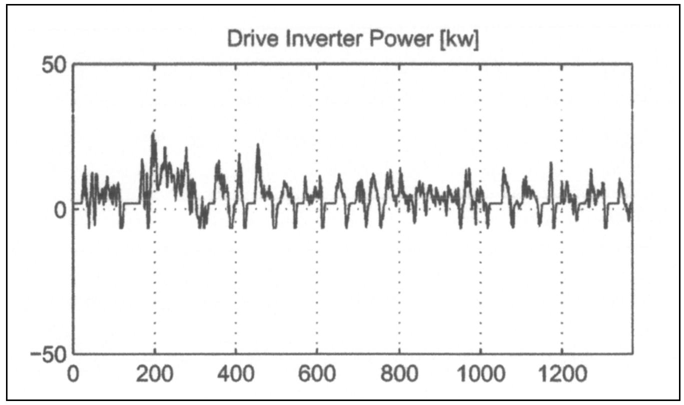

Figure 9 - Inverter Power

Figure 10 shows the optimal schedule for producing electrical power using the engine. This plot shows $P_e(t)$.

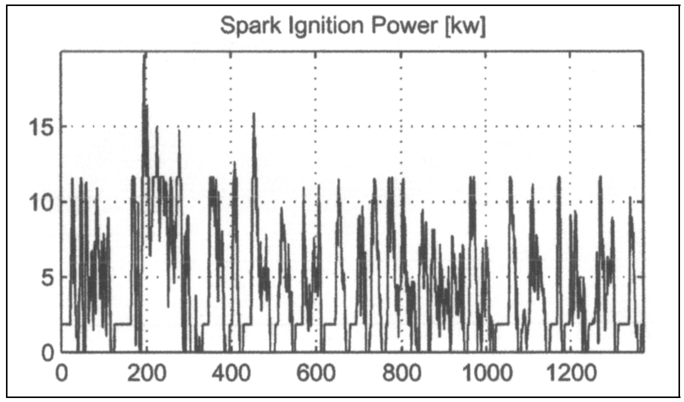

Figure 10 - Engine Power

The fuel use, $f(t)$, as a result of optimal engine operation, is shown in Figure 11.

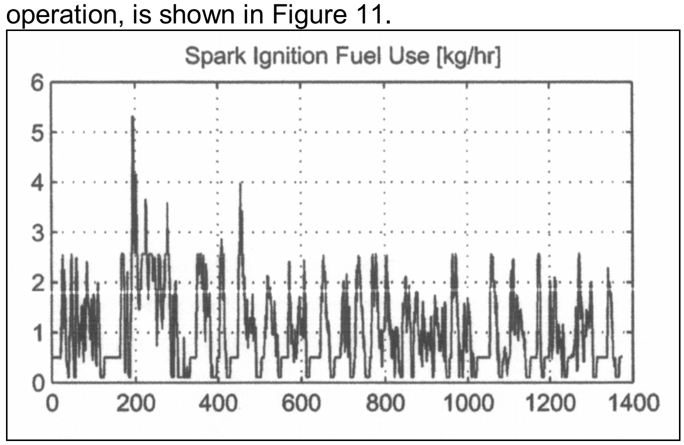

Figure 11 - Engine Fuel Use

The optimal braking power is shown in Figure 12. This plots $P_b(t)$. A counter intuitive result occurs in this plot. The brakes are applied. Since the optimization problem is to minimize fuel consumption, it would seem that the brakes would not be used. It is counter intuitive that energy available through regeneration would be lost by applying brakes. For the system studied in this example, the brakes were applied because of a combination of constraints, The engine's rate of change is constrained by equation (10) and (11). The battery is limited to accepting a maximum amount of power by equation (17). By solution of the LP we find that the optimal fuel economy occurs when the engine is operated such that some braking occurs.

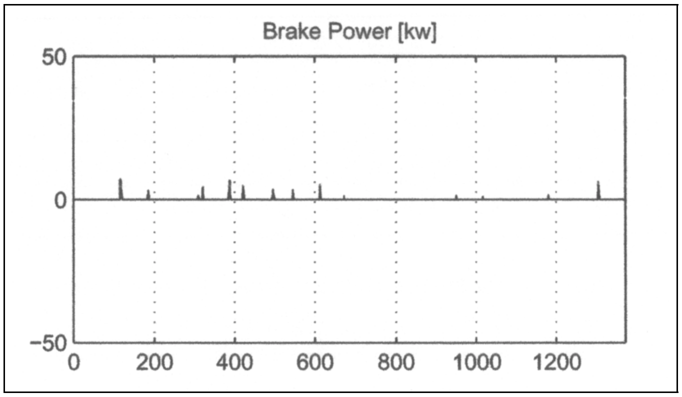

Figure 12 - Braking Power

Figure 13 shows the resulting power on the terminals of the battery, $P_s(t)$.

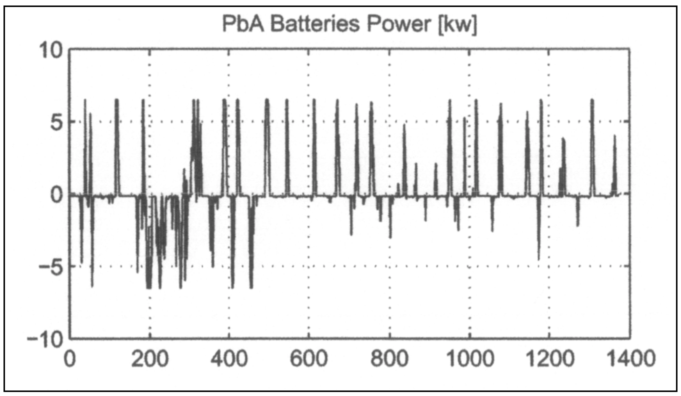

Figure 13 - Battery Power

Figure 14 shows the instantaneous energy in the battery, $E(t)$. An interesting result is that only a fraction of the battery's capacity is used to achieve optimal fuel economy.

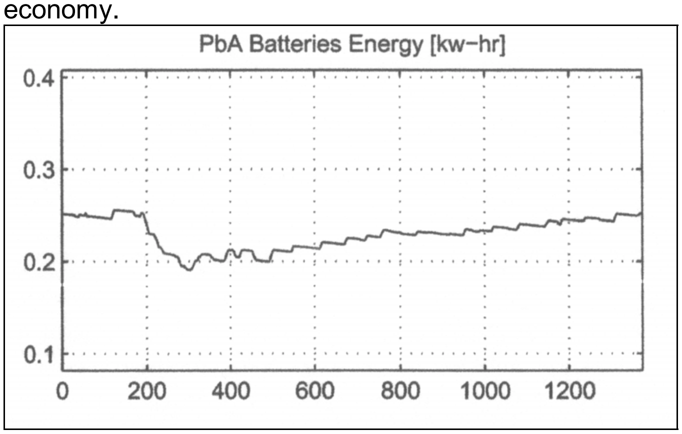

Figure 14 - Battery Energy

DISCUSSION: FINDING OPTIMAL COMPONENT SIZES

SIZING COMPONENT USING THE CONVEX OPTIMIZATION RESULTS

Given the method developed in section 2, the optimal component size for maximum fuel economy can be found. If the assumption is made that the component sizes drive the constraints used in equations (25) through (32), then the component sizing problem reduces to a 2 variable optimization problem.

One difficulty that emerges is that the component sizes affect the relationship between vehicle velocity and motor shaft power in $f_v(\cdot,\cdot)$. This is because the mass of the components changes the vehicle mass (m) in equation (7). However, if, in addition to changing the constraints on the solution as the component sizes are changed, trajectory specified by $P_m(t)$ is recalculated, then the problem can be solved using the algorithm illustrated in Table 2.

Table 2 - Component Sizing Algorithm

Vary Engine Size from Minimum to Maximum
    Vary Battery Size from Minimum to Maximum
    Set Constraints based on Engine Size and
        Battery Size
    Determine $P_m(k)$ based on Engine Size and
        Battery Size
    Find Minimum Fuel Consumption
Select Engine Size and Battery Size that has lowest fuel consumption

More sophisticated techniques can be used to search the space of component sizes and specifications. However, this technique was chosen for illustrative purposes.

NUMERICAL EXAMPLE

This example is a contour map of the MPG versus engine size and battery size. Figure 8 was generated for the same vehicle used in the first numerical example. Again, the sizing is based on PNGV recommendations. In this case, the optimal sized components for this combination of engine and battery occurs for about a 15 kW engine with about 1.5 kW-hrs of PbA batteries. The dark band in the lower left half of the plot separates the region of feasible component sizes from the infeasible component sizes.

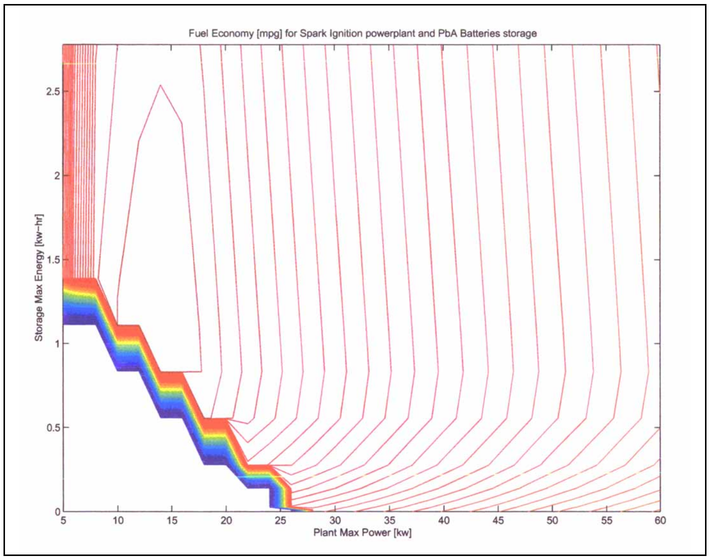

Figure 15 - Example of MPG map for Spark Ignition Engine and Pba Batteries

DISCUSSION: EVALUATING THE CAUSAL CONTROLLER

Having found the optimal solution using noncausal techniques, i.e. using past and future information, the global minimum fuel consumption is known. However, to actually implement a system, a causal control law must be designed. Once the causal control law is designed, it can be compared to the optimal to see how well it performs. Figure 16 illustrates a simple control law that was chosen for evaluation. The control law maintains the energy in the battery at a constant level using a linear feedback law. The output of the controller is slew rate limited and clamped to the engine's maximum power level. The slew rate limiting and clamping is done to duplicate the constraints applied to the optimization problem.

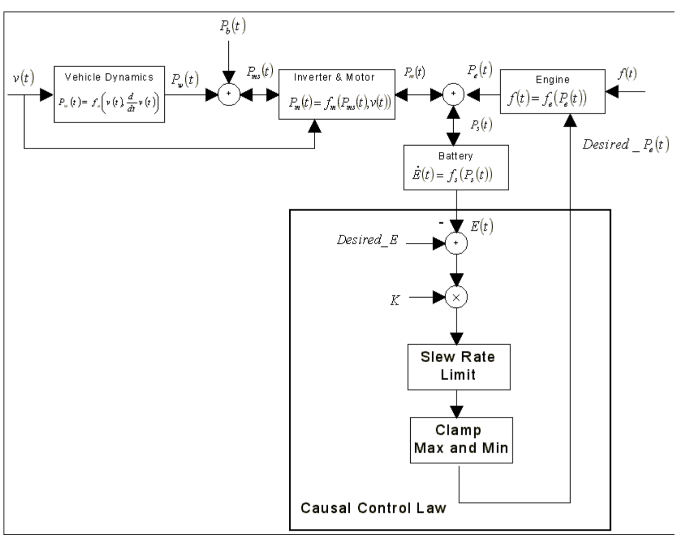

Figure 16 - Causal Control Law

To calculate a figure of merit for the causal controller, three values were computed. The first was the minimum achievable fuel consumption using convex optimization. This is referred to as F_{opt}. Next the model was simulated using the proposed causal control law. The fuel consumption achieved here is referred to as F_{causal}. Finally, the model was simulated without use of the batteries. Since this result is the same as directly driving the inverter using the engine, this result is referred to as F_{direct}. This provides three fuel consumption numbers that can be used to determine a figure of merit as shown in equation (50)

$$FOM = \frac{F_{direct} - F_{causal}}{F_{direct} - F_{opt}} \quad (50)$$

The values for FOM range over $\{-\infty, 1\}$. 1 is the best that can be achieved by any controller. 0 is the result achieved by a controller that uses as much fuel as directly driving the inverter. Negative numbers indicate that the controller is less efficient than directly driving the inverter.

Using a vehicle configuration with a 50 kW engine with a 1.5 KW-Hr battery, the results obtained are summarized in Table 3.

Table 3 - Figure of Merit for Causal Controller

F_{opt}	458 grams
F_{causal}	469 grams
F_{direct}	498 grams
$FOM = \frac{F_{direct} - F_{causal}}{F_{direct} - F_{opt}}$	0.73

One startling conclusion that came from this example was that through selection of a simple control law, 73% of the possible performance, available through control law selection and tuning, was achieved. Even using perfect future knowledge, a control law can only achieve an additional 27% improvement over the simple control law.

EXTENSIONS

The techniques presented in this paper do not cover all of the topics of interest in hybrid vehicle optimization. What has been presented is the core of a technique that can be extended to answer a significant portion of the optimization questions that arise in hybrid vehicle design. The paragraphs that follow present additional extensions that can be added to increase the utility of these techniques.

- Optimizing Parallel Hybrid Vehicle Models with both discrete gear ratios and continuously variable gear ratios.
- Improving accuracy through use of more sophisticated approximation for continuous functions, derivatives and integrators.
- Adding emissions to the optimization objective.
- Optimizing over a set of drive cycles instead of a single drive cycle
- Finding optimal solutions which include turning the engine off.
- Modeling variable energy storage efficiencies as a function of charge in the battery.
- Modeling thermal transients that affect fuel economy and emissions.

These extensions have not been implemented yet. However, the preliminary design of the models and optimization problems has been done. Initial results indicate that all of these extensions can be incorporated into the optimization.

CONCLUSION

The problem of finding the minimum fuel consumption for a specific hybrid propulsion system has been cast as a linear program. This problem has been solved to find the ultimate limit of performance for a series hybrid propulsion system architecture independent of any control law. The result obtained is the global optimal solution. There is no lower fuel consumption possible. This result has been used to select component requirements and to evaluate control law performance.

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DEFINITIONS, ACRONYMS, ABBREVIATIONS

Convex Function: A real valued function f that satisfies $f(\theta \cdot x+(1-\theta)\cdot y)\le \theta \cdot f(x)+(1-\theta)\cdot f(y)$ for all x and y in its domain, and all θ between 0 and 1.

Convex Optimization: A mathematical optimization problem in which a convex objective function is minimized, subject to any number of linear equality constraints, and any number of inequality constraints of the form $f_i(x) \le 0$, where f_i are convex functions:

minimize $f_0(x)$

subject to:

$A \cdot x = b$

$f_i(x) \le 0, i = 1..m$

where

$f_0, \ldots, f_m$ **are convex functions**

Linear Program: A mathematical optimization problem which a linear objective is minimized subject to any number of linear equality and inequality constraints:

minimize $c^T \cdot x$

subject to:

$A_{eq} \cdot x = b_{eq}$

$A_{ineq} \cdot x \le b_{ineq}$