Volterra Series: Engineering Fundamentals

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ABSTRACT

In the last century engineers have achieved great success in the analysis, control, and design of circuits and systems which are *linear* and *time-invariant*. For such systems we have the *convolution formula* for the output $y(t)$ in terms of the input $u(t)$ to the system:

$$y(t) = \int h(\tau) u(t-\tau) d\tau \tag{1}$$

A *Volterra series expansion* is a representation for *nonlinear systems* analogous to (1):

$$y(t) = h_0 + \sum_{n=1}^{\infty} \int \cdots \int h_n(\tau_1, \ldots, \tau_n) u(t-\tau_1) \ldots u(t-\tau_n) d\tau_1 \ldots d\tau_n \tag{2}$$

The purpose of this thesis is to address some fundamental engineering issues surrounding the Volterra series (2). These issues are:

(I) When does (2) make sense and what exactly does it mean?

We show that (2) can be interpreted as a *Taylor series*, and so it is not surprising that (2) makes sense for inputs u smaller than a positive number which has the interpretation of the *radius of convergence* of the Volterra series (2).

(II) For what nonlinear systems is the expansion (2) appropriate?

Unlike (1), which is valid for essentially *all* linear time-invariant operators arising in engineering, the Volterra series expansion (2) is only appropriate for *some* nonlinear operators. We show that it is appropriate precisely for those operators with *fading memory*.

(III) How can the kernels h_n be measured in the laboratory?

Measuring the kernels by classical methods is extremely slow. We develop a new quick method for measuring the kernels and apply it to various real systems.