A Theoretical Model for QCN

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Overview

- The stability ("unit step response") of congestion control algorithms are analyzed theoretically in the following way
  - Write down equations describing evolution of algorithm
    - Usually, these are nonlinear delay-differential equations
  - Analyze these equations for stability
    - Usually, linearize equations around operating point and analyze linear system

- The reason QCN equations were hard to get were that the Fast Recovery cycle is different from the usual source behavior (there is usually no Target Rate--Current Rate)
  - We show how the equations can be obtained
  - And check their accuracy using simulations

Fluid Model for QCN

- We will model only the key features of the QCN protocol. Namely, we do not consider:

  - Timer, HAI, extra fast recovery, window jittering, drift increase.

- Switch behavior is not too different from what we have seen for BCN => Easy to describe

- But source behavior appears to have a new ‘memory’ element in the Fast Recovery phase. It’s not possible to model this with a single variable, namely the current rate at the source

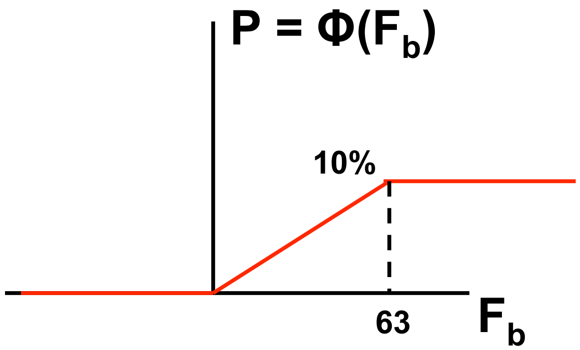
- This motivates using two variables at the source: Current Rate, and Target Rate

Fluid Model for QCN

- Target Rate (TR) is the rate that the source tries to reach by successive phases of fast recovery
  - Anytime the source sends 100 packets, and it receives no congestion signals, CR increases to halve the distance between CR and TR, i.e.: $CR \leftarrow (CR + TR)/2$
  - Anytime the source receives a congestion signal, it multiplicatively decreases CR, i.e.: $CR \leftarrow (1-G_d F_b)\ CR$
- Upon receiving congestion signal, TR drops to CR, i.e.:
  $TR \leftarrow CR$
- In Active Increase, after sending 100 packets and not receiving congestion signals: $TR \leftarrow TR + \alpha$
  - $\alpha = 5$ Mbps

Fluid Model for QCN

- Assume N flows pass through a single queue at a switch. State variables are $TR_i(t)$, $CR_i(t)$, q(t), p(t).



$$\frac{dTR_i}{dt} = -(TR_i(t) - CR_i(t)) \times CR_i(t-\tau)p(t-\tau) + (1-p(t-\tau))^{500} \times \alpha \times \frac{CR_i(t-\tau)}{100}$$

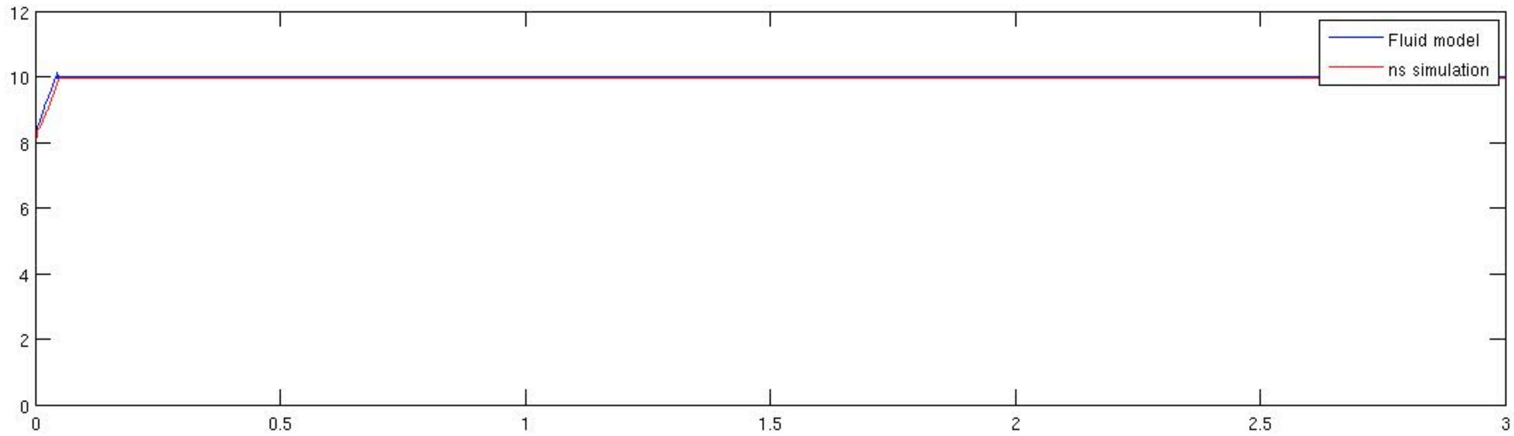
$$\frac{dCR_i}{dt} = -(G_d F_b(t-\tau) CR_i(t)) \times CR_i(t-\tau)p(t-\tau) + \frac{TR_i(t) - C_i(t)}{2} \times \frac{CR_i(t-\tau)p(t-\tau)}{\frac{1}{(1-p(t-\tau))^{100}} - 1}$$

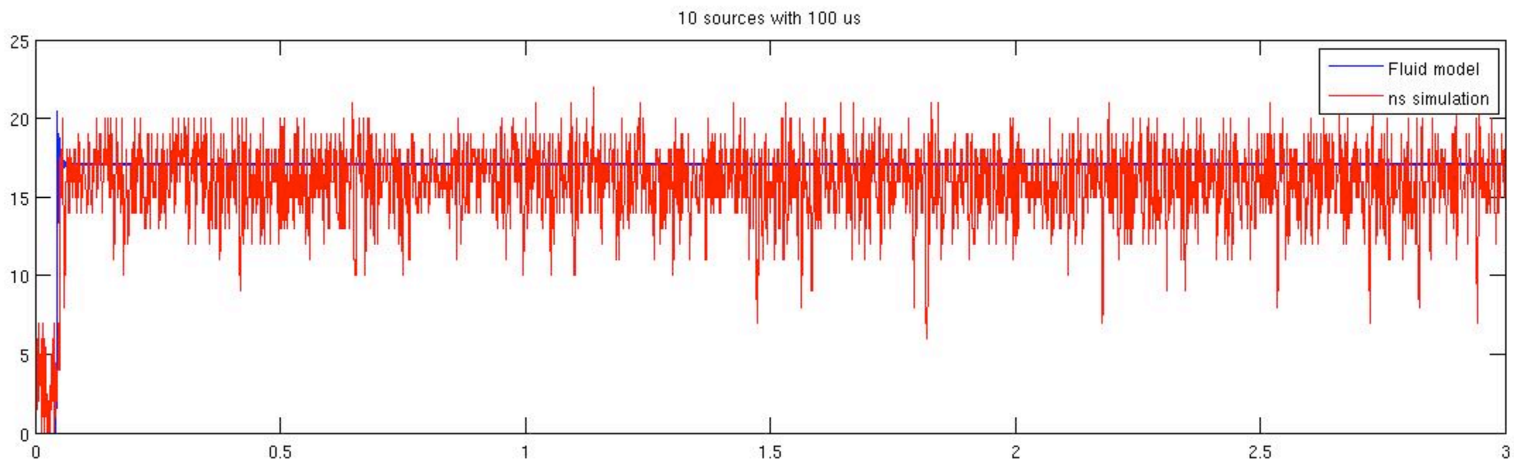
$$\frac{dq}{dt} = \sum_{i=1}^{N} CR_i(t) - C$$

$$F_b(t) = q(t) - Q_{eq} + \frac{w}{Cp(t)} \times \left(\sum_{i=1}^{N} CR_i(t) - C\right)$$

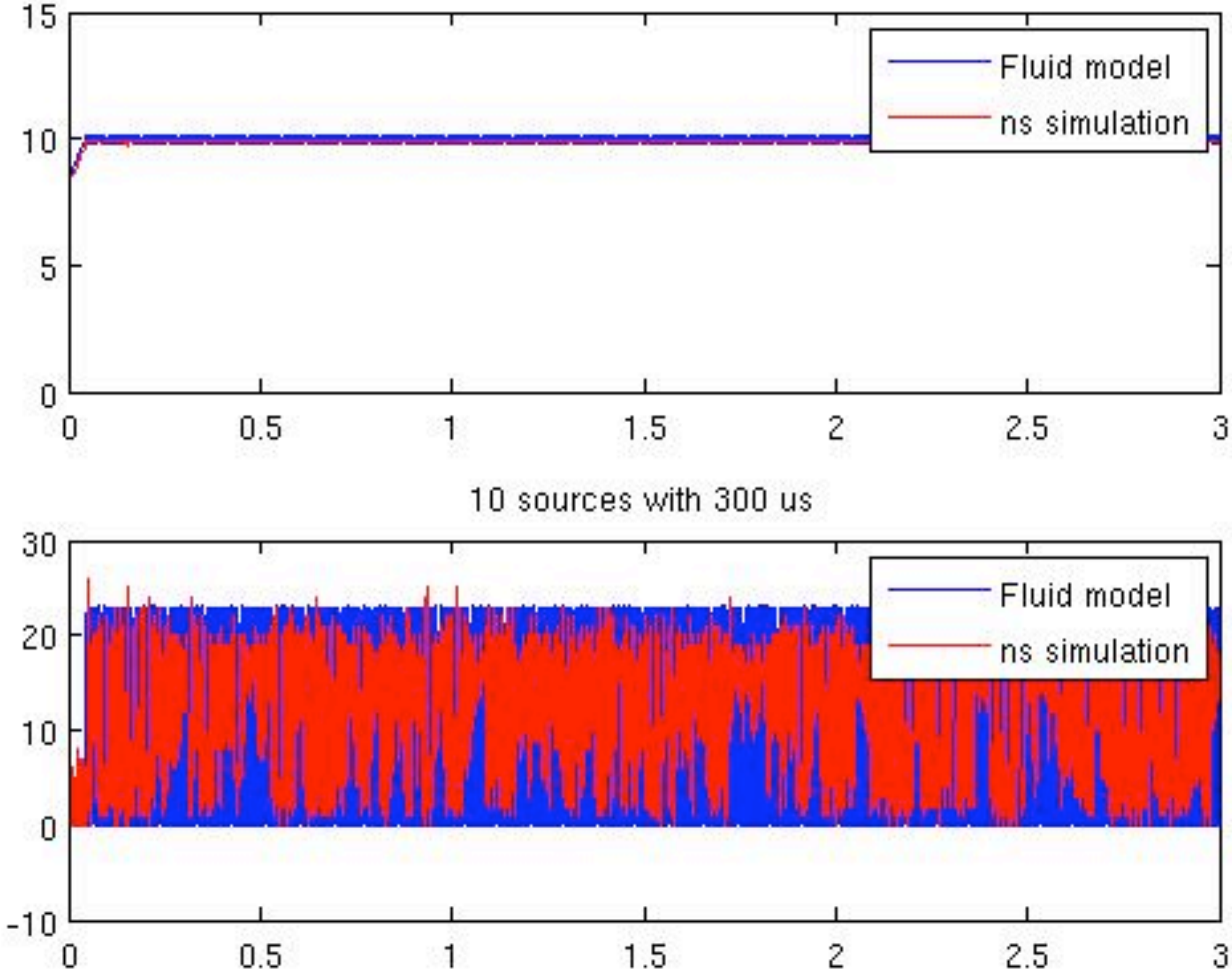
$$\frac{dp}{dt} = (\Phi(F_b(t)) - p(t)) \times 500$$

10 sources, 100 us

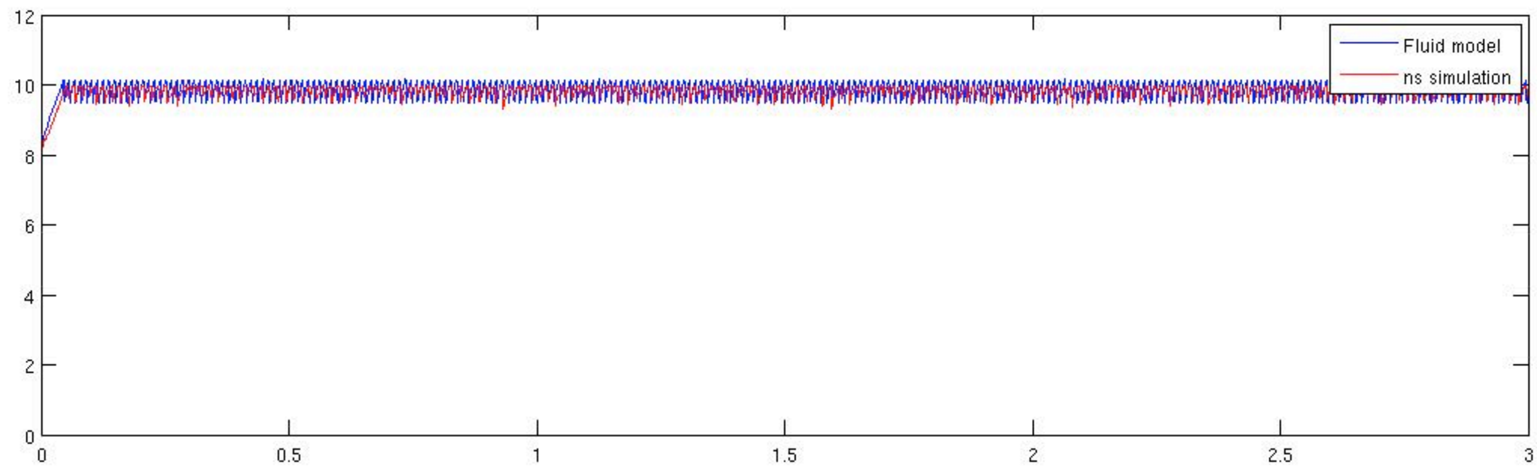


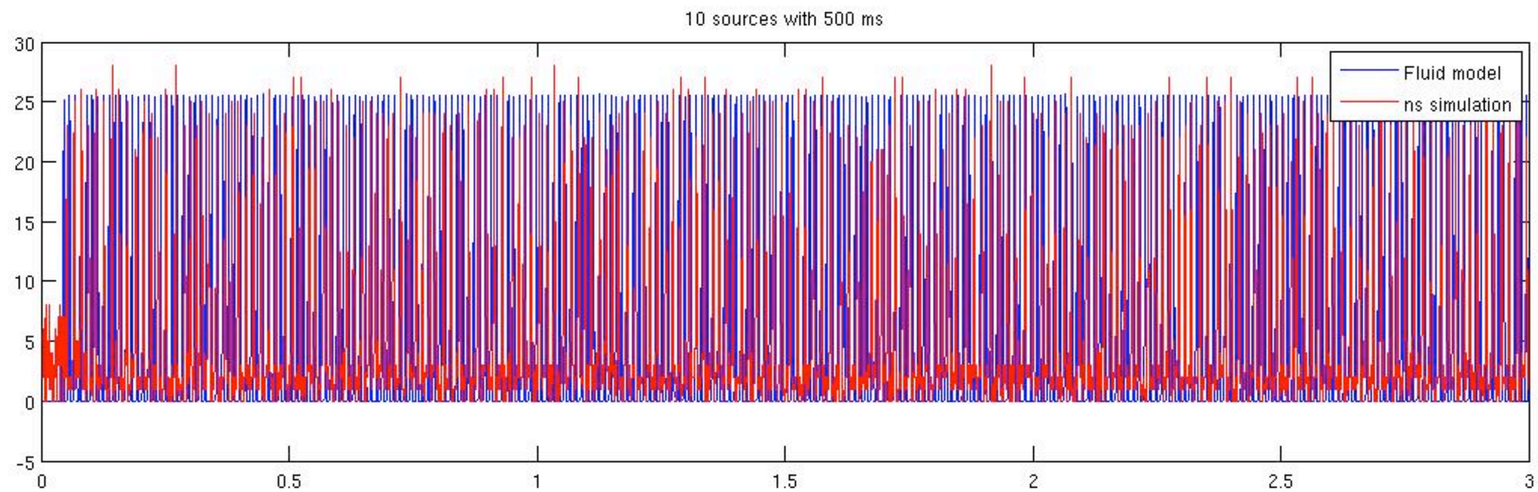


10 sources, 300 us

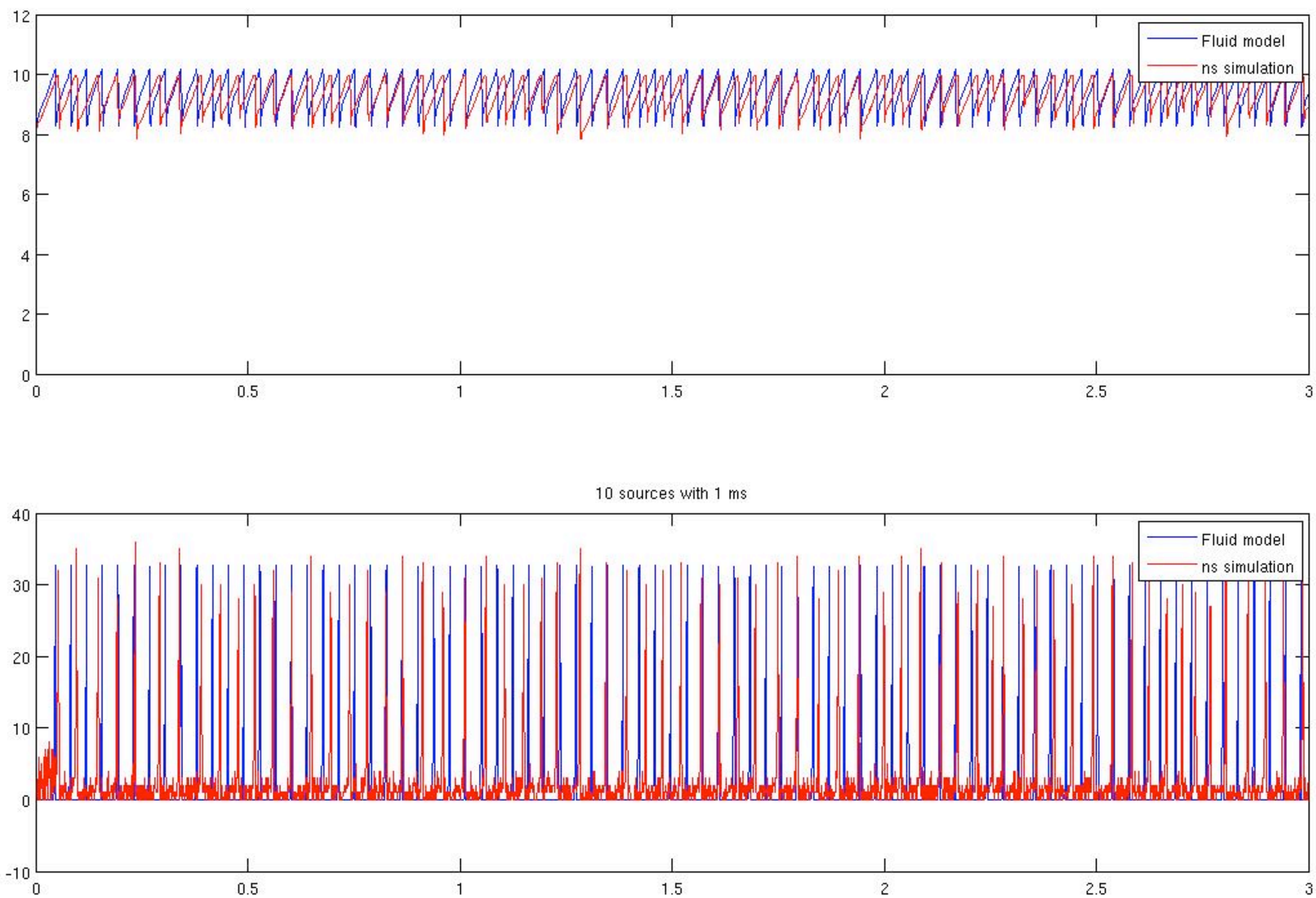


10 sources, 500 us





10 sources, 1 ms



10 sources, 2 ms