The Entropy and Delay of Traffic Processes in ATM Networks

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Abstract: In this paper we discuss the application of some recent results obtained from studying the asymptotics and the entropy of flows in large queueing systems to the analysis of entropy variations of traffic processes in ATM networks. We argue that certain important queueing nodes/networks (e.g. $\cdot$/M/1, $\cdot$/GI/∞ nodes and Jackson networks) are entropy increasing in nature, in that the entropy of the arrival process to such a node/network is strictly less than the entropy of the departure process except when the arrival process is Poisson (which has maximum entropy). The entropy increasing nature of $\cdot$/GI/∞ queues is established for a class of input processes. Based on these entropy maximization results and with the aid of some simulations, we investigate the connection between the entropy and delay of ATM traffic processes, and examine the effectiveness of entropy as a measure of "burstiness" of traffic streams in ATM virtual circuits.

1 Introduction

High speed packet-switched networks based on technologies such as Asynchronous Transfer Mode (ATM) [20] are widely believed to be the most promising candidates for providing services in the evolving Broadband-Integrated Services Digital Network (B-ISDN). In an ATM network sources generate information in the form of small fixed size packets which are multiplexed with the packet streams of other sources as they proceed from source to destination over virtual circuits (see Figure 1). Traffic processes in ATM networks are characterized by their highly intermittent and bursty nature. Since fluctuations induce congestion, these rapid fluctuations of network traffic are undesirable as they impose fundamental limitations on the utility of network resources.

Several methods have been proposed to regulate the burstiness of incoming traffic in ATM networks. Perhaps the most popular of these are the Leaky Bucket algorithm and its variants [3, 16]. At a theoretical level, research effort has been expended to give mathematical meaning to the intuitive notion of "a burst of traffic". This has led to the introduction of various measures of source burstiness like "equivalent capacity", "effective bandwidth", "index of dispersion", and to the modelling of bursty traffic as "Markov modulated fluids" and "Markov modulated Poisson processes" [6, 7, 8, 9, 11, 21]. The idea behind the implementation of flow control schemes is simple: a bursty source is required to buffer its traffic temporarily and release it into the network smoothly by spacing out the packets. This not only enhances network utilization [4], but also saves the network the cost of providing more buffer space to bursty traffic and assigns this cost directly to bursty sources. For these reasons and because of the extremely high speeds and the asynchronous nature of packet transmission in ATM networks, traffic shaping takes place, primarily, at the boundaries [3] and not much is known about traffic flow patterns *inside* networks.

In current admission control schemes [15], traffic descriptors (bandwidth requirements, duration of call, etc.) provided by a source are presented to all switching nodes on a circuit from the source to the destination. Service is provided to the source if each node is able to meet its requirements. Since traffic characteristics at internal nodes may be quite different than at the entry points of the network, this may cause either an over-controlled or an under-controlled network depending on whether the actual internal burstiness levels are lower or higher than that presented by the source at the network entry points. There is thus a need to obtain a characterization of internal traffic flow patterns in large queueing systems and to better understand network-wide dynamics.

Some recent results [12, 15, 17] indicate that under reasonable assumptions on arrival, service and interspersing cross traffic distributions, it is possible to approximately estimate *internal traffic characteristics*. For example, Ohba et. al. [15] derive end-to-end delay distributions for through traffic (modelled as a renewal process) with interfering cross traffic streams (consisting of Bernoulli batch arrivals and Interrupted Poisson Processes); while Lau and Li [12] use the nodal decomposition method to study the amount of distortion suffered by a process as it goes through a queue. Plotkin and Varaiya [17] report some interesting results stemming from the use entropy as a "measure of disorganization" of through traffic streams and investigate a possible connection between entropy and mean queue sizes. The model they consider is essentially the one depicted in Figure 1. Based on simulations, they observe an increase or decrease in the entropy of traffic streams depending on the statistics of through and cross traffic. They also observe a correlation between entropy and average queue-sizes in the tandem structure.

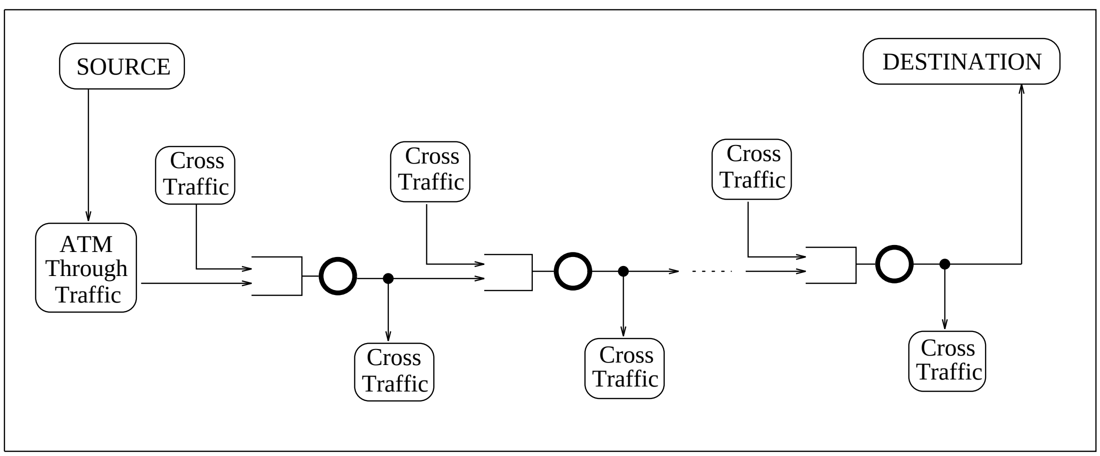

Figure 1: An ATM virtual circuit

In this paper we discuss the application of some recent results [13, 18] obtained from studying the asymptotics and the entropy of traffic flows in large queueing networks to the problem of traffic shaping in ATM networks. Although these results are interesting in their own right, our purpose in this paper is to discuss the entropy of flows in ATM networks. Therefore, we merely recall the bare essentials of [13] and [18] as needed. Furthermore, as we are interested in presenting the main ideas, we have frequently sacrificed generality and precision for brevity and simplicity.

In Section 2, we will first argue that the exponential server ($\cdot$/M/1) queue and other quasi-reversible nodes/networks [10] act as "entropy increasing operators" in that the entropy of the departure process from a $\cdot$/M/1 queue or quasi-reversible node/network is strictly greater than the entropy of the arrival process except when the arrival process is Poisson (which has maximum entropy among all processes of a given average rate). In Section 3, we establish the entropy increasing nature of $\cdot$/GI/∞ queues for special inputs. Based on the entropy increasing nature of $\cdot$/M/1 queues and simulation experiments, we produce some examples in Section 4 to show that, in general, there is no connection between entropy and delay. This will then suggest that entropy, although a useful traffic descriptor, does not adequately capture the notion of "burst". We conclude the paper by exploring this lack of correlation between entropy and burstiness.

2 An Entropic Analysis of Queueing Systems

In this section we briefly outline an entropic approach to analyzing queueing systems. We will be primarily interested in queueing systems which possess the Poisson-in-Poisson-out property although we will touch upon other more general queueing systems also. Networks of "quasi-reversible nodes" are prime examples of queueing systems possessing the Poisson-in-Poisson-out property and have found wide applicability in the modelling and analysis of computer, communication, and manufacturing networks. Well-known examples of quasi-reversible nodes are the $\cdot$/M/1, $\cdot$/M/k, $\cdot$/GI/∞, $\cdot$/GI/1-LCFS, $\cdot$/GI/1-Processor Sharing nodes; while Jackson and Kelly networks are examples of quasi-reversible networks. See [10] for a comprehensive and elegant treatment of quasi-reversible networks.

Although the transient and equilibrium behaviour of quasi-reversible nodes/networks under Poisson process inputs is well understood, not much is known about the behaviour of these networks under *non-Poisson process inputs*. In particular, there are few general results on the distributions of departure processes from quasi-reversible networks under non-Poisson process inputs. Some such results, also important to us for the sequel, may be found in [13], [18] and [19]. In [13] it was established that the departure process from a series of independent and identical $\cdot$/M/1 queues when fed by an arbitrary stationary and ergodic arrival process converges to a Poisson process, while [18] establishes similar results for departures from a series of Jackson networks. In [19], it is shown that successive departures from a series of $\cdot$/GI/∞ queues under arbitrary stationary and ergodic input processes converge either to a Poisson process or to a ν-Poisson process (defined in Section 3).

To understand the underlying reason for the occurrence of the Poisson process as the limit in the above queueing tandems and to gain some insight into the asymptotic behaviour of traffic processes in large queueing networks, we adopt the view that queueing nodes/networks act as "Markov operators" on the space of realizations of point processes. That is, a knowledge of the joint distribution of the arrival and service processes at a queueing node is sufficient to generate the departure process distribution; no other information is needed. In particular, the distributions of the arrival process at other nodes where it has been processed *prior* to visiting the node in question are irrelevant. Then, observing successive departure processes from a series of identical queues will be like watching a homogeneous discrete-time Markov chain evolve. This allows one to bring in such notions of standard Markov chain theory as the existence and uniqueness of invariant distributions and "relative entropy". The Markov operator view of queueing allows us to suggest that quasi-reversible nodes/networks increase the entropy of a process passing through them. Since among processes of a given average rate the Poisson process has maximum entropy, we argue that it is not surprising that it occurs as the limit from passing arbitrary arrival processes through a tandem of such "entropy increasing" nodes/networks.

We proceed to make the foregoing discussion about Markov operators and entropy precise in the context of the $\cdot$/M/1 queue. Before this, we recall the definition and some results concerning the entropy of point processes from [5] and [14]. To keep the discussion to a minimum and to avoid introducing unnecessary notation we restrict ourselves to the task of mentioning the salient features concerning the entropy of point processes. Further information may be found in the references cited above.

2.1 The Entropy of Point Processes

The entropy of a point process is defined in the following manner. Observation of the process conveys information of two kinds: the actual number of points observed and the location of these points given their number. This suggests defining the entropy of a realization $\{x_1, ..., x_M\}$ as follows

$$H = H(M) + E(H(x_1, ..., x_M|M))$$

Starting with this idea, after some manipulations and simplifications, the *entropy* of a point process which has as its state space a finite interval $(0, T]$ of $\mathbb{R}$ is given by the following sum [14]

$$H_{(0,T]} = H^N_{(0,T]} + H^P_{(0,T]},$$

where $H^N_{(0,T]}$ is the *numerical entropy* and refers to the *number* of points in $(0, T]$; and $H^P_{(0,T]}$ is the *positional entropy* and refers to the positions of these points in $(0, T]$ given their number. It is fairly easy to see ([5], page 571) that subject to certain natural conditions, the numerical entropy is maximized by the Poisson random variable; whereas the positional entropy is seen to be maximized by the uniform distribution. These two observations taken together imply that among point processes of a given average rate, the point process entropy is maximized by the Poisson process.

The *entropy rate* of a point process is defined as

$$HER = \lim_{T\to\infty} H_{(0,T]}/T,$$

provided the limit exists. This limit is shown to exist for stationary and ergodic point processes [5] and is again maximized by the Poisson process.

A quantity, important for the sequel, that we do not discuss here is the "relative entropy" of a point process with respect to the Poisson process. This requires a discussion of the conditions under which a point process is absolutely continuous with respect to a Poisson process. As this will take us far afield, we refer the interested reader to [5].

We are now in a position to discuss the entropy increasing nature of $\cdot$/M/1 queues.

2.2 Entropy and the $\cdot$/M/1 Queue

In this subsection, we elaborate on some of the thinking behind the view that a $\cdot$/M/1 queue with service rate 1 acts as a Markov operator on the space of stationary and ergodic processes of rate $\lambda < 1$.

Let $(\Omega, \mathcal{F}, P)$ be a probability space. Recall from [2] that the pair $(\mathbf{M}, \mathcal{M})$, the *canonical space of point process*, is the space of realizations $\mathbf{M}$ endowed with an appropriate σ-algebra $\mathcal{M}$; and a point process is a measurable map from $(\Omega, \mathcal{F}, P)$ to $(\mathbf{M}, \mathcal{M})$. Let $\mathcal{D}$ be the space of distributions of point processes. That is, if $\mathbf{P}$ is a point process and μ^P its distribution, then

$$P(\{\omega : \mathbf{P}(\omega) \in A\}) = \mu^P(A) \;\; \text{for all} \;\; A \in \mathcal{M}.$$

Let $\mathcal{D}_s^e(\lambda) \subset \mathcal{D}$ be *the space of distributions of stationary and ergodic point processes of rate* λ. It follows from [1] that a $\cdot$/M/1 queue with mean service rate 1 maps elements of $\mathcal{D}_s^e(\lambda)$ into $\mathcal{D}_s^e(\lambda)$. Furthermore, by Theorem 2 of [1], this map $\mathcal{T} : \mathcal{D}_s^e(\lambda) \longrightarrow \mathcal{D}_s^e(\lambda)$ is well-defined and is the input-output map of the $\cdot$/M/1 node. For the Markov operator perspective, we think of $\mathcal{T}$ as the "one-step transition operator" which gives the departure process distribution in terms of the arrival process distribution.

Now consider a series of independent $\cdot$/M/1 queues (all with mean service rate equal to 1) fed by an independent arrival process $\mathbf{A}^1$. Let $\mu^1 \in \mathcal{D}_s^e(\lambda)$ $(\lambda < 1)$ be the distribution of $\mathbf{A}^1$. Let $\mathbf{A}^2$ be the departure process from the first node and let $\mu^2 = \mathcal{T}(\mu^1)$ be its distribution. Proceeding thus, let $\mathbf{A}^n$ be the departure process from the $n-1^{th}$ node and let μ^n be its distribution. Note that $\mu^n = \mathcal{T}(\mu^{n-1}) = \mathcal{T}^{n-1}(\mu^1)$.

In terms of the notation established above, we may express the result of [13] concerning the Poisson limit of departures from a series of $\cdot$/M/1 queues as

$$\lim_{n\to\infty} \mathcal{T}(\mu^n) = \pi^\lambda,$$

where π^λ is the distribution of a rate λ Poisson process. Now, from the Markov operator point of view, this means that the distribution of successive departures is converging to the one with maximum entropy (the Poisson process), *independent* of the initial distribution μ^1. So at a single stage in the series of $\cdot$/M/1 nodes, it is reasonable to expect that the relative entropy with respect to the invariant distribution, the Poisson process, of the arrival process is strictly less than the relative entropy of the departure process with respect to the Poisson process. Because the Poisson process has maximum entropy (and maximum entropy rate) among all processes of a given average rate [5], one may further expect that the *absolute* entropy rate of the arrival process is less than the *absolute* entropy rate of the departure process. We now formally state the above observation as:

Problem: *Let* $\mathbf{A}$ *be a stationary and ergodic process of rate* $\lambda < 1$ *arriving to a* $\cdot/M/1$ *queue of mean service rate 1 and let* $\mathbf{D}$ *be the corresponding departure process. Then, the entropy rate* HER_A *of* $\mathbf{A}$ *is strictly less than the entropy rate* HER_D *of* $\mathbf{D}$*, unless* $\mathbf{A}$ *is Poisson, in which case* $HER_A = HER_D$*.* ■

Remark: Using the chain of arguments that led to the formulation of the above problem for $\cdot$/M/1 nodes, one may expect a similar thing to be true of other Poisson-in-Poisson-out nodes. For example, it is not unreasonable to expect that the following Poisson preserving nodes are also entropy increasing in nature: $\cdot$/M/k, $\cdot$/GI/∞, $\cdot$/GI/1-LCFS, $\cdot$/GI/1-processor sharing, etc. Indeed, in Section 3 we establish the entropy increasing nature of $\cdot$/GI/∞ queues for special inputs. We also point out that should all quasi-reversible nodes/networks be shown to be entropy increasing, then this gives the hope that passing an arbitrary arrival process through a series of such nodes (the nodes need not be identical) will still result in a Poisson process limit.

3 Entropy and the $\cdot$/GI/∞ Queue

In this section we shall show that $\cdot$/GI/∞ queues increase the entropy of certain special processes passing through them. Our aim is to illustrate the main idea and therefore we have sacrificed precision for simplicity. Full details will be provided in subsequent publications.

Given a probability measure ν on $[0,\ 1]$, define measures $\nu^n, n \in \mathbb{Z}$ with support in $[n,\ n+1]$ as translates of ν by n units of time. That is, $\nu^n(S) = \nu(S-n)$ for all Borel sets S in $[n,\ n+1]$. Note that $\nu^0 = \nu$.

Definition 1: Given a probability measure ν on $[0,\ 1]$, a process $\mathbf{P}_\nu$ is said to be a ν*-Poisson process of rate* α if the following conditions hold.
i) The number of points of $\mathbf{P}_\nu$ in $[n,\ n+1]$ is an i.i.d. sequence, as n varies over $\mathbb{Z}$, with each marginal being distributed as a Poisson, parameter α, random variable.
ii) Each point of $\mathbf{P}_\nu$ in $[n,\ n+1]$ is distributed over the interval $[n,\ n+1]$ according to ν^n, independent of all other points.

Note that according to the above definition $\mathbf{P}_\nu$ is the usual Poisson process with parameter α if ν is the uniform distribution over $[0,\ 1]$, and it is a batch process of i.i.d. Poisson random variables if ν is a point mass at some $x \in [0,\ 1]$. Hence $\mathbf{P}_\nu$ is, in general, not a time stationary process. However, by shifting the paths of $\mathbf{P}_\nu$ uniformly over the interval [0, 1], we obtain a *stationary* ν*-Poisson process.*

In this section we will be exclusively concerned with $\cdot$/GI/∞ queues with ν-Poisson process inputs. We shall prove the following theorem.

Theorem 1: *Let* $\mathbf{A}$ *be a* ν*-Poisson process of rate* α *that is inputted to a* $\cdot$*/GI/*∞ *queue with service time distribution* μ_s *(support of* $\mu_s \subset [0, \infty)$*). Let the corresponding departure process be* $\mathbf{D}$*. Then the entropy rate of* $\mathbf{D}$ *is greater than or equal to the entropy rate of* $\mathbf{A}$*, with equality iff (1)* μ_s *periodic with period 1 (i.e. the possible values of* $\mu_s \subset \mathbb{Z}^+$*), or (2)* ν *is uniform on* $[0,\ 1]$.

Remarks: (a) The cases where there is no increase of entropy rate are the cases where $\mathbf{A}$ is an invariant distribution to the $\cdot$/GI/∞ queue. That $\mathbf{A}$ is invariant to a $\cdot$/GI/∞ queue whose service times are periodic with period 1 follows from Lemma 2 in the Appendix, accounting for Case (1). In Case (2) $\mathbf{A}$ is the usual Poisson process and hence is invariant.

(b) This is a very special case of the general entropy increasing result for $\cdot$/GI/∞ queues. That is, in this case the "numerical entropy" has already increased to the maximum (since the number of points in intervals $[n, n+1]$ as n ranges over $\mathbb{Z}$ is i.i.d. Poisson - maximum entropy). The only thing that can happen is for the "positional entropy" to increase. Our proof consists of showing this.

(c) We need some results concerning invariant distributions of $\cdot$/GI/∞ queues in order to prove Theorem 1. These are presented in the Appendix.

Proof of Theorem 1

Two queueing nodes in series are said to be interchangeable if for *any* input process, the law of the overall departure process from the series is invariant with respect to the relative position of the two queues. It is fairly easy to see that any two $\cdot$/GI/∞ queues in series are interchangeable.

Now consider the schematic shown below. In Tandem 1 $\mathbf{A}$ is first fed to a $\cdot$/GI/∞ queue whose service times are periodic with period 1 (hence the symbol δ_N - which is meant to suggest that the support of the service time distribution is contained in the natural numbers $I\!N$). By Lemma 2 in the Appendix, $\mathbf{A}$ is invariant to this queue and hence the departure process is equal to $\mathbf{A}$ in distribution. This is then passed through a second $\cdot$/GI/∞ queue whose service times are distributed as μ_s. This results in the process $\mathbf{D}$ as the output. Since any two $\cdot$/GI/∞ queues are interchangeable, the logic of Tandem 2 is evident.

By looking at Tandem 2 we infer that $\mathbf{D}$ is invariant to a $\cdot$/GI/∞ queue whose service is

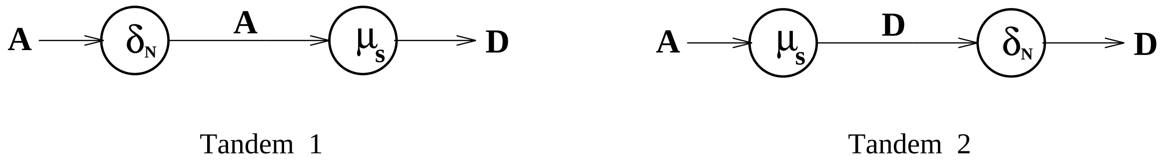

Figure 2: Interchanging two ·/GI/∞ queues

periodic with period 1. Now, by Lemma 3 in the Appendix, this implies that **D** is a γ-Poisson process where γ is some probability measure on [0, 1]. Since the numerical entropies of **A** and **D** are equal, it suffices to show that the entropy of the distribution γ is greater than the entropy of the distribution ν. This then proves Theorem 1.

Specialize to the case where ν has density $f_a(t)$ on [0, 1] and μ_s has density $f_s(t)$ on $[0,\ \infty)$. As will be evident from the proof, this is only a simplifying assumption and not a real restriction. Let the density of γ be $f_d(t)$. The ν-Poisson process **A** can be thought of as the departure process resulting from passing a δ_0-Poisson process (δ_0 denotes the point mass at 0; therefore, a δ_0-Poisson process is an i.i.d. sequence of Poisson random variables living on the integers) through a ·/GI/∞ queue whose service time distribution is $f_a(t)$. Since **D** is obtained by passing **A** through a ·/GI/∞ queue whose service time distribution is $f_s(t)$, we can think of **D** as the departure process resulting from passing a δ_0-Poisson process through a ·/GI/∞ queue with service time distributed as $g(t) = f_a(t) * f_s(t)$. From $g(t)$ we obtain $f_d(t)$ by a "folding over" of $g(t)$. The following example serves to explain the fore-going discussion.

Example: (Please refer to Figure 3). Suppose that **A** is a Poisson process and we pass it through a rate μ ·/M/∞ server queue. Then **D** is also a Poisson process. This makes $f_a(t) = f_d(t) \equiv 1$ on [0, 1], $f_s(t) = \mu e^{-\mu t}$. Now

$$g(t) = f_a(t) * f_s(t) = (1 - e^{-\mu t})\ \mathbf{1}_{\{0 \le t \le 1\}} + (e^{\mu} - 1)\ e^{-\mu t}\ \mathbf{1}_{\{t>1\}}$$

and

$$\begin{aligned} f_d(t) &= \sum_{n=0}^{\infty} g(t+n)\ \mathbf{1}_{\{0\le t\le 1\}} \\ &= 1 - e^{-\mu t} + (e^{\mu}-1)\left[e^{-\mu(t+1)} + e^{-\mu(t+2)} + \cdots + e^{-\mu(t+n)} + \cdots\right] \\ &= 1 - e^{-\mu t} + (e^{\mu}-1)\ e^{-\mu t}\left[e^{-\mu} + e^{-2\mu} + \cdots + e^{-n\mu} + \cdots\right] \\ &\equiv 1, \quad \text{as expected.} \end{aligned}$$

Continuing on with our proof, since the support of $f_a(t)$ is contained in [0, 1], we may

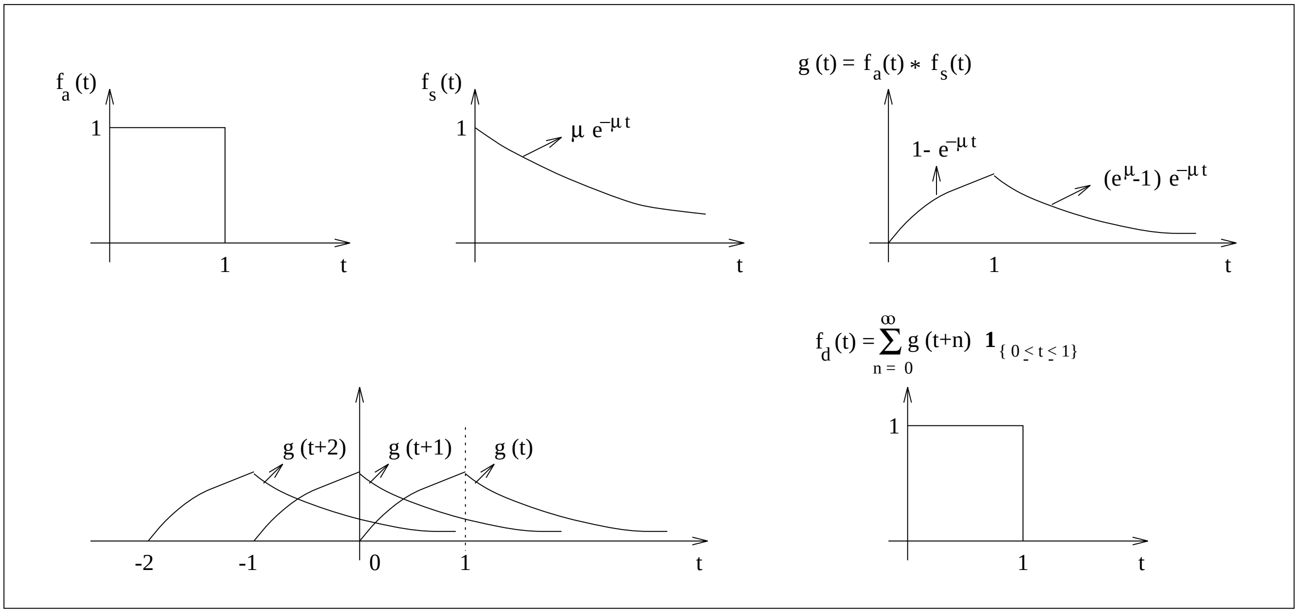

Figure 3: An Example

write

$$g(t) = \int_0^\infty f_a(s)\ f_s(t-s)\ ds\ = \int_0^{1\wedge t} f_a(s)\ f_s(t-s)\ ds$$

$$\Longrightarrow g(t+n) = \int_0^{1\wedge(t+n)} f_a(s)\ f_s(t+n-s)\ ds.$$

Now

$$\begin{aligned} f_d(t) &= \sum_{n=0}^{\infty} g(t+n)\ \mathbf{1}_{\{0\le t\le 1\}} \\ &= \int_0^t f_a(s)\ f_s(t-s)\ ds\ + \int_0^1 f_a(s)\ f_s(t+1-s)\ ds\ + \int_0^1 f_a(s)\ f_s(t+2-s)\ ds\ + \cdots \\ &= \int_0^1 f_a(s)\ \left\{ f_s(t-s)\ \mathbf{1}_{\{t\le 1\}} + f_s(t+1-s) + f_s(t+2-s) + \cdots \right\}\ ds \\ &= \int_0^1 f_a(s)\ \{f(t-s)\}\ ds, \end{aligned}$$

where $f(t-s) = f_s(t-s)\ \mathbf{1}_{\{t\le 1\}} + f_s(t+1-s) + f_s(t+2-s) + \cdots$. So, the crux of the proof has been reduced to the following. $f_d(t)$ has been written as an integral of $f_a(s)$ with kernel $f(\cdot)$. We will show that the *new measure* $f_d(\cdot)$ is given as a convex combination of the *old measure* $f_a(\cdot)$ with weights provided by the function $f(\cdot)$. We do this by showing that the kernel $f(\cdot)$ is "doubly stochastic"; i.e., $\int_0^1\ f(t-s)\ ds = \int_0^1\ f(t-s)\ dt = 1$. Heuristically, this will then mean that $f_d(\cdot)$ is "more uniform" than $f_a(\cdot)$ and therefore has more entropy. We may then infer the desired result via Lemma 1 of the Appendix.

For fixed $t \in [0,\ 1]$,

$$\begin{aligned}
\int_0^1 f(t-s)\ ds &= \int_0^t f_s(t-s)\ ds\ + \int_0^1 f_s(t+1-s)\ ds\ + \cdots + \int_0^1 f_s(t+n-s)\ ds\ + \cdots \\
&= \int_0^t f_s(s)\ ds\ + \int_t^{t+1} f_s(s)\ ds\ + \cdots + \int_{t+n-1}^{t+n} f_s(s)\ ds\ + \cdots \\
&= \int_0^\infty f_s(s)\ ds\ = 1, \quad \text{as required.}
\end{aligned}$$

For fixed $s \in [0,\ 1]$,

$$\begin{aligned}
\int_0^1 f(t-s)\ dt &= \int_s^1 f_s(t-s)\ dt\ + \int_0^1 f_s(t+1-s)\ dt\ + \cdots + \int_0^1 f_s(t+n-s)\ dt\ + \cdots \\
&= \int_0^{1-s} f_s(t)\ dt\ + \int_{1-s}^{2-s} f_s(t)\ dt\ + \cdots + \int_{n-s}^{n+1-s} f_s(t)\ dt\ + \cdots \\
&= \int_0^\infty f_s(t)\ dt\ = 1, \quad \text{as required.}
\end{aligned}$$

Lemma 1 of the Appendix tells us that the entropy of $f_d(\cdot)$ is greater than the entropy of $f_a(\cdot)$. Now, the two processes **A** and **D** have the same numerical entropy, whereas the positional entropy of **D** is greater than that of **A**. Therefore, the entropy of **D** in the interval $[0, N]$ (N is an integer) is greater than that of **A** in the same interval, i.e. $H_{[0,N];D} \geq H_{[0,N];A}$. Dividing by N and passing to the limit, we see that Theorem 1 is proved. ■

4 The Entropy and Delay of Traffic Flows in ATM Networks

The popularity and success of "entropy" as a "measure of randomness" make it a natural candidate to capture the notion of randomness of traffic streams in ATM Networks. Given that randomness increases delay, this association of entropy with randomness automatically leads to the question of whether there is a connection between entropy and delay. We now produce some simulation results that, in general, indicate a lack of correlation between entropy and mean waiting times (and hence also queue-sizes, via Little's formula - $L = \lambda W$). Essentially, we look for the obvious counter-examples; that is, we take a series of ·/M/1 queues and input two types of arrivals - (1) a highly bursty arrival process, and (2) the deterministic arrival process. Assuming that the ·/M/1 queue is entropy increasing, in both cases we obtain a series of departure processes that are increasingly more entropic. If there is a connection between delay and entropy, then we should observe, simultaneously in both cases, either a monotonic increase or a monotonic decrease of waiting times. However, average waiting is seen to *monotonically decrease* with bursty arrivals whereas it is seen to *monotonically increase* in the case of deterministic arrivals (Figure 4).

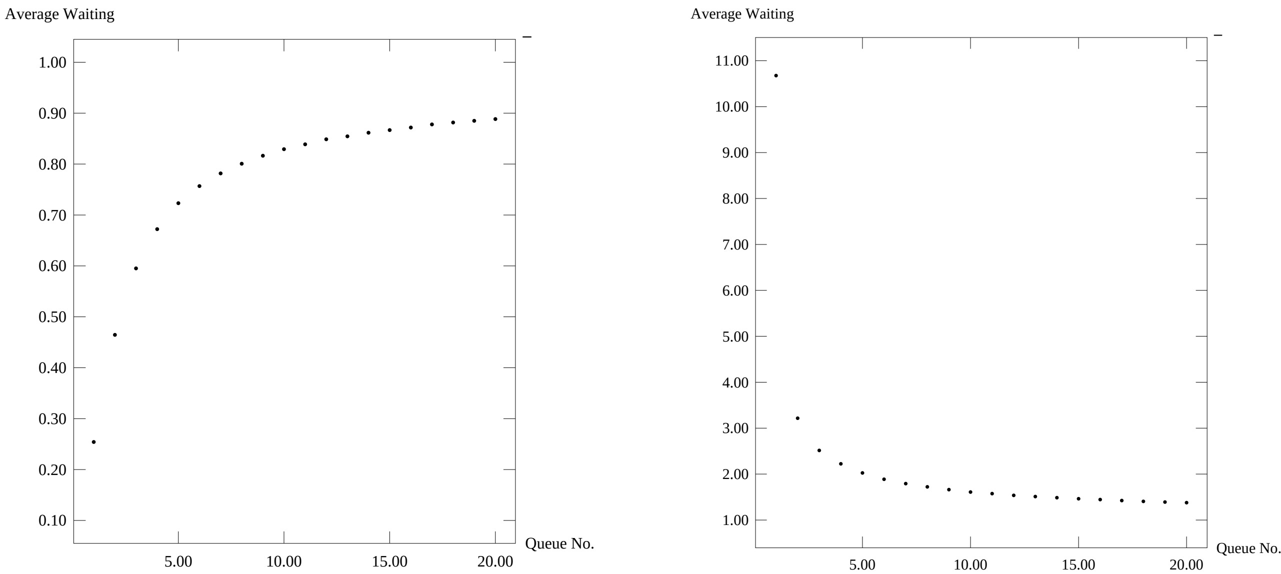

Figure 4: Average waiting times at 20 ·/M/1 queues in series with deterministic input (left) and bursty input (right).

This lack of correlation between entropy and delay is not surprising since the delay a process suffers at a queue is related to its burstiness and bursty traffic is not the most entropic. If at a given time a source emitting packets intermittently sends out a lot of packets in quick succession, then we may infer that the source is in a burst phase. Therefore, the chance of observing a packet in the next infinitesimal duration is very high. Conversely, if the source has not emitted packets for a while, then it is very probable that the source is in an idle phase and the chance of observing a packet in the next infinitesimal duration is almost zero. Thus, observing the past history of a bursty source provides a lot of information regarding its probable future behaviour. However, if the source is emitting packets according to a Poisson process, then observing the past sheds no light on the future behaviour of the source (due to the memoryless nature of the inter-arrival time distribution). And this perfect lack of information from the past is what makes the Poisson process the most entropic.

Although the foregoing discussion shows a lack of correlation between entropy and delay, it does not take away from the effectiveness of entropy as a traffic descriptor. Indeed, if one adopts the Markov operator perspective, the very act of repeatedly subjecting an arbitrary arrival process to the same operation (like simple queueing, queueing in the presence of cross traffic, Bernoulli splitting, etc.) suggests that the process may be tending toward to a limit. Therefore, one expects to see a gradual increase in the relative entropy of the arbitrary arrival process with respect to the limiting process.

5 Appendix

Lemma 1: *Let $f_a(t)$ and $f_d(t)$ be the densities of two probability measures on $[0,1]$ that are related by the equation $f_d(t) = \int_0^1 f_a(s)\ f(t-s)\ ds$, where the kernel $f(t,s) = f(t-s)$ satisfies $\int_0^1 f(t-s)\ dt = \int_0^1 f(t-s)\ ds = 1$. Then*

$$-\int_0^1 f_d(t)\ \log\left(f_d(t)\right)\ dt\ \geq -\int_0^1 f_a(t)\ \log\left(f_a(t)\right)\ dt.$$

Proof:

$$\begin{aligned}
-\int_0^1 f_d(t)\ \log\left(f_d(t)\right)\ dt &= -\int_0^1 \left[\int_0^1 f_a(s)\ f(t-s)\ ds\right]\ \log\left[\int_0^1 f_a(s)\ f(t-s)\ ds\right]\ dt \\
&\geq -\int_0^1 \left(\int_0^1 f(t-s)\ dt\right) f_a(s)\ \log\left(f_a(s)\right)\ ds \\
&= -\int_0^1 f_a(s)\ \log\left(f_a(s)\right)\ ds
\end{aligned}$$

where the inequality is Jensen's. ■

Lemma 2: *Consider a $\cdot/GI/\infty$ queue whose service is periodic with period 1. If ν is a probability measure on $[0,\ 1]$, then a ν-Poisson process of rate α, $\mathbf{A}$, is an invariant distribution for this queue.*

Proof: Presented in [19].

The only other result invoked in the proof of Theorem 1 is Lemma 3. This is stated below and is the converse of Lemma 2.

Lemma 3: *If a point process $\mathbf{A}$ is invariant to a $\cdot/GI/\infty$ queue whose possible service times are periodic with period 1, then $\mathbf{A}$ is either a ν-Poisson process or a stationary ν-Poisson process, where ν is a probability measure on $[0,1]$.*

Proof: Presented in [19].

Conclusion

In this paper we have discussed the use of entropy as a traffic descriptor and have investigated its effectiveness in capturing the burstiness of traffic processes. A direct use of entropy as a measure of burstiness seems suspect, although using entropy methods in analyzing queueing systems seems quite fruitful. We have also mentioned several results and problems concerning the entropy increasing nature of nodes/networks that are interesting in their own right.

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