# Throughput-Delay Trade-off in Energy Constrained Wireless Networks

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The random network model assumed in this paper is a generalization of the model in [1] that incorporates transmission energy consumption. We assume a random network of $n$ nodes distributed uniformly at random on a unit torus with each node having a randomly chosen node as its destination. We assume the *relaxed protocol model* where a transmission from node $i$ to node $j$ is successful if, for any other node $k$ that is transmitting simultaneously,

$$d(k,j) \geq (1+\Delta)d(i,j) \quad \text{for} \quad \Delta > 0,$$

where $d(i,j)$ is the distance between nodes $i$ and $j$. Time is slotted for transmission and the duration of the time slots do not scale with $n$. Each node has an average transmission power constraint $P$ when it transmits. We assume that the signal from a source attenuates with distance $r$ as $1/r^{\alpha/2}$, for some $\alpha \geq 2$ so that when a node transmits at power $P$ the received power at a distance $r$ is $Pr^{-\alpha}$. Further assuming that the channel between any transmitter-receiver pair is discrete-time AWGN with noise power $N$ and average signal power $P$, the transmission rate is given by

$$R(P,r) = \frac{1}{2}\log\left(1 + \frac{Pr^{-\alpha}}{N}\right).$$

**Definition of throughput:** A throughput $\lambda > 0$ is said to be feasible/achievable if every node can send at a rate of $\lambda$ bits per second to its chosen destination. We denote by $T(n)$, the maximum feasible throughput with high probability (*whp*). In this paper, $T(n)$ will be the maximum throughput with delay and/or energy-per-bit scaling constraints.

**Definition of delay:** The delay of a packet in a network is the time it takes the packet to reach the destination after it leaves the source. The average packet delay for a network with $n$ nodes, $D(n)$, is obtained by averaging over all packets, all source-destination pairs, and all random network configurations.

**Definition of energy-per-bit:** The energy-per-bit for a network with $n$ nodes, $\mathcal{E}(n)$, is the average energy-per-bit required to communicate between an S-D pair, averaged over all $n$ S-D pairs, and all random network configurations.

In this model, the throughput, delay and energy-per-bit for a communication scheme are related through the scheme's average transmission range, i.e., average hop distance.

**Lemma 1.** *In a fixed random network, for any communication scheme with average transmission range* $r(n)$*,*

$$\mathcal{E}(n) = \Omega\left(r(n)^{\alpha-1}\right).$$

The above lemma can be used to establish a minimum delay scaling for a given energy-per-bit scaling constraint. Further, using a trade-off scheme similar to Scheme 1 in [1], we obtain the following result.

**Theorem 1.** *The optimal trade-off between energy-per-bit and delay scaling is given by* $\mathcal{E}(n) = \Theta(D(n)^{1-\alpha})$*. Further, the optimal throughput-delay scaling trade-off at this minimum energy-per-bit scaling is*

$$T(n) = \Theta(D(n)/n) \quad \textit{for} \quad T(n) = O\left(1/\sqrt{n\log n}\right).$$

It turns out that if there is no constraint on energy the optimal throughput-delay scaling is $T(n) = \Theta(D(n)\log D(n)/n)$, which is only marginally better than that with the minimum energy-per-bit scaling constraint. Worse still, the energy-per-bit must scale up very fast as $\Theta\left(D(n)/\log D(n)\right)$ to achieve this marginally higher throughput. Moreover the throughput-delay trade-off with minimum energy-per-bit scaling is equivalent to the throughput-energy-per-bit trade-off with minimum delay scaling.

For mobile networks, we consider the same model as above with the additional feature that each node moves with velocity $v(n)$ according to an independent Brownian motion. For mobile networks the trade-off extends beyond that of fixed networks allowing higher throughputs with lower energy-per-bit by using the mobility of the nodes at the cost of higher delay.

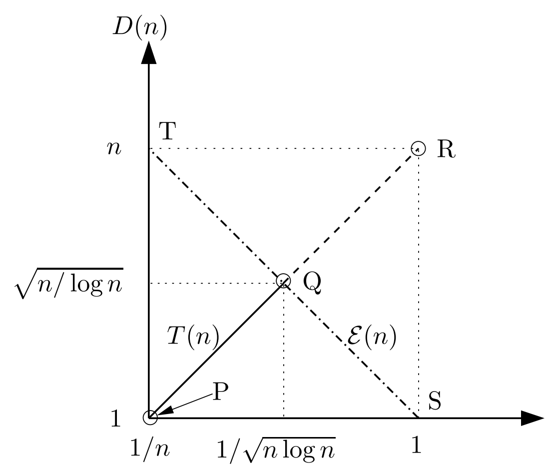


Figure 1: Optimal throughput-delay-energy trade-off in random wireless networks assuming $\alpha = 2$ and $v(n) = \Theta\left(1/\sqrt{n}\right)$. The scales of the axes are in terms of orders in $n$.

Figure 1 summarizes our results for the case of $\alpha = 2$ and $v(n) = \Theta\left(1/\sqrt{n}\right)$. For fixed networks, segment SQ gives the optimal energy-per-bit-delay tradeoff and segment PQ gives the optimal throughput-delay tradeoff at the minimum energy-per-bit scaling. Mobility provides additonal trade-off ranges represented by segments QT and QR.

## References

[1] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-Delay Trade-off in Wireless Networks", *IEEE INFOCOM*, 2004.